MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY,

CONTAINING

PAPERS,

ABSTRACTS OF PAPERS,

AND

REPORTS OF THE PROCEEDINGS

OF

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MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. XL. NOVEMBER 14, 1879. No. 1.

LORD LINDSAY, M.P., F.R.S., President, in the Chair.

William Charles Armstrong, Esq., Wanderers' Club, Pall Mall, S.W.;
Prof. J. J. Åstrand, Observatory, Bergen, Norway;
Thomas W. Bithrey, Esq., 45 Stepney Green, E.;
The Rev. James Law Challis, M.A., Vicarage, Stone, near Aylesbury;
George Howard Darwin, Esq., M.A., Trinity College, Cambridge;
Henry T. Vivian, Esq., 34 Wellington Road, Camberwell, S.E.; and
Robert Rumsey Webb, Esq., M.A., St. John's College, Cambridge;

were balloted for and duly elected Fellows of the Society.

On the Adjustment to Position of an Equatoreal Telescope. By the Rev. James Pearson, M.A., F.R.A.S.

Having recently had occasion to set up for the first time an Equatoreal Telescope, I adopted a method which I have not seen described elsewhere, and which has proved very successful. It is independent alike of meridian observations and spirit-levels, and for distinction's sake I may term it the "method of co-ordinates." An upright pyramidal stone of great height compared with the circumference of its base having been firmly embedded in the ground, with its principal faces north and south

approximately, the mounting of the Equatoreal was firmly attached to it by means of bolts, screws, and clamps.

The mounting, which, as well as the telescope, was made by Cooke & Sons, of York, consists of a polar axis carrying a single hour-circle, and a cradle-piece so constructed that the telescope may be separately affixed or withdrawn at pleasure. The cradle-piece forms an appendage to the declination-axis, at right angles to the former, which carries the declination-circle.

The telescope, then, being placed on its mounting, both were approximately set in position, *i.e.* the polar axis in the direction of the pole and the axis of the telescope with its plane of motion coincident with the plane of the meridian. This being effected, the telescope was pointed to a star, and then the mounting was moved about on its bearings by means of foot-screws, and round a vertical screw, forming an axis, until the declination-circle indicated the true North Polar Distance of the star when in the centre of the field of view. A marine chronometer, tested to Greenwich Mean Time by means of a Dipleidoscope, was employed to determine the Right Ascension of the meridian of the place at a given instant, and thence, by subtraction of the star's Right Ascension, its Hour-angle was found for the same instant. For convenience, the instant selected was the commencement of some hour on a common watch, its error being compared with the standard chronometer, and thus the Hour-angle of the star at any following instant was obtained at a glance by adding the minutes and seconds elapsed on the watch since the commencement of the hour nearest to it.

This done, two coordinates were obtainable for the same instant, viz. the true North Polar Distance of the star, and its true Hour-angle; and thus, by moving once more the mounting on its bearings, the hour-circle was made to indicate the true Hour-angle, and then the position was made permanent by the screws.

The adjustment was found so far satisfactory that planets and stars could be made visible in the day time by its means, which is a competent test of its accuracy.

Note on the Difference of Variation of Gravity at Revel and St. Petersburg; and on Grischow's Pendulum Observations at other Stations. By Major J. Herschel, R.E., F.R.S.

The difference to which I wish to draw attention is that exhibited on comparing the observations at the two stations named, by Grischow* in 1757, and by Sawitsch in 1865.

As such a comparison will seem at first sight derogatory to the modern observation, I will give my reasons for thinking that the more ancient are not devoid of reliance.

* *St. Petersb. Acad. Sci. Novi Comm.* vii., pp. 447–451.

Grischow employed two pendulums, one of which had seen service with La Condamine (who had had it made) in Peru, and the other with La Caille at the Cape of Good Hope. The former was swung in connection with machinery which counted its oscillations; the other was swung free, and its rate was determined by coincidences with the pendulum of an astronomical clock, every twelve or thirteen minutes.

Grischow observed these two at five stations, and were it not for the unaccountable difference between his results and those of Sawitsch, I venture to say that they should enlist immediate confidence on the sole ground of their concurring testimony.

I do not propose to go into any details of reduction here; but in order to show the grounds of this confidence, I give the vibration-numbers, reduced to 13°·3 R. or 62° F., which is about the average temperature of the whole series. The observations at Arensburg extended over nearly a twelvemonth, and supply good temperature factors, viz. 1·1 and 0·7 respectively for 1° R. For the purpose of comparison it is necessary to reduce the one to the unit of the other, or both to a common unit. I prefer the latter method, and adopt the Equatoreal Seconds Pendulum. By means which it is not necessary to describe, I estimate that the one (A) would make 98758·0, and the other (B) 86350·0 vibrations, in air, at 13°·3 R., at the Equator, or near it; and these are the numbers I use to reduce to the terms of an Equatoreal Seconds Pendulum vibrating 86400.

The following short table exhibits the data* and comparison. (I have added La Caille's observations at Paris as quoted by Grischow,* but reduced to 13°·3 R.):—

	A	B	A	B	A – B
St. Petersburg	98950·1	86519·9	86568·0	86570·0	–2·0
Revel	941·8	510·2	560·8	560·3	+0·5
Arensburg	937·4	504·9	556·9	555·0	+1·9
Pernau	940·6	509·4	559·7	559·5	+0·4
Dorpat	939·6	510·6	558·9	560·7	–1·8
Paris	900·5	453·1	524·7	503·2	...

There is no selection of observations here. The figures represent the whole of Grischow's published observations; and I think that on the face of it they are entitled to confidence. I exhibit them solely to justify the inquiry which I shall now proceed to make being listened to. I base nothing upon them as yet.

Returning now to the *un*reduced observations at St. Petersburg and Revel, these are:

At St. Pet.	By A 98947·1	At 16°·0 R.	By B 86516·9	At 17°·6 R.
„ Revel	„ 941·8	„ 13·3 „	„ 510·4	„ 13 „
Diff.	+5·3	+2·7	+6·5	+4·6

* See also *St. Petersb. Acad. Sci. Nova Acta*, vii., p. 215 *et seq.*

or with the temperature factors 1·1 and 0·7 respectively.

St. Pet. – Revel	+ 8·3	and	+ 9·7

which, when reduced to seconds, become

+ 7·3	and	+ 9·7

agreeing of course with the former reduction.

Now Sawitsch's observations, as last published (see *Monthly Notices* for March 1879)—observations which I am unwilling even to seem to question—give the following:

Length of seconds pendulum at St. Petersburg	441·0254
„ „ at Revel	441·0125

This difference corresponds to a change of Vibration-number of an Equatoreal Seconds Pendulum

$$= +\tfrac{1}{2}\frac{\cdot 0129}{441\cdot 0}.86570 = +1\cdot 3 \text{ sec.}$$

I am quite unable to discover any reasonable explanation of the discrepancy. Naturally Sawitsch's has much the higher probability; and I have therefore examined the older observations carefully. I argued thus: Grischow's results are with entirely different pendulums, it is true; but do they depend on the same clock rates? Yes. Then the clock rate must be in error. In effect, I at once found an error of 10 sec. (see *St. Petersb. Acad. Sci. Novi Comm.*, vii., p. 451, l. 5) in the mean daily rate of the clock. But this proved to be only a misprint: the more correct rate had been used. Nor am I able to believe readily in the clock rate being wrong. The swings of A extend continuously over many days at both stations, the clock is rated by transits daily of the Sun and stars, and an undetected error of 6 or 8 seconds a day is scarcely conceivable.

I submit the difficulty therefore for the scrutiny of others, with the hope that some solution will be found.

A like difficulty attends—but in a less degree—the results at Dorpat. To show this I will suppose that my table above is accepted, subject to a possible constant but unknown correction to each of the second pair of columns, to allow for the provisional character of the adopted equatoreal numbers. I now show the same figures at four stations, of which the Local-numbers—by which term (with capitals) I denote the Vibration-numbers which we otherwise know, with more or less certainty, would be shown by an Equatoreal Seconds Pendulum—are annexed for comparison.

		A	B	Mean.	Loc. No.
1757	St. Pet. (Grischow)	86568·0	86570·0	86569·0	86569·5
1757	Revel „	60·9	60·3	60·6	68·2
1758	Dorpat „	58·9	60·7	59·8	64·0
1756	Paris (La Caille)	24·7	...	24·7	24·2

To guard against any suspicion that these figures—the very remarkable agreement of which with each other in certain respects, and equally remarkable disagreement in others, is patent—are the result of discriminative selection (a kind of survival of the fittest, in fact), I beg to say that they are the direct consequence of adopting the Equatoreal-numbers 98758·0 and 86350·0. Each pendulum *had* an equatoreal vibration number which was not known. We have a right to assume any number for any pendulum, in any given condition, which will best satisfy the observations. I propose these, and show how well they fit the observations. That is evidence in their favour. If they are right they are a key to help to discriminate and to indicate facts. Thus it appears, beyond any doubt, that the condition of B, when in Grischow's hands, was not the same as when in La Caille's; and the Vibration-number is accordingly dropped, as not belonging to the differential series.

I have gone further in this examination than I intended, in the hope of finding other reasons, in the discordance *inter se* of the results, for their non-appearance in any collection of pendulum observations. That the Revel and Dorpat results are now apparently irreconcilable with the more recent results of 108 years later is true; but was this foreseen? And is there even now the *smallest* ground for doubting the Arensburg value?

Calcutta, May 24, 1879.

Note on the Semi-diameter of the Moon.

By E. Neison, Esq.

The results obtained by Professor Pritchard from the measurement of the photographs of the Moon taken at Oxford, and given in the *Monthly Notices* for June, p. 447, are of considerable interest. Unfortunately, as Professor Pritchard gives no details of the method he employed, beyond stating that it is very similar to that employed by Wichmann (*Ast. Nach.*, 1847), it is impossible to say how far he has succeeded in eliminating the systematic errors which are incidental to this work. Professor Pritchard makes the semi-diameter of the Moon equal to

$$15'\ 34'\!\!.175.$$

From a careful discussion of nearly eleven hundred observations made with apertures of different sizes, at the Observatories of Greenwich, Oxford (Radcliffe), and Washington, I deduced the value

$$15'\ 33''\!\!.37 + 4''\!\!.10\ (1 + 0''\!\!.70 \times \text{aperture in inches}).$$

For the eight-inch Transit-Circle of the Greenwich Observatory this corresponds to a semi-diameter of

$$15' \ 33''.99,$$

the result from 350 measures being

$$15' \ 33''.97.$$

For the five-inch Transit-Circle of the Radcliffe Observatory the formula gives a semi-diameter of

$$15' \ 34''.28,$$

whilst the mean of over 200 measures, between 1862 and 1877, gives

$$15' \ 34''.35.$$

It may be added that this empirical formula agrees closely with the theoretical formula, founded on the assumption that the differences in semi-diameter obtained from instruments of different aperture vary as the diffraction disks and the amount of light.

Applying this formula to the semi-diameter of the Moon in a 13-inch Reflector, the result is obtained of

$$15' \ 33''.78$$

for the value of the semi-diameter. Adding the photographic irradiation, which my experiments indicate to be about $+''0.30$, the resulting photographic semi-diameter should be

$$15' \ 34''.08.$$

This agrees very closely with the result obtained by Professor Pritchard.

How far this coincidence is fortuitous cannot be said; for until we know the method by which these measures have been reduced, it is impossible to say how far the above result indicates the true photographic semi-diameter, or how far it may be vitiated by the effect of the inequalities on the limb. In the method employed by Wichmann the effect of these inequalities is not eliminated; and if Professor Pritchard has employed the same method they will not have been eliminated from the above result.

Professor Pritchard compares the remarkably accordant results obtained by himself with the results obtained by Wichmann, apparently with the view of showing that the results obtained by measuring photographs will be far superior to the results of measurements with the Heliometer. The comparison, however, fails completely. Professor Pritchard's column III. gives the difference between his own separate and mean values, whereas his

column IV. gives the result of comparing the separate Heliometer determination with the very faulty semi-diameter (Burckhardt's) employed in the *Berliner Jahrbuch* for 1844–1845. To render the comparison fair, Professor Pritchard should have compared his separate results with the semi-diameter from Burckhardt's Tables. He would then have found his separate results as utterly discordant as those of the Heliometer appear to be.

As Professor Adams has pointed out, Burckhardt's tabular semi-diameter is vitiated by a large error, an error which may amount to some 2″, whilst his mean semi-diameter is more than 1″ too small. Correcting Burckhardt's semi-diameter for the effect of these errors, all the large discordances in column IV. disappear, leaving the separate results of the Heliometer sensibly as accordant as those from the photographs.

Professor Pritchard remarks that the mean semi-diameter (Hansen's) employed in the *Nautical Almanac* appears to be

$$15' \; 34''.10.$$

I do not know how Professor Pritchard has arrived at this conclusion, but it seems quite erroneous. In the *Nautical Almanac* Hansen's tabular semi-diameter is employed unaltered. Hansen clearly states, in the introduction to his Tables, that he adopts 57′ 0″ for the mean value of the Moon's horizontal equatoreal parallax, and therefore, as Table XXII. shows, the mean value of the tabular semi-diameter is

$$15' \; 33'.47,$$

no less than

$$0''.63$$

smaller than supposed by Professor Pritchard. Instead, therefore, of the photographic semi-diameter differing from the *Nautical Almanac* semi-diameter by the very small amount of

$$+0''.075,$$

it differs by the rather considerable amount of

$$+0''.705,$$

or nearly ten times as much.

As Hansen's Tables are not very intelligible on a mere cursory inspection, it is important that this correction should be made, to prevent erroneous ideas as to the real value of Hansen's semi-diameter.

Comparison of Hansen's Coefficients of the Moon's Latitude with those of Plana and others. By R. Wilding, Esq.

The values of the coefficients of the terms in the expressions for the coordinates of the Moon having now been found by a number of independent calculators, I venture to suggest that, for the practical purpose of forming lunar tables, perhaps the best values to be adopted would be those which are found to agree most nearly, *especially* if obtained by methods entirely different in principle. When the results, under such circumstances, *very* nearly agree, it seems highly probable that they are correct except as regards variations which might be accounted for by the fact that different elements of the orbit are assumed in their calculation. In cases where the coefficients are given *literally* the latter discrepancy may be made to disappear.

A comparison is made by Hansen, in his "*Darlegung &c.*," between his coefficients of the Moon's longitude and those found by Plana and Damoiseau; but I am not aware that any similar one is made of his latitude with that of other writers, and I have therefore endeavoured to do so in the present paper.

For this purpose, Hansen's expression for the latitude has to be reduced to a series of terms having for arguments angles increasing *as* the time. His mode of deducing the latitude is peculiar. First, a kind of *mean sine* of the latitude is taken equal to (sine *constant* inclination) × (sine of *true* distance of Moon from *true* ascending node), and this is afterwards corrected by the addition of a number of small equations, the result being the *true* sine of the latitude. From this the latitude itself is then obtained.

From the above explanation it will be seen that the reduction of the whole to terms having arguments varying *as* the time is a matter of some complexity, and one in which there is great liability to error; so I must request that *too much* reliance should *not* be placed upon my figures, although I cannot help saying that they *appear* very correct, and have been honestly worked out.

The very remarkable agreement with those of Delaunay which they present induces me to give his coefficients also for comparison. It is true that both Hansen and he used the same elements (Airy's); but the processes employed were so vastly different that the close accordance of the results seems to indicate that they are both extremely correct.

Both Clausen (*Astron. Nach.*, No. 406) and Pontécoulant (*Théorie Analytique*) have compared Burckhardt's coefficients with those of Damoiseau and others; but, as Burckhardt's coefficients of *precisely* the same arguments differ very considerably as given by these two authorities, it is clear that one or both reductions are erroneous. Pontécoulant's reduction of Burckhardt's latitude seems certainly wrong, and has the effect of making it appear that Burckhardt's tables of latitude are much worse than they really are.

The arguments are those of Delaunay.

$D = ☽ - ☉$
$F = ☽ - ☊$
$l = ☽$'s mean anomaly
$l' = ☉$'s „ „
} All mean angles.

Argument.	Delaunay.	Hansen.	
	″	″	
F	18461·3	18461·6	
F + l	+ 1010·2	+ 1010·3	
F − l	− 999·8	− 999·9	
2D − F	+ 623·6	+ 623·6	
2D + F − l	+ 199·3	+ 199·6	
2D − F − l	+ 166·4	+ 166·6	
2D + F	+ 117·2	+ 118·4	Dam. 117″·3; Plana 117″·6; Pont. 117″·5.
F + 2l	+ 62·0	+ 62·0	
2D − F + l	+ 33·4	+ 33·0	
F − 2l	− 32·3	− 31·8	
2D − F − l'	+ 29·7	+ 29·7	
2D − F − 2l	+ 15·3	+ 15·9	
2D + F + l	+ 15·1	+ 15·1	
2D − F + l'	− 12·2	− 12·2	
2D + F − l − l'	+ 9·0	+ 8·9	
2D − F − l − l'	+ 7·5	+ 9·3	Dam. 7″·4; Plana 8″·2; Pont. 8″·3.
2D + F − l'	+ 8·0	+ 8·0	
F + l − l'	+ 4·6	+ 6·8	Dam. 6″·8; Plana 6″·4; Pont. 6″·3. (I may have taken out Delaunay's coeff. wrongly.)
4D − F − l	+ 6·5	+ 7·7	Dam. 6″·3; Plana 6″·6; Pont. 6″·6.
F − l + l'	− 5·4	− 5·6	
F + l'	− 6·6	− 6·5	
F + D	− 5·3	− 5·5	
3F	− 6·3	− 6·3	
F − D	+ 4·8	+ 4·8	
F + l + l'	− 5·3	− 5·3	
F − l − l'	+ 5·0	+ 5·1	
F + 3l	+ 4·0	+ 4·0	
4D − F	+ 3·7	+ 3·6	
4D + F − l	+ 2·8	+ 3·0	
3F − l	− 2·8	− 2·8	
F − l'	+ 4·9	+ 4·9	
3F − 2D	− 2·2	− 3·4	Dam. −2″·3; Plana −2″·4; Pont. −2″·3.
4D + F − 2l	+ 2·3	+ 2·4	

Argument.	Delaunay.	Hansen.	
	$''$	$''$	
$2D-F-3l$	+ 1·5	+ 2·1	
$2D-F+2l$	+ 2·1	+ 2·2	
$2D+F-2l$	− 1·7	− 1·4	
$2D-F+l-l'$	+ 1·8	+ 0·3	Dam. 1″·6; Plana 1″·6; Pont. 1″·6.
$2D+F+2l$	+ 1·5	+ 1·5	
$2D+F+l'$	− 1·2	− 1·2	
$2D+F-l+l'$	− 1·8	− 1·6	
$4D+F$	+ 1·1	+ 1·2	
$2D+F+l-l'$	+ 1·1	+ 1·1	
$F-D+l$	+ 0·4	+ 0·5	
$2D-F-2l'$	+ 1·1	+ 1·1	
$3F+l$	− 1·0	− 1·0	
$2D-F+l+l'$	− 0·8	− 0·8	
$F+2l-l'$	+ 0·7	+ 0·8	
$F-3l$	− 1·6	− 1·6	
$2D-F-l+l'$	− 1·2	− 0·9	Dam. −0″·8; Plana −1″·9 Pont. −2″·0.
$D+F+l$	− 0·6	− 0·7	

A close agreement is found to exist between the remaining small coefficients which are omitted. The coefficient of the term having for argument, Moon's true longitude, is also omitted, as I do not find that Delaunay has given his value of it.

Note on the Ellipticity of Mars, and its effect on the Motion of the Satellites. By Professor J. C. Adams.

One of the results of Professor Asaph Hall's able discussion of his observations of the satellites of *Mars* is to show that the orbits of both the satellites are at present inclined at small angles to the plane of the planet's equator. It becomes an interesting question to inquire whether this state of things is a permanent one. The plane of *Mars'* orbit is inclined to its equator at an angle of 27° or 28°. If then the planes of the orbits of the satellites retain constant inclinations to the orbit of the planet, as they would do if the Sun's disturbing force were the only force tending to alter those planes, their inclinations to the plane of *Mars'* equator, and still more their inclinations to each other, would in time become considerable.

In No. 2280 of the *Astronomische Nachrichten*, Mr. Marth has found the motions of the nodes of the orbits of the satellites on the orbit of the planet due to the Sun's action, and he con-

cludes that, if there is no force depending on the internal structure of *Mars* which counteracts or greatly modifies the Sun's action, the nodes of the orbits will be in opposition to each other a thousand years hence, when the mutual inclination of the satellites' orbits will amount to about 49°.

In this case the near approach to coincidence between the planet's equator and the planes of the orbits of the satellites, which is observed to exist at the present time, would be merely fortuitous; but this appears *à priori* to be very improbable.

It is well known that, if there were no external disturbing force, the ellipticity of a planet would cause the nodes of a satellite's orbit to retrograde on the plane of the planet's equator, while the orbit would preserve a constant inclination to that plane. Laplace has shown that, when both the action of the Sun and the ellipticity of the planet are taken into account, the orbit of the satellite will move so as to preserve a nearly constant inclination to a fixed plane passing through the intersection of the planet's equator with the plane of the planet's orbit, and lying between those planes, and that the nodes of the satellite's orbit will have a nearly uniform retrograde motion on the fixed plane. The angles which this fixed plane makes with the planes of the planet's equator and its orbit respectively will depend on the ratio between the rates of the above-mentioned retrogradations of the nodes produced by the Sun's action and by the ellipticity of the planet. If the latter of these causes would produce a much slower motion of the nodes than the former, as in the case of our Moon, the fixed plane will nearly coincide with the planet's orbit; but if, as in the case of the inner satellites of *Jupiter*, the ellipticity of the planet would produce a much more rapid motion of the nodes than the Sun's action, then the fixed plane will nearly coincide with the planet's equator.

The ratio of the motion of a satellite's node to that of the satellite itself, when the Sun's action is the disturbing force, varies, *ceteris paribus*, as the square of the satellite's periodic time, that is as the cube of its mean distance from the planet. On the other hand, the ratio of the same two motions, when the ellipticity of the planet is the disturbing cause, varies inversely as the square of the mean distance. Hence, for different satellites of the same planet, the motion of the nodes caused by the ellipticity will bear to the motion caused by the Sun's action the ratio of the inverse fifth powers of the mean distances.

Now, the distance of the inner satellite of *Mars* from the planet's centre is only about $2\frac{3}{4}$ radii of the planet, a greater comparative proximity than is known to exist elsewhere in the Solar System, and the distance of the outer satellite from the same centre is only about 7 radii of the planet, while the periodic times of both are very small compared with the periodic time of *Mars*. Hence the effect of a given small ellipticity of *Mars* on the motion of the nodes of the satellites will be greatly magnified.

It is true that the ellipticity of *Mars* is still unknown, and is probably too small to be ever directly measurable; but we are not without means of determining, within not very wide limits, its probable amount, and we shall presently see that, in all probability, in the case of both the satellites the motion of the nodes produced by the ellipticity greatly exceeds the motion caused by the Sun's action, so that the fixed planes for both satellites are only slightly inclined to the planet's equator.

From measures of the planet's diameter and of the greatest elongations of the satellites, combined with the known time of rotation of *Mars* and the periodic times of the satellites, it is found that the ratio of the centrifugal force to gravity at *Mars'* equator is about $\frac{1}{220}$. Hence it follows that if the planet were homogeneous its ellipticity would be about $\frac{1}{176}$. If, instead of the planet being homogeneous, its internal density varied according to the same law as that of the Earth, so that the ellipticity would bear the same ratio to the above-mentioned ratio of centrifugal force to gravity at the equator as in the case of the Earth, then the ellipticity would be about $\frac{1}{228}$. In all probability the actual ellipticity of *Mars* lies between these limits.

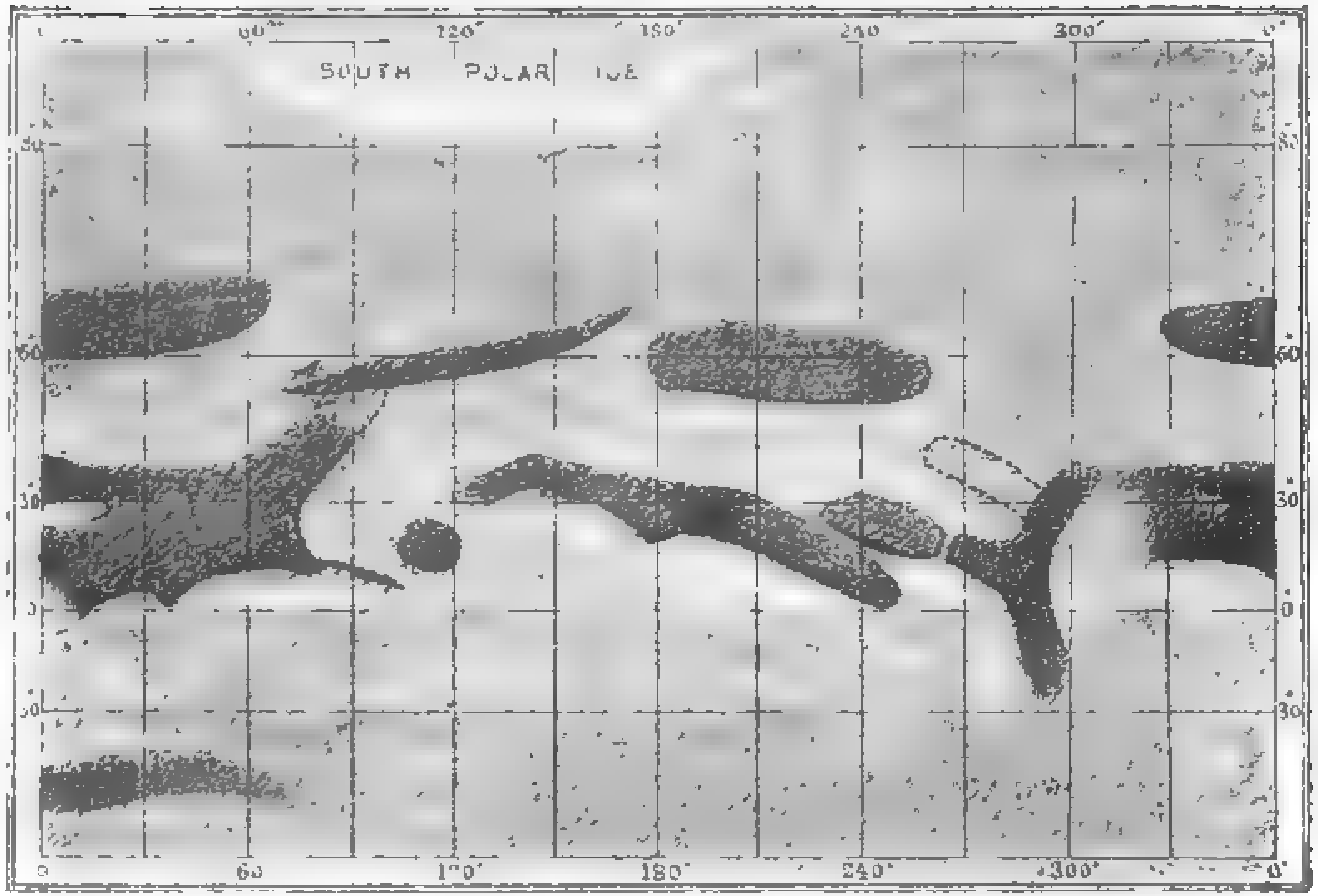
The following Table shows the annual motions of the nodes of the two satellites, caused by the Sun's action and by the planet's ellipticity respectively, for the above values of that ellipticity, and also for the ellipticity $\frac{1}{118}$, which has been deduced from Professor Kaiser's observations, although I have no doubt that this value is too great. The Table likewise contains the corresponding inclinations of the fixed planes, so often mentioned above, to the planet's equator.

Satellite I.			*Satellite II.*		
Annual motion of the node due to the Sun's action, $0^{\circ}\cdot06$.			Annual motion of the node due to the Sun's action, $0^{\circ}\cdot24$.		
Supposing ellipticity =			Supposing ellipticity =		
$\frac{1}{118}$	$\frac{1}{176}$	$\frac{1}{228}$	$\frac{1}{118}$	$\frac{1}{176}$	$\frac{1}{228}$
the annual motion of the node due to that ellipticity will be			the annual motion of the node due to that ellipticity will be		
$33^{\circ}\cdot3$	$18^{\circ}\cdot2$	$11^{\circ}\cdot3$	$13^{\circ}\cdot4$	$7^{\circ}\cdot3$	$4^{\circ}\cdot5$
Corresponding inclinations of fixed plane to planet's equator:			Corresponding inclinations of fixed plane to planet's equator:		
$17''$	$31''$	$50''$	$27'$	$50'$	$1^{\circ}\ 19'$

From this it may be inferred that the orbit of the 1st satellite preserves a constant inclination to a plane which is inclined less than 1′ to the plane of *Mars'* equator, and that the orbit of the 2nd satellite preserves a constant inclination to a plane which is inclined about 1° to the plane of the same equator.

The ellipticity will also cause rapid motions in the apses of

CHART OF MARS ON MERCATOR'S PROJECTION.



the orbits of the satellites, particularly in that of the first; and as this orbit appears from Professor Hall's determination to have a sensible eccentricity, it will be possible, by future observations, to determine the motion of the apse, and therefore the ellipticity of the planet. If further observations show that the orbits of the satellites are sensibly inclined to their fixed planes, the motion of their nodes will supply another means of determining the ellipticity of the planet.

On the Physical Configuration af Mars.

By Professor Wm. Harkness, U.S. Navy.

(*Communicated by authority of Rear-Admiral John Rodgers, U.S. Navy; Superintendent U.S. Naval Observatory.*)

I have constructed the accompanying map of *Mars* from eight sketches taken by myself between August 18 and October 18, 1877. More sketches would have been exceedingly desirable, but the unsteady state of the atmosphere prevailing during the opposition rendered it impossible to obtain them. The telescope employed was the 26-inch Equatoreal of this Observatory. Eye-pieces magnifying up to 400 diameters were tried each night, but it was usually found that a power of 175 gave the best result. This was due partly to the southern declination of the planet, and partly to the bad seeing before mentioned. After each sketch was finished, Professor Hall very kindly compared it with the planet, and in every case he agreed that the sketch showed all that was certainly visible upon the planet, and nothing more. As seen through the telescope, the colour of *Mars* was a golden yellow; except the polar spot, which was pure white, and the markings, which were a light indigo blue.

When laid down upon Mercator's projection the several sketches agreed well with each other, and resulted in the accompanying map; the latitudes and longitudes of which depend upon the Ephemeris given by Mr. Marth in the Royal Astronomical Society's *Monthly Notices* for April 1877, vol. xxxvii., pp. 301–304. The features indicated upon the map by dotted lines were seen only once, and appear inconsistent with other features which were seen oftener. The south polar spot was sensibly circular in form, and, according to my sketches, had a diameter of about $14\frac{1}{2}°$. Professor Hall determined the position of its centre to be latitude, $84°\cdot8$ south; longitude, $118°$ west.

Owing to the meagreness of the data upon which it is based, this map cannot be regarded as complete; but it is hoped that it may be greatly improved during the Opposition of next autumn. In its present state it somewhat resembles the map given by Dr. Kaiser in the third volume of the *Leiden Observations*.

U.S. Naval Observatory, Washington,
1879, June 17.

On a Photograph of the Solar Spectrum, showing Dark Lines of Oxygen. By Professor J. C. Draper.

(*Communicated by W. H. M. Christie, Esq.*)

I send by mail, with this letter, a photograph of a solar spectrum of the third order, in the vicinity of G. It is an original photograph of the diffraction grating spectrum itself. It has not been enlarged nor manipulated in any way whatever. It was taken on October 1, at the City College, and though the sky was not perfect, it is a good picture. If you think it good enough, I should be very much obliged if you would present it at a Meeting of the Royal Astronomical Society, for the inspection of members who take an interest in these matters.

The apparatus was essentially that described in the *American Journal of Science* for October 1868, page 256. It consisted of Heliostat with silvered glass mirrors, slit, silvered glass concave mirror, silvered glass Rutherfurd grating of over 17,000 lines to the inch, and a camera box. The light passed from the grating to the collodion plate, with nothing intervening except the glass bearing the scale, as described below. In the case of that part of the spectrum which does not bear the scale, nothing whatever except air intervened between the grating and the collodion on the plate.

The slit aperture with which the picture was taken was exceedingly fine, probably ($\frac{1}{3000}$) one three thousandth of an inch in width, and possibly less. Even with exposures of six and seven minutes, and a very sensitive collodion, the fineness of the slit has rendered it necessary to force the development of the image, hence come the minute spots on the film.

As the red of the second order overlaps this portion of the third order, one of your half-prisms, made for me in New York, was used for the separation of these orders in focussing. It was placed in the aperture of the focussing glass, and almost touched the clear glass plate of the camera, which occupied the position ordinarily held by ground glass.

The picture may be studied either as a positive or as a negative. In the first case, it is to be laid on a black background of velvet or cloth, and viewed by reflected light, when the dark lines represent the absorption lines and the white lines the bright or coloured spaces in the solar spectrum. In the second case it is to be held between the eye and the light of a white cloud (at night a light, with a ground-glass shade), and viewed by transmitted light. Thus examined with a suitable magnifying power, the clear spaces represent the dark lines of the spectrum, and the dark lines of photographic action, the bright parts of the spectrum. This must be borne in mind when making examinations in connection with the descriptions given hereafter.

The cross lines near the centre of the plate indicate the position of the line of no deviation for the apparatus described. The

capital letter G is written on the back of the plate bearing the collodion film. It is over the line G of the solar spectrum, and a little to the left of the line of no deviation. On the right of the line of no deviation, and beginning at the cross lines, five bands appear, the third and fourth containing what I believe to be dark lines of oxygen. The collodion film is protected by a second plate of glass, as the picture is not varnished. The two plates of glass are separated by a mat, made of blotting paper. The glass on which the collodion film rests, being ordinary photographic glass, is not suited for measurement of the spectrum lines by projection.

To examine the picture, hold it in such a position that the letter G reads properly. Then, using a moderate magnifying power ($2\frac{1}{2}$ to 3 inches focus), as before described, and viewing by transmitted light, a scale bearing divisions and figures will be seen above the line which runs the whole length of the photograph, and which divides it into two portions.

The manner in which this scale was introduced was as follows: On a slip of the flattest glass I could find, a scale of 200 half-millimetres was ruled. This was fitted in the plate-holder of the camera, in front of and close to the collodion film, with the ruled side of the plate looking toward the collodion. It was carefully adjusted to intercept one-half of the spectrum. Its shadow was thus cast with this half of the spectrum upon the collodion film, and so the scale was printed by the spectrum itself. The line which traverses the whole length of the spectrum is the edge of the glass upon which the scale is ruled. The scale I have introduced here merely as a means of indicating positions. Its real purpose is to verify readings of wave-lengths made by the projection process I have heretofore described.

The necessity of avoiding the intervention of anything between the grating and the collodion film is shown by the injury to definition and obscurity produced in the upper half of the spectrum by the glass on which the scale is ruled. In the case in question the obscurity is increased by vapours which arise from the collodion and condense on the glass scale.

To study the picture intelligently, take one of Dr. Rutherfurd's admirable photographs of this part of the spectrum; a series of five bands will be found on the less refrangible side of G. In the photograph I have used, their positions, as read by the scale attached, are as follows:—

1st	4312·60	to	4314·15	These readings may vary slightly in different photographs, owing to difference in stretching of the paper used for the photographs and the scale; but they serve to indicate the bands in question.
2nd	4314·60	„	4316·10	
3rd	4316·20	„	4317·95	
4th	4318·15	„	4320·00	
5th	4320·25	„	4322·15	

Take H. Draper's photograph, published in *Nature* for August 30, 1877, the five bands are easily identified, and the oxygen

lines in the spectrum below correspond to the third and fourth of these bands. In his article discussing the oxygen lines, he gives their wave-lengths as 4317 and 4319. In support of the opinion, that it is in this part of the spectrum that oxygen lines should be found if any exist, I refer to the diagram given in the *Observatory* for June 1879, page 47. The wave-length values of these oxygen lines, as given by the authorities there presented, are:—

Ångstrom.	J. C. Draper.	Plucker.	H. Draper.	Huggins.
4316·20 4316·95	4316·50	4317·	4317·	4318·
4318·85 4319·45	4319·75	4320·	4319·	

All agree in placing them within the limits of the third and fourth bands of Rutherfurd's photograph.

Take my photograph of the 3rd order, October 1, 1879. The five bands are easily identified, and present the following scale readings:—

1st	97·95	to	100·10
2nd	100·45	„	102·35
3rd	102·75	„	104·70
4th	105·40	„	107·25
5th	107·70	„	110·10

Remembering that the picture is now studied as a negative, and that the clear spaces are the dark lines of the solar spectrum, it will be seen that each of these five bands contains dark lines. In some instances also, and notably in the case of the line separating the 3rd and 4th bands, the separating line is double. An experience gained by the study of more than 200 photographs of this region leads me to think that many of the lines given in the following Table of the lines contained in these bands are really double:—

	Scale Number.		Scale Number.
Lines in 1st band	98·65 99·25 100·00	Separating lines	104·85 105·30
Separating line	100·30	Lines in 4th band	105·80 106·20 106·60 107·00
Lines in 2nd band	100·85 101·65	Separating line	107·50
Separating line	102·60	Lines in 5th band	108·10 108·60 109·70
Lines in 3rd band	103·20 103·55 104·20	Separating line	110·20

As regards the photographic intensity of these bands when compared with that of the lines between 70 and 80 of the scale, no one can have any doubt of its superiority in the latter case. To a careless examiner it may appear stronger in the first, but this is a deception caused by the great width of the bands.

In answer to objections that may be advanced against these lines in the 3rd and 4th bands as oxygen lines, on account of their lack of intensity, I submit the appearances offered by the picture. I would also suggest that, since both oxygen and nitrogen are really the only non-metallic gaseous elements that approach a permanently gaseous state, we should be prepared from this fact alone to expect them to produce only faint absorption lines in the solar spectrum.

It also seems to me that the picture really gives us the true spectrum of oxygen, for this region, under the conditions existing in the solar envelopes, viz. that it is a spectrum of six faint lines in two bundles, corresponding to those in the 3rd and 4th bands of the picture.

If we admit that the electric oxygen bands corresponding to this position are really composed of lines, and that the attenuated condition of gaseous oxygen in the Sun explains the faintness of oxygen absorption lines, we have no difficulty in accepting the opinion that oxygen is present in the solar envelopes, and that it acts there in the same manner as any other elementary body, and produces its proper dark absorption lines in the solar spectrum.

As regards the appearance of the picture from a photographic point of view, I would say that, if the slit is opened sufficiently, there is no difficulty in obtaining a more pleasing picture with a brief exposure. But, on opening the slit, purity of spectrum disappears, and as the slit is widened the lines fade away, until at last nothing remains but the five unlined bands, as is shown in H. Draper's photograph in *Nature* of August 30, 1877.

That there is nothing peculiar about this photograph of the 3rd order is shown by the fact, that in photographs of the 1st order the resolution of these five bands is foreshadowed, in those of the 2nd order it is partly accomplished, and in the 3rd order the dissection is completed, as you see.

New York, 1879, *Oct.* 9.

On the Working of the Speculum for Mr. Common's 37-inch Silver-on-glass Reflector. By Mr. G. Calver.

The Speculum recently finished for the 37-inch Reflector gave me an opportunity of coping with the difficulties to be encountered in making a large speculum of this kind. I have found some of these difficulties to be less than I anticipated; and I believe considerably larger instruments might be undertaken with a reasonable prospect of success.

There never was any doubt whether large glass specula could be as easily mounted as metal specula, for they have the great advantage of being lighter; but the chief question was whether they could be annealed so as to stand the usual treatment in working. If they would stand this, there is no fear of their durability when mounted.

To decide whether the speculum should be metal or glass was, to a certain extent, to venture a risky experiment. For my own part, I had reason to believe that large disks could be obtained properly annealed. Two firms had guaranteed me disks of three or four feet, and since that time they have offered to undertake a five-foot disk.

One obstinate fact, and one I could get no solution of, was the failure of the four-foot French speculum. But in spite of this failure, a 37-inch glass disk was decided on, to be of about $4\frac{1}{2}$ inches thickness.

There was the question, too, of silvering. Mirrors are usually silvered by being suspended face downwards; but I was quite sure that it could be done face upwards and without even taking it from its cell, and that I could thus avoid a great difficulty with respect to silvering.

I had long been in the habit of silvering mirrors face upwards when testing them during the figuring, as it saved both time and trouble, and I consequently devised a plan for silvering the large mirror which unfortunately was the cause of the blowing up of the first disk.

The cell was perforated as if for a Gregorian. I was going to cut a $2\frac{1}{2}$-inch hole through the glass, and make a gutta-percha plug to fit. This hole was to serve to let off the water and solutions when silvering. A band of stiff paper, coated with something not acted on by the chemicals, was to form a band to hold the solutions. I had tried many things; beeswax is good when pure, but is mostly sold adulterated with oil and rosin; solid paraffin was found the best of all.

I made a strong machine suited to my method of working. I had flatted the back of the disk and got the concave ready for fining, and was, at the same time, edging it and cutting the hole through the centre; but when less than $\frac{1}{4}$ inch in depth had been cut, it burst into hundreds of fragments. The breaking, doubtless, was owing to the disturbance of internal tension.

I was not sure whether removing the skin from the edge or disturbing the centre was the cause of the blowing up, and I therefore tested this question on an $18\frac{1}{2}$-inch disk, which, on attempting to cut a hole through it, blew up also. I have since had a large disk *cast* with a centre in it, and it answers perfectly; and had the 37-inch been cast in the same manner, it would have answered the proposed plan for silvering.

The 37-inch was ground and polished with a tool 36 inches in diameter and weighing nearly 3 cwt., and the focus—17 feet $7\frac{1}{2}$ inches—out within an inch or two of what was intended.

Before polishing was commenced, a portion of the workshop was covered with calico to exclude dust, room only being left for working. A wooden tunnel, commencing at the door and extending 40 feet on the ground level, was covered with sailcloth to keep out light and air-currents.

The machine was made so that its revolving table could be turned from the horizontal to the vertical position; the speculum was worked in its cell; the whole weighing nearly 11 cwt.

When testing commenced I sat in this dark tunnel to test at the centre of curvature, and the speculum was focussed by an assistant with screws, so that I received in my eye the image of a very minute pinhole in a lamp screen close by my head.

During the polishing and figuring I carefully studied the behaviour of the disk—for flexure—for distortion of figure by contraction and expansion during changes of temperature—and I found the disk as perfect as a 6-inch one, and without a single infirmity. I soon found it would admit of a perfect and permanent figure, and it gave in the early stage a perfect and symetrical image of an artificial star, which was magnified many hundred times by an eye-piece. These results were gratifying and satisfactory; they at once removed all doubts of ultimate success.

The work of correcting was tedious and trying, especially in the latter stages, when for every few minutes' polishing the whole preparations for testing had to be repeated, and the settling of the mass into its normal state had to be patiently waited for, and often days passed before further advance could be made.

When the figure had advanced as far as necessary for testing at the centre of curvature, the wooden structure was thrown open at the outer end for the purpose of testing on a distant terrestrial object.

The test objects for daylight purposes consisted of a hole punched through a sheet of metal, with a reflector so placed at the back as to reflect the light of the clouds through, so that it appeared as a bright white spot: a spot of whiting and printed paper was used for definition. When the Sun shone I obtained its image by means of a prism with one of its surfaces polished to a small spherical curve. This was made use of because I could not use a bulb, having to look in the direction of the Sun, and not with the Sun behind me.

For testing at night a lamp covered with a metal screen with a hole through it was used. For every advance towards the correct figure it was first tested at the centre of curvature, and then on the distant object. In the latter case it was, of course, used as a telescope proper, by placing a plane at its working focus for parallel rays.

The plan of figuring was that of local figuring and correcting. The polish and surface was obtained with the large and heavy polisher, and corrected with a number of polishers of various sizes and forms to suit every stage of the progress and the tem-

perature of the air &c. If an error of irregularity of figure set in, it was polished out with the large polisher.

The machine I employed was on a principle which was a modification of that used by the Earl of Rosse. I have made five different machines, one of which was on the principle used by Mr. Lassell—an excellent principle; but I have long since come to the conclusion, that no machine can do the final work like the trained hand, and I was gratified to learn, when in conversation with Professor Draper, that his experience agreed precisely with my own on this point.

The fact that, when polishing specula, we use a tool that is somewhat elastic (for there is nothing equal to pitch as a material for the polisher) shows that it is the *form of the polisher* we want to aim at, and the curve of the glass will follow. Therefore the machine, its rates of revolution and strokes, the size and weight of the polishers, their consistency (which depends on the temperature in which they are worked, and the friction) should all be arranged so as to give to the polisher the figure or curve we desire to give the glass surface.

I may state that the 37-inch disk, as an experimental one, answered all expectations and fulfilled all necessary conditions; and I may also state, as the result of my experience, that I see no obstacle to the construction of glass mirrors of very large sizes.

The 37-inch was silvered by Martin's process, and in this way:—

The surface was washed with some of the potash solution, it was then rinsed and sponged with a handful of pure cotton wool, and finally rinsed.

A band of stout brown paper was ready; that part of it to go next the edge for about two turns was painted with hot melted paraffin. This paper band was wound round, and tightly bound round with strong cord, leaving a rim standing up about two inches above the surface to hold the solutions. Water was ther poured on to cover and keep the surface till the bath was ready

The solutions were filtered through cotton wool. I make th solutions very strong; that is, I dissolve the chemicals in $\frac{1}{4}$ or $\frac{1}{8}$ the quantity of water usually given. Thus the filtering is quickly done and the bulk is small. The quantity of water required t make the bath of sufficient depth can easily be added. The bat was about one inch deep at the edge.

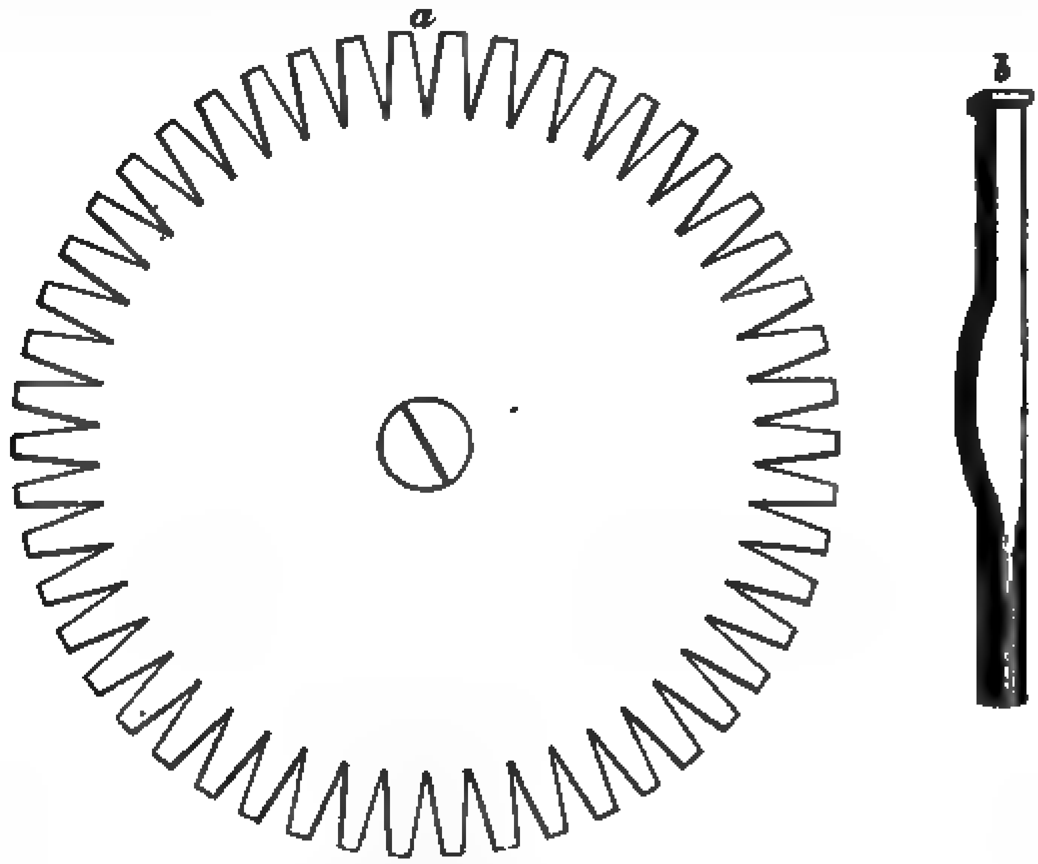
When all was ready, the speculum was tilted and the wa shot off; quickly brought to the level again, and the solutio poured on. The sinking particles are thus subtracted by filt tion, and the light particles will float harmlessly, and nearly the silver goes to the glass surface. It was well silvered twenty minutes, then well washed and sponged over wit handful of cotton wool, finally rinsed, left to dry and polish. was conveniently silvered with the help of one assistant.

Description of a Self-Registering Micrometer. By Mr. A. Bowden.

The general shape of this instrument is that of an ordinary micrometer, with the following modifications.

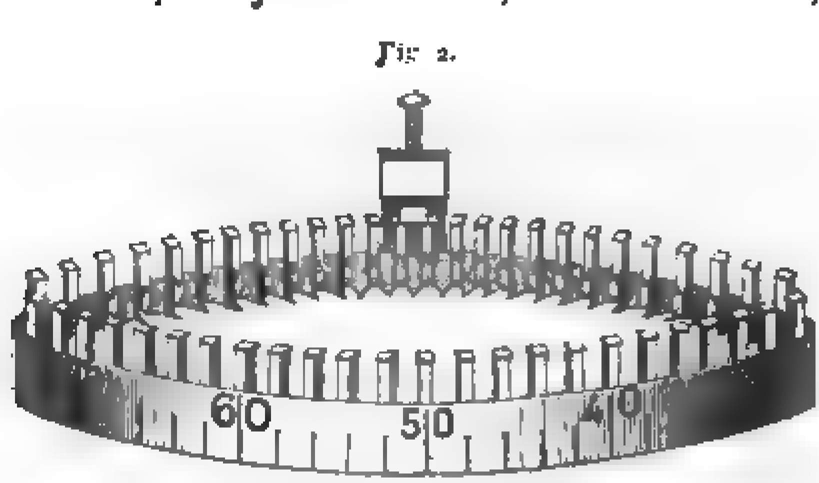
The graduated band has a plate within its circumference at both upper and lower edges; through which are drilled holes immediately within the band and behind each alternate division, so that in a centesimally divided micrometer there are fifty pairs of holes. Through each of these pairs runs a pin (called an *indicator*) shaped as in Fig. 3, *b*, with the points towards the milled

Fig. 3.



head of the micrometer, and the heads standing up on the same side as, and just behind, the index. Through the arm which carries the index, and just behind it, is drilled a hole, through

Fig. 2.



which passes a pin (called the *motor*), having a flat round head

with a square base, and which is kept up, so that the base shı just clear the indicator heads, by a small spiral spring.

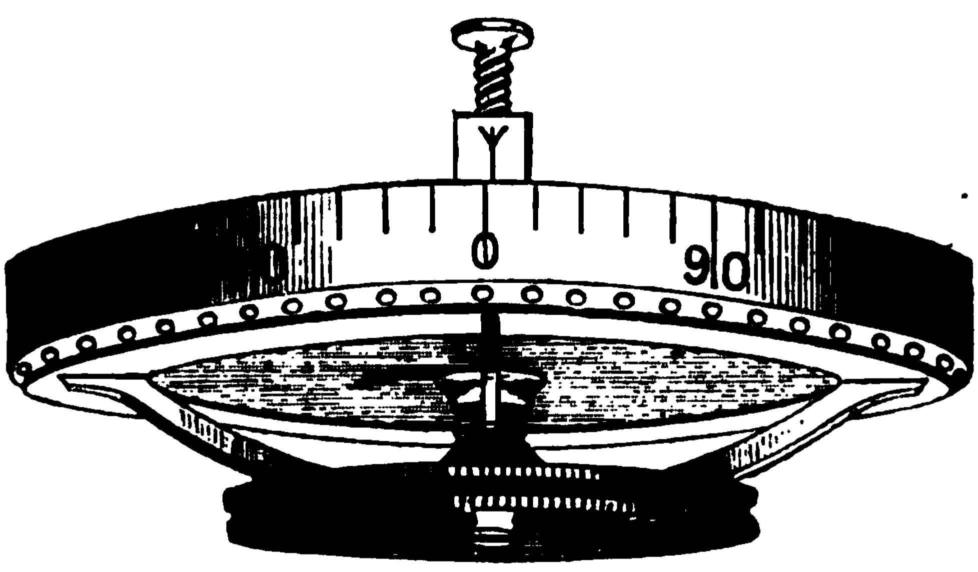
Fig. 2 represents the micrometer head (with the top pla off) ready for use.

When a bisection has been made, the observer presses gent on the head of the motor, thereby driving down the indicat below it, and making its point, which was previously flush wi the outer surface of the lower plate, protrude below the ban where it remains. On releasing the motor it springs up, and fresh bisection may be made. When a sufficient number bisections have been made, the observer notes the integı number of revolutions in the usual way, and the fractional parl to hundredths, are taken down from the divisions behind whi the indicators protrude.

The relative sizes of the base of the motor and the heads the indicators are such that, if the micrometer is set so that tl index points to any reading not more than half a division either side of the even division behind which the indicator stand one indicator only will be driven down; but if the reading beyond this on either side, two will be projected, and the divisi between these two will be recorded.

To restore the indicators to their normal positions, a flat riı is carried on the shaft of the micrometer, between the band aı the milled head (Fig. 1). This is held down towards the mill

Fig. 1.



head by a spiral spring, and, when pressed up, pushes the inc cators back into position. To keep them in position, whether ı or down, a thin steel plate, shaped as in Fig. 3, *a*, is fixed betwe the two plates of the micrometer head, so that an edge of a too comes behind each indicator. When the indicator is press down, the small protuberance behind it bears on the tooth, whi yields and allows it to pass, but immediately straightens aı prevents its return till the pressure is given at the other end l the ring described above, when the same action occurs in tl opposite direction.

In a micrometer head having a diameter of two inches, tl distance between the centres of the indicators would be near

$0^{in}\cdot1$. If it is possible to make their distances as little as $0^{in}\cdot05$, so as to have an indicator for each division, readings would be recorded to within $0^{r}\cdot0025$; but even supposing it can only be read to $0^{r}\cdot005$, which, if $1^{r} =$ say $20''$, would give $0''\cdot1$, this, though not quite so exact as reading from the vernier, practically as accurately records the position of the star in the field, and possesses the advantage of allowing the bisection to be repeated 3, 5, or 7 times.

Radcliffe Observatory, Oxford.

Observations of the Spectrum of Comet 1879 *d* (*Palisa*). By Lord Lindsay.

The spectrum of this comet was observed at Dun Echt on two nights by Ralph Copeland and J. G. Lohse. It consisted of the usual three bright bands, the middle one being the brightest, while the one of shortest wave-length was by much the faintest. The resulting wave-lengths are:

	Sept. 23. mmm.	No. of Obs.	Oct. 10. mmm.	No. of Obs.
Band 1, centre	552·7	(3)	549·8	(1)
„ 2, towards red edge	513·1	(1)	...	...
„ „ centre	511·0	(1)	512·4	4)
„ 3 „	468·9	(3)	462·1	(1)

Dun Echt, 1879, *Nov.* 11.

Observations of Comet 1879 *d* (*Palisa*), *made at Dun Echt Observatory with the Filar Micrometer of the* 15-*inch Refractor.*

By Lord Lindsay.

Date.		Dun Echt Mean Time.			☄ − ✱ Δα		☄ − ✱ Δδ		α☄			δ☄			Observer.
1879.		h	m	s	m	s	′	″	h	m	s	°	′	″	
Aug.	26	11	2	47·1	+3	47·98	−1	54·5	10	35	22·22	+48	19	31·8	J. G. L.
„	27	12	14	46·3	−5	3·25	−2	55·7	10	42	24·31	48	5	2·5	R. C.
„	28	15	15	35·4	−	1·84	−2	48·8	10	50	0·55	47	47	46·4	J. G. L.
Sept.	3	14	40	58·2	−1	39·18	−6	45·8	11	30	46·95	45	39	40·6	R. C.
„	4	13	48	14·8	−	35·68	+3	42·8	11	37	20·26	45	13	12·1	J. G. L.
„	13	10	59	11·7	−3	10·02	−4	50·9	12	36	7·39	39	51	10·3	R. C.
„	23	7	53	47·3	−	34·01	−7	25·3	13	34	13·24	31	29	40·7	„
„	25	8	27	41·5	+	28·53	−7	19·0	13	44	55·30	29	33	30·6	J. G. L.
Oct.	2	8	45	24·0	−	59·71	+1	32·0	14	18	52·07	22	32	15·1	R. C.
„	3	7	41	3·8	+	34·91	+5	51·9	14	23	7·22	21	33	46·9	J. G. L.
„	6	7	38	35·5	−	24·25	+5	42·1	14	35	57·24	18	30	6·9	„
„	19	6	54	3·3	...		+	52·1	...			5	48	11·7	R. C.
„	„	7	5	40·0	+2	1·60	...		15	23	42·26	...			„
„	20	7	2	8·5	+	8·69	−3	28·6	15	26	56·66	+4	52	59·1	J. G. L.

Adopted Mean Places of Comparison Stars, 1879·0.

	α h	α m	α s	Reduction. s	δ °	δ ′	δ ″	Reduction. ″	
Aug. 26	10	31	32·28	+1·96	+48	21	36·6	−10·3	Arg.-Oeltz. 11006.
„ 27	10	47	25·66	1·90	48	8	8·6	10·4	„ 11221.
„ 28	10	50	0·50	1·89	47	50	45·7	10·5	$\frac{\text{Arg.-Oeltz. 11258} + ✱ \text{ No. 99 } \textit{Ast. Nach.} \text{ No. 538}}{2}$.
Sept. 3	11	32	24·37	1·76	45	46	37·4	10·9	$\frac{\text{Arg.-Oeltz. 11940} + ✱ \text{ No. 128 } \textit{Ast. Nach.} \text{ No. 538}}{2}$.
„ 4	11	37	54·18	1·75	45	9	40·3	11·0	$\frac{\text{Arg.-Oeltz. 12025} + ✱ \textit{ Ast. Nach.} \text{ No. 995}}{2}$.
„ 13	12	39	15·80	1·61	39	56	11·8	10·6	Greenwich Nine-Year 1175.
„ 23	13	34	45·58	1·66	31	37	16·0	10·0	$\frac{\text{Lalande 25259 + B. W. xiii. 685 + B. W. xiii. 686}}{3}$.
„ 25	13	44	25·08	1·68	29	40	59·5	9·9	$\frac{\text{B. W. xiii. 921 + B. W. xiii. 922}}{2}$.
Oct. 2	14	19	50·00	1·78	22	30	52·3	9·2	B. W. xiv. 405.
„ 3	14	22	30·51	1·80	21	28	4·4	9·3	B. W. xiv. 457.
„ 6	14	36	19·64	1·85	18	24	33·9	9·1	B. W. xiv. 746.
„ 19	15	21	38·57	2·09	5	47	28·4	8·8	Lamont $\frac{1730 + 1731}{2}$.
„ 20	15	26	45·86	2·12	4	56	36·3	8·6	Micrometrically measured from B. W. xv. 501.
(„ 20)	15	28	31·62	+2·12	+5	8	15·6	−8·5	B. W. xv. 501; not used as comparison star.

Note.—Rounding off the above quantities has in some cases given rise to discordances of one unit in the last figure.

Remarks
- Sept. 23. Comet about 3′ diameter, *vgmb*M; but no nucleus with power 307.
- Oct. 19. Comet bright, round, gradually much brighter in the middle; no nucleus.
- Oct. 20. Comet very low; images woolly and unsteady.

On the Evidence of a past Connection between four widely separated Southern Stars. By E. J. Stone, M.A., F.R.S.

I have found it desirable, in the formation of my Cape Catalogue, to examine the cases in which large proper motions have been assigned to Southern Stars in the British Association Catalogue. In the greater number of cases these large proper motions have not been confirmed; but in the following group of stars the proper motions are large and fairly well determined. These proper motions appear to me otherwise interesting, and to indicate the existence, in the past, of some close connection between stars which are now widely separated.

The following data are extracted from the Cape Catalogue now passing through the press:—

1880.

Name.	Mag.	R.A. h m s	Precess. s	P.M. s	N.P.D. ° ′ ″	Precess. ″	P.M. ″
ζ Toucani	4·	0 13 48·60	+2·8953	+0·280	155 34 49·03	−20·018	−1·
e Eridani	4·5	3 15 8·16	+2·1169	+0·266	133 31 46·81	−13·214	−0·
ζ^1 Reticuli	5·6	3 15 9·95	+1·0954	+0·194	153 2 5·58	−13·212	−0·
ζ^2 Reticuli	5·6	3 15 36·39	+1·0980	+0·190	152 57 53·21	−13·184	−0·

The angular proper motions of these stars are 2″·13, 2″·99, 1″·47, and 1″·45 respectively. It will be remarked that the proper motions are exceptionally large in both elements, and that they have the same signs for all the stars. In my opinion these facts are not sufficient to prove that the stars have belonged to a stellar system, but they undoubtedly raise a strong presumption that such may have been the case.

We have observations of these stars extending over a period of nearly 130 years. To avoid the complications introduced by changes in the axes of reference, I shall refer the motion of each star to the other stars, and discuss the changes of angular distance.

1st.—For the distances of the close pair of stars ζ^1 and ζ^2 *Reticuli* we have

Epoch.	Distance. ° ′ ″	
1750	0 5 5	Lacaille.
1825	0 5 8·08	Brisbane.
1840	0 5 9·27	Maclear.
1880	0 5 8·40	Stone.

From which we deduce for the epoch $1880+t$

Distance = 0° 5′ 9″·39 + 0″·03 × t, when Lacaille's observations are used.

Distance = 0° 5′ 8″·79 + 0″·0059 × t, when Lacaille's observations are excluded.

Whilst the proper motions of these stars, therefore, have carried each of them through an angular space of 189″, the angular distance of the stars has changed but very little indeed. The slight change which has taken place shows that the angular separation of these stars was formerly less than at the present time. There are only one or two stellar parallaxes at present known as large as 0″·5; but with such a parallax as this these stars must, in the last 130 years, have been carried progressively through at least 34,700,000,000 miles without any corresponding separation of their centres.

2nd.—For the distances between the star *e Eridani* and the mean position of the two stars ζ^1, ζ^2 *Reticuli*, we have

Epoch.	Distance. °	′	″
1750	19	28	2·55
1825	19	28	8·00
1840	19	28	8·70
1880	19	28	12·65

From these we deduce for the epoch $1880+t$

	°	′	″	″
Distance =	19	28	12·29	$+0''\cdot0762 \times t$, including Lacaille.
Distance =	19	28	12·57	$+0''\cdot0877 \times t$, excluding Lacaille.

The rate of separation between the star *e Eridani* and the centre of gravity of ζ^1 and ζ^2 *Reticuli* is very small compared with the proper motions of the stars themselves, but the distance is increasing.

The star *e Eridani* therefore participates in the large proper motion of ζ^1 and ζ^2 *Reticuli*, but is very slowly separating from that system.

3rd. For the distance between ζ *Toucani* and ζ^1 and ζ^2 *Reticuli* we have

Epoch.	Distance. °	′	″
1750	19	25	16
1825	(19	25	6)
1840	19	25	15
1880	19	25	18

The distance deduced from Brisbane's observations is discordant. The right ascensions of Brisbane's Catalogue are exceedingly rough, and the distance is here greatly affected by errors in right ascension. The general result, however, is the same whether Brisbane's observations be included or excluded. Rejecting Brisbane's distance, we have for the epoch $1880+t$

	°	′	″	″
Distance =	19	25	17	$+0\cdot006 \times t$, including Lacaille's observations.
Distance =	19	25	18	$+0\cdot075 \times t$, from the Cape observations alone.

The distance is increasing, but very slowly. The distance is nearly the same as in case (2) and the velocity of separation is the same if determined from the Cape observations which are alone corrected for proper motions to the epochs.

In this case also we have the stars participating in a large common proper motion, but slowly separating from each other.

4. The following are the distances between the stars ζ *Toucani* and *e Eridani*:—

Epoch.	Distance. ° ′ ″	Obs.—Comp. ″
1750	33 4 31	+0.9
1825	33 4 56·5	−0·6
1840	33 5 2·0	−0·5
1880	33 5 16·9	0·0

From which we obtain for the epoch $1880+t$

$$\text{Distance} = 33^\circ \; 5' \; 16''\!\cdot\!9 + 0''\!\cdot\!36 \times t.$$

The distance is increasing as in the other cases considered. The rate of change is larger than in the other cases, but it is still much smaller than the common proper motion of the group, and the distance itself is much greater than in the cases (1), (2), and (3.)

It appears therefore—

α. The four stars of the group under consideration have proper motions much larger than the average proper motions of stars.

β. That the stars have a common proper motion of more than a second of arc.

γ. That each star of the group is moving away from every other star of the group by quantities which are small compared with the common proper motion of the group.

δ. That, roughly speaking, the velocities of separation are larger, the larger the present angular separation of the stars.

The two stars ζ^1 and ζ^2 *Reticuli* have been shown to be separating very slowly indeed, and it has then been shown that the triangle formed by joining ζ *Toucani*, *e Eridani*, and the mean position of ζ^1 and ζ^2 *Reticuli* is slowly expanding. It is clear, therefore, that in the remote past the stars of the group must have been much closer together than at present. By a stellar system I mean a group of stars in which the several stars are bound together by much closer gravitational bonds than those which connect the members of one system with the members of another system. Have the stars in the group under consideration ever formed such a stellar system? It appears to me at least exceedingly probable that such must have been the case; but I fear the solution of the problem cannot be carried much further than I have already done.

If we suppose two stars to pass through a point and to

separate by moving uniformly along two straight lines, then the distance d can be expressed as follows:—

$$\tan d = \frac{at + bt^2}{1 + \alpha t + \beta t^2}.$$

We have no knowledge of the time t at which the angular separation of stars is supposed to be small, and the formula therefore contains five unknown quantities. But our observations, which only extend over 130 years, are fairly represented by a formula only containing tha first power of t. We are therefore only really given the distance d and its rate of change for the present epoch. If the stars have ever been sufficiently close to form a stellar system, our accurate astronomy is too much a thing of yesterday to furnish data for the determination of the time at which this must have taken place. The points to which I have called attention appear, however, to point clearly in the direction indicated; but, with respect to time, all we can safely say is that the stars cannot have been sufficiently near to sensibly affect each other's motions within the last 300,000 years.

5. I shall next give the distances, for the epoch $1880 + t$, between the star *β Hydri* and the group of stars considered above.

The following data are extracted from my Cape Catalogue:—

Name.	Mag.	R.A. 1880.	Precess.	P.M.	N.P.D. 1880.	Precess.	P.M.
		h m s		s	° ′ ″		″
β Hydri	3	0 19 25·32		+0·720	167 55 48·68		−0·32

The distances between *β Hydri* and

	°	′	″	″
ζ Reticuli =	20	1	12	$-0{\cdot}33 \times t$;
ζ Toucani =	12	21	24·63	$+0{\cdot}892 \times t$;
e Eridani =	38	29	33	$-0{\cdot}44 \times t$.

The star *β Hydri* has therefore a large velocity of separation from *ζ Toucani*, and is moving towards *ζ Reticuli* and *e Eridani*. *β Hydri* cannot therefore have been a member of the same stellar system as *ζ Reticuli*, *ζ Toucani*, and *e Eridani*.

It will be remarked, however, that *β Hydri* has large proper motions in right ascension and north polar distance of the same signs as those of the stars contained in the group under consideration, and that these conditions are therefore not alone sufficient to prove that stars whose proper motions satisfy them belong to a system.

I have examined several other cases, but I have not met with any other stars belonging to the same stellar system.

If we assume the masses of these fourth and fifth magnitude stars to be equal to that of our Sun, and the annual parallaxes $= 0''{\cdot}5$, the equation of *vis viva* will show that no changes of

proper motion affecting the tenths will have taken place since the angular separation of the stars amounted to a couple of minutes. It appears to me that such a system as that under consideration in the present paper may have originated from a star, or a system of two stars like α^1 and α^2 *Centauri*, with a large proper motion being brought, by reason of that large proper motion, sufficiently near another system to disturb the relative motions of each, and that both systems have thus been changed from close into open orbits. I consider that we now see each star moving with the common motion of the centre of gravity combined with the velocity due to its motion about the centre of gravity of the whole system, which latter velocity has for a long period past been sensibly an uniform velocity in a straight line. It would be interesting to examine the annual parallaxes of these stars.

1879, *October* 25.

Conjunction of Mars and Saturn, June 30, 1879, observed at the Observatory, Melbourne. By R. L. J. Ellery.

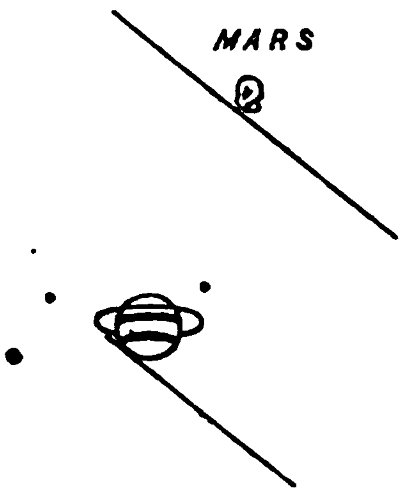
Observations with South Equatoreal, 8″ Aperture, and Parallel Wire Micrometer. Power 250.

Melbourne Mean Time.				Distance by Micrometer.	Distance, corrected, of Semi-diam.	Position of *Mars*.	
	h	m	s	″	″	°	′
June 30	16	25	33	113·19	110·50	294	37
		48	3	99·85	97·25	312	24
		58	19	94·99	92·39	322	24
	17	2	22	93·18	90·58	327	42
		6	37	91·28	88·68	331	12
		12	32	91·66	89·06	336	4
		14	12	90·52	87·92	338	7
		16	20	90·13	87·53	...	
		19	40	90·33	87·73	...	
		22	29	90·71	88·11	347	52
		24	41	91·47	88·87	...	
		27	0	91·47	88·87	...	
		28	25	92·23	87·63	354	2
		32	19	94·14	91·54	...	
		33	38	94·71	92·11	357	42
		35	35	95·85	93·25	359	22
		38	42	97·38	94·78	...	
		43	11	102·52	99·92	4	22

Remarks.—The measures having been made from south limb of *Mars* to south limb of *Saturn*, and the first column of distances corrected for wire zero only, a correction of $-2''\cdot60$ for diff. of semi-diameter is applied, which gives the distances in second column. No correction for parallax or refraction has been applied.

Observations by Mr. Ellery with the 8-inch Refractor.

Measures with Parallel-wire Micrometer, power 250.
Colour Comparison with negative eye-piece, 220.



Saturn.—Green yellow or *green cream colour*, sombre and dull in tone. *Divisions in ring* and *dark band* on body of planet well marked. Four satellites visible during first measures; but only three, as shown (two of which assumed to be *Titan* and *Rhea*), during the latter part of the observation.

Mars.—Very red-yellow, especially at terminator near North Pole. Greenish dark spot about centre of planet. *South Polar Ice Cap* like a bead of bright silver apparently projecting beyond the limb.

Colours of planets contrast strongly in the field of the telescope. *Saturn's* colour quite remarkable.

Measures were made from south (bright) limb of *Mars* to south limb of *Saturn*, and diameter of *Saturn's* ball measured (mean of 5 measures) 16″·1.

The sky was perfectly clear throughout the observations. Atmosphere quiet and steady; images sharp, well defined, and undisturbed.

Observations by Mr. Turner with the Great Telescope.

Saturn is of a pale greenish yellow tint.

Mars is a bright orange-red. Gibbous in shape, the bright limb of *Mars* being nearest to *Saturn*, thus: 

The illuminated limb of *Mars* is of a bright reddish yellow—the more central and far-off portions of a bright yellow without red. The North Pole of *Mars* is of a *bright* reddish yellow—the South Pole of a *very pale yellow*, or whitish yellow tone.

The colours of *Mars* are altogether *much more brilliant* than that of *Saturn*. *Saturn* has a dull sickly appearance, but *Mars* is bright and brilliant.

In the Finder the contrast of colour is, if anything, more striking than in the Great Telescope.

One dark band is seen on *Saturn*, below the ring, and three satellites, as shown above.

Sky beautifully clear, atmosphere steady, and altogether a splendid morning for such an observation.

Powers used 235 and 255.

Observations by Mr. White with the North Equatoreal of $4\frac{1}{2}$ inches aperture and 60 inches focal length.

At $16^h\ 55^m$ examined the planets in a dark field, with negative eye-piece magnifying 140 times. *Mars* looked bright red, slightly tinged with orange. *Saturn* pale buff colour.

At $17^h\ 0^m$ put in Simms's Position Wire Micrometer and measured the apparent distances between the centres of the illuminated disks of the planets and the position angle of the line joining them. The eye-piece used was a positive one and magnified 82 times; the field was brightly illuminated during these measures. The following are the results, uncorrected for parallax or refraction:—

Melbourne Mean Time. h m s	Angle of Position. ° ′	Distance. ″	Melbourne Mean Time. h m s	Angle of Position. ° ′	Distance. ″
17 3 25	327 5	...	17 23 1	347 55	88·2
4 28	329 53	89·9	25 28	349 51	88·8
7 3	...	90·2	27 56	353 0	89·9
8 59	333 9	90·1	30 13	354 52	89·9
11 3	337 39	87·1	32 41	357 37	91·6
12 55	338 26	86·4	35 29	0 3	94·5
15 33	340 15	88·0	39 54	2 51	95·9
17 45	343 9	87·7	17 43 52	6 31	99·5
20 44	345 39	88·7	18 6 38	21 22	121·9

The mean height of the barometer during the observations was $30^{in}\cdot259$, attached thermometer $48^\circ\cdot5$ Fahr. and temperature of the air $35^\circ\cdot0$ Fahr.

The Astronomer Royal presented to the Society a series of 28 Photographs of Drawings of *Mars*, made at the Royal Observatory, Greenwich, during the Opposition of 1877.

MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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LORD LINDSAY, M.P., F.R.S., President, in the Chair.

Observations of the Solar Eclipse 1879, *July* 18, *made at the University Observatory, Strasburg.* By Prof. A. Winnecke.

The morning of July 19 was extremely cloudy. Different observers with smaller telescopes got neither the beginning nor the end of the eclipse. I was myself a little more fortunate. The Refractor by Merz which I used has an aperture of 101^{mm}; and a polarisation eye-piece, power 105, was applied. At $3^h 50^m 15^s$ Strasburg sidereal time, clouds left the Sun; the eclipse had just commenced.

The end was, between thick clouds, very uncertainly observed at $4^h 27^m 50^s{\cdot}5$ Strasburg sidereal time.

From this account it is clear that, in the ordinary way of observing a partial solar eclipse, this one would have been nearly lost for Strasburg; but fortunately we have taken measures with the Heliometer. I had asked Herr Hartwig, Assistant of the Observatory, to profit by every bit of clear sky; he was aided by Dr. Küstner, who read the microscopes of the scales and of the position-circle.

Herr Hartwig obtained seven measures of the length of the chord joining the cusps, and seven angles of position of it; the power of the eye-piece was 170.

Diminishing the *Nautical Almanac* R.A. of the Moon by $10''{\cdot}0$ and correcting by $-1''{\cdot}90$ the *Nautical Almanac* semidiameter of the Sun, and by $-2''{\cdot}30$ the *Nautical Almanac* semi-

diameter of the Moon (see *Monthly Notices*, vol. xxxvi., p
Herr Hartwig obtained the equations:

Measures of length of chord connecting the cusps.

$$
\begin{aligned}
0 &= -23''09 + 4.016\,dr + 4.019\,dR + 1.132\,dx + 3.722\,dy + i.\\
0 &= -18.88 + 4.037\,dr + 4.090\,dR + 0.684\,dx + 3.853\,dy + i.\\
0 &= -18.59 + 4.263\,dr + 4.266\,dR + 0.566\,dx + 4.106\,dy - i.\\
0 &= -25.44 + 4.570\,dr + 4.573\,dR + 0.476\,dx + 4.435\,dy - i.\\
0 &= -28.39 + 5.046\,dr + 5.047\,dR + 0.384\,dx + 4.930\,dy - i.\\
0 &= -28.17 + 6.026\,dr + 6.015\,dR + 0.270\,dx + 5.924\,dy - i.\\
0 &= -31.90 + 7.605\,dr + 7.605\,dR + 0.164\,dx + 7.537\,dy + i.
\end{aligned}
$$

Measures of angle of position of the chords.

$$
\begin{aligned}
0 &= +3''10 - 0.246\,dx + 0.075\,dy + \gamma - \delta.\\
0 &= -4.99 - 0.252\,dx + 0.045\,dy + \gamma + \delta.\\
0 &= -3.66 - 0.239\,dx + 0.033\,dy - \gamma + \delta.\\
0 &= +4.08 - 0.214\,dx - 0.020\,dy - \gamma - \delta.\\
0 &= +5.59 - 0.210\,dx + 0.016\,dy - \gamma - \delta.\\
0 &= -3.62 - 0.168\,dx + 0.008\,dy - \gamma + \delta.\\
0 &= +4.19 - 0.133\,dx + 0.003\,dy + \gamma - \delta.
\end{aligned}
$$

In these equations the symbols denote:

dr = Correction to the adopted semi-diameter of the Sun.

dR = „ „ „ Moon.

dx = „ „ excess of Moon's R.A. over Sun's R.A.

dy = „ to excess of Moon's Decl. over Sun's Decl.

i = „ of adopted point of coincidence of images.

γ = Distance of optical centres of the halves of the object-glass.

δ = Error committed by the observer in rotating the object-glass in a great circle.

By treating the fourteen equations according to the me of least squares, Herr Hartwig obtained the final equations:

$$
\begin{aligned}
2.857\,dx + 15.968\,dy + 0.284\,i + 0.191\,\gamma + 0.135\,\delta - 86''073 &= 0.\\
15.968\,dx + 181.442\,dy - 4.283\,i + 0.046\,\gamma - 0.028\,\delta - 895.069 &= 0.\\
0.284\,dx - 4.283\,dy + 7.000\,i + 0.000\,\gamma + 0.000\,\delta + 26.720 &= 0.\\
0.191\,dx + 0.046\,dy + 0.000\,i + 7.000\,\gamma - 1.000\,\delta - 0.390 &= 0.\\
0.135\,dx - 0.028\,dy + 0.000\,i - 1.000\,\gamma + 7.000\,\delta - 28.930 &= 0.
\end{aligned}
$$

Hence :

$$
\begin{aligned}
dx &= +5''\cdot173 - 0\cdot224\,dr - 0\cdot229\,dR.\\
dy &= +4\cdot448 - 1\cdot006\,dr - 1\cdot006\,dR.\\
i &= -1\cdot306 + 0\cdot000\,dr - 0\cdot001\,dR.\\
\gamma &= +0\cdot474 + 0\cdot013\,dr + 0\cdot013\,dR.\\
\delta &= +4\cdot119 + 0\cdot002\,dr + 0\cdot002\,dR.
\end{aligned}
$$

The probable error of an observation is $1''\cdot48$; it would have been sensibly smaller had there not been so many clouds, which are very dangerous for double-image measures. Nevertheless, the position of the Moon is rigidly determined.

Taking the *Nautical Almanac* R.A. of the Sun as correct, we have:—

Correction to Moon's R.A.	$-4''\cdot47 - 0\cdot224\,dr - 0\cdot229\,dR$	$\pm 1''\cdot26$	prob. error.
Correction to Moon's Decl.	$+4\cdot45 - 1\cdot006\,dr - 1\cdot006\,dR$	$\pm 0\cdot16$	"

With these corrections and the values of the diameters adopted in the former calculations, the end of the eclipse is $4^h\ 28^m\ 19^s\cdot5$ Strasburg sidereal time. I thus lost the Moon very early; but it must be remembered that the rate of separation of the limb of the Sun and the Moon is extremely slow, the distance of the centres changing about $1''$ of arc in every 10^s of time.

Strasburg, 1879, *Nov.* 5.

On the Polarisation of the Solar Corona.

By Arthur Schuster, Ph.D., F.R.S.

1.—Introductory.

Accurate measurements of the polarisation of the light sent out by the solar corona will most likely give us some important information on the nature of the Sun's surroundings. It is therefore a matter of interest to calculate the polarisation due to the scattering of light by a particle placed in the neighbourhood of a luminous sphere.

Before we can apply the theoretical results to the actual phenomenon, we must take account of all the particles along one given line of sight. We cannot do this, as we do not know the law of distribution of the scattering particles round the luminous sphere. We are therefore obliged to discuss various possible laws, and to see how the theoretical results agree with observed facts. A further complication is introduced by a large and unknown quantity of unpolarised light mixed up with the corona and most likely due to incandescence. Our problem is an inverse one, and seems at first sight very hopeless. From the observed polarisation of light we are to find out what part of it is due to

scattering particles, and, as will be seen, we cannot do this without finding out at the same time in what way the scattering particles are distributed round the Sun and in what way the light due to other causes varies with the distance from the Sun. I began the calculation in the hope of getting a rough idea only of the amount of polarisation which we might expect. But it appeared that even such observations as we can make during the short time available during a total solar eclipse may yield most important information as to the constitution of the solar corona. I shall show that combined measurements of the polarisation at different distances from the Sun and of the decrease in intensity of the total light of the corona with increasing distance from the Sun will be sufficient to determine all our unknown quantities. Even if such measurements are incomplete, we may gain a rough idea of these quantities, and even a solitary observation like that of Mr. Winter during the eclipse of 1871 will give some results. We may hope to obtain, by means of the polariscope, answers to the following questions:—

In what way is the scattering matter distributed in the solar atmosphere?

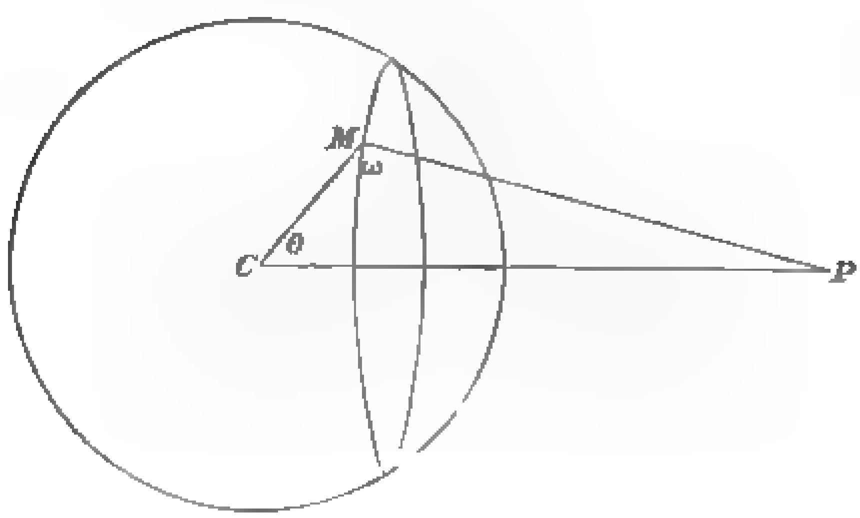
What part of the light sent out by the corona is due to scattering matter?

Is the scattering matter projected outwards from the Sun, or is it falling into the Sun from outside?

2.—Light emanating from a Luminous Sphere is scattered by a Small Particle.

I shall begin by calculating the phenomena of polarisation shown by a single scattering particle when placed at different distances from the Sun.

Fig. 1.



In the figure let

C = centre of luminous sphere.

M = any point on the surface of the sphere.

P = position of scattering particle.

Also adopt the following notation :—

O = position of observer's eye.
PM = y.
PC = r.
ω = angle PMC.
θ = „ MCP.
ϕ = „ between planes MCP and CPO.
χ = „ CPO.
η = „ between planes CPO and MPO.
α = „ MPO.

We fix our unit of length to be the radius of the luminous sphere.

We have to resolve the light sent out by the element of surface at M and scattered at P into two components, one polarised in the plane MPO and the other at right angles to that plane. Let B^2 be the intensity of that component of the scattered ray which is polarised in the plane MPO; then $B^2 \cos^2 \alpha$ will be the component which is polarised at right angles to that plane.* The intensity resolved in any plane forming an angle δ with MPO will be

$$B^2 (\cos^2 \delta + \cos^2 \alpha \sin^2 \delta).$$

We shall refer the polarisation, however, not to the plane MPO, which varies with the position of M, but to the plane CPO, which forms an angle η with MPO. Putting $\epsilon = \delta - \eta$, we find the component polarised in a plane forming an angle ϵ with the radial plane

$$B^2 \cos^2 (\eta + \epsilon) + \sin^2 (\eta + \epsilon) \cos^2 \alpha = B^2 S, \tag{1}$$

where

$$S = 1 - \sin^2 \alpha \sin^2 (\eta + \epsilon). \tag{2}$$

In the spherical triangle formed at P by the lines PC, PM, PO we have

$$\sin \alpha \sin \eta = \sin \omega \sin \phi, \tag{3}$$

$$\sin \alpha \cos \eta = \cos \omega \sin \chi - \sin \omega \cos \chi \cos \phi, \tag{4}$$

or

$$\sin^2 \alpha \sin^2 (\eta + \epsilon) = \sin^2 \phi \cos^2 \epsilon \sin^2 \omega + \sin^2 \epsilon \cos^2 \omega \sin^2 \chi + \cos^2 \phi \sin^2 \epsilon \sin^2 \omega \cos^2 \chi + X, \tag{5}$$

* *See* a Paper by Lord Rayleigh, "On the Light from the Sky; its Polarisation and Colour," *Phil. Mag.*, 1871, vol. xli., p. 107.

where X consists of three terms involving ϕ only in the form $\sin\phi$, $\cos\phi$, $\sin\phi\cos\phi$ respectively. These terms will disappear in any integration with respect to ϕ between the limits 0 and 2π.

If I represents the light sent out normally by the luminous sphere, the intensity of the light falling on the point P from an element of surface at M will be $\mathrm{I}d\Omega$, where $d\Omega$ is the solid angle subtended at P by the element of surface $d\mathrm{S}$. Instead of integrating over the luminous sphere, we can integrate over the unit sphere surrounding P. We have

$$d\Omega = \sin\omega\, d\omega\, d\phi = -d\phi\, d\cos\omega.$$

The intensity of the light scattered by P will therefore be

$$\mathrm{B}^2\mathrm{I}\int_{\cos\omega_1}^{1}\int_0^{2\pi}\mathrm{S}\, d\cos\omega\, d\phi,$$

where ω_1 is the angle between CP and the tangent to the sphere from P. But

$$\frac{1}{\pi}\int_0^{2\pi}\mathrm{S}\, d\phi = (1+\sin^2\epsilon\sin^2\chi)+\cos^2\omega\,(1-3\sin^2\epsilon\sin^2\chi).$$

Integrating this with respect to $\cos\omega$, between the proper limits, and writing again ω for ω_1, we find for the intensity of vibration resolved in any direction determined by ϵ

$$\frac{\mathrm{IB}^2}{\pi}\left[\left(\frac{4}{3}-\cos\omega-\frac{\cos^3\omega}{3}\right)-\sin^2\epsilon\sin^2\chi\,(\cos\omega-\cos^3\omega)\right]. \qquad (6)$$

Fixing our unit of intensity so that $\dfrac{\mathrm{B}^2\mathrm{I}}{\pi}=\tfrac{1}{2}$, we have for the light polarised in the radial plane ($\epsilon = 0$)

$$2g_0 = \frac{1}{3}(1-\cos\omega)(4+\cos\omega+\cos^2\omega), \qquad (7)$$

and for the light polarised in a perpendicular plane ($\epsilon = \tfrac{1}{2}\pi$)

$$2g_1 = 2g_0 - \sin^2\chi\,(1-\cos\omega)(\cos\omega+\cos^2\omega). \qquad ($$

It is usual in polariscopic measurements to express the de[
of polarisation p by the ratio of the polarised to the total li
which in our notation would be

We shall find it more convenient to express the polarisation by the ratio of the polarised light to the stronger component, or by

$$q = \frac{g_0 - g_1}{g_0}.$$

Between q and p we have the relation

$$\frac{1}{p} = \frac{2}{q} - 1,$$

which shows that the two quantities will always increase and decrease together. We find, therefore, for the light scattered by a particle,

$$q = 3 \sin^2 \chi \frac{\cos \omega + \cos^2 \omega}{4 + \cos \omega + \cos^2 \omega}. \qquad (9)$$

For a given value of χ this quantity evidently increases with decreasing ω, or with increasing distance from the Sun. The fraction is always positive, and therefore the polarisation is always radial.

If the scattering particle is situated close to the surface of the Sun, $\cos \omega = 0$ and

$$g_0 = g_1.$$

Hence the light is altogether unpolarised. If the point is at an infinite distance from the sphere, $\cos \omega = 1$ and

$$\frac{g_0 - g_1}{g_0} = \sin^2 \chi;$$

that is to say, the luminous sphere behaves like a luminous point, as is evident *à priori.*

We can put (6) under the form

$$\frac{1}{3}(2 - 3 \cos \omega + \cos^3 \omega) + \frac{1}{2}(1 - \sin^2 \epsilon \sin^2 \chi) \cos \omega \sin^2 \omega. \qquad (10)$$

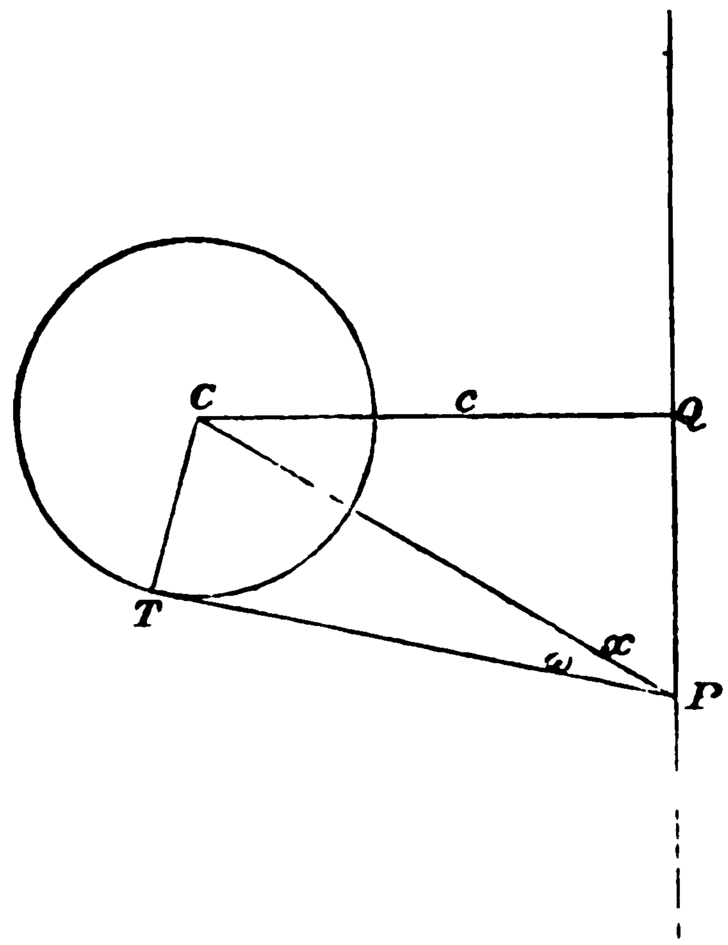
As $\sin^2 \omega = \frac{1}{r^2}$, the second term represents light which would be scattered by the particle if the luminous sphere were replaced by a point placed at its centre and sending out light of an intensity $\frac{\cos \omega}{2B^2}$. The first term represents light which is altogether unpolarised and which might therefore be due to the self-luminosity of the particles.

This way of expressing the scattered light shows well how, on the one hand, at an infinite distance the sphere can altogether be replaced by a luminous point; while, on the other hand the

polarised part disappears altogether if the particle is in contact with the Sun.

Let us now consider in what way the polarisation of the scattered light changes as the scattering particle is moved along a line of sight. Consider the line of sight, for instance, to be the tangent to the luminous sphere. Close to the Sun, as I have shown, the light is entirely unpolarised. As the particle is moved either way along the tangent, the polarisation increases. At large distances it must vanish again, and hence it must pass a maximum. We shall investigate the position of this maximum.

Fig. 2.



Let QP be a line of sight, P being the position of the particle and CQ = c being the perpendicular from C to QP,

$$\sin \chi = \frac{\mathrm{CQ}}{\mathrm{CP}};$$

and if PT is the tangent to the sphere from P,

$$\mathrm{CP} = \frac{\mathrm{CT}}{\sin \omega} = \frac{1}{\sin \omega};$$

therefore

$$\sin \chi = c \sin \omega.$$

The expression (8) now becomes

$$q = 3c^2 \sin^2 \omega \, \frac{\cos \omega + \cos^2 \omega}{4 + \cos \omega + \cos^2 \omega}$$

To find the maximum, we now vary ω, putting the differential coefficient of (12) equal to zero; we thus obtain an equation of the fifth degree. But as both the numerator and its differential coefficient are divisible by $(1+\cos\omega)$, the equation reduces to one of the fourth degree; which is

$$\cos^4\omega+\cos^3\omega+8\cos^2\omega-2\cos\omega-2=0.$$

This equation has two real roots; of these one is negative and does not concern us; the other is equal to 0·597309. This, then, is the value of $\cos\omega$ for which the polarisation is a maximum along a given line of sight. The locus of the maximum is a sphere concentric with the luminous sphere, and having a radius

$$\frac{1}{\sin\omega}=1\cdot2469,$$

or nearly one and a quarter times the radius of the luminous sphere. If any line of sight cut this sphere, a particle situated on it (the sphere) will polarise the light more than any other particle placed on the line of sight. If the line of sight lies wholly outside the sphere, then the particle which lies on the perpendicular from the centre of the sphere to the line of sight will be the one which will polarise the light most. Substituting the value of $\cos\omega$ into (11), we find the amount of maximum polarisation for different values of c. Thus, if $c=1$, or if the line of sight is a tangent to the Sun, the maximum value is $q=0\cdot372$, which gives $p=0\cdot229$. Three minutes away from the Sun we have already $q=0\cdot524$. The line of sight which touches the sphere of maximum polarisation forms an angle of $3'\ 57''$ with the nearest point on the Sun's limb. The maximum polarisation for that line of sight is $q=0\cdot578$.

In order to show the rapid way in which the polarisation increases as the particle is moved away from the Sun, I have calculated in Table I. the values of q for light scattered from a particle in a direction at right angles to the line drawn from it to the centre of the luminous sphere. The angle which the line of sight makes with a line drawn to the Sun's centre is given in the first column. The solar diameter is supposed to be 16′. It will be seen that, at a distance of four solar radii from the Sun's centre, the Sun very nearly behaves as a luminous point.

3.—A Luminous Sphere surrounded by an Atmosphere of Scattering Particles.

We now pass to the consideration of an atmosphere of scattering particles round the luminous sphere. It will be convenient to substitute for the variable ω the angle χ, connected with the former by

$$\sin\omega=k\sin\chi,$$

where

$$kc = 1.$$

Putting

$$\Delta\chi = \sqrt{1 - k^2 \sin^2 \chi},$$

we find by (8) and (7)

$$2 (g_0 - g_1) = k^2 \sin^4 \chi \, \Delta\chi,$$
$$6g_0 = 4 - \Delta\chi \, (4 - k^2 \sin^2 \chi).$$

Let h be measured along a line of sight from Q (Fig. 2); then

$$\frac{c}{h} = \tan \chi,$$

$$-dh = \frac{c}{\sin^2 \chi} \, d\chi.$$

We wish to compare the intensities of light received along different lines of sight; or, to speak more correctly, we wish to compare the intensities received within cones of small, but given, aperture $d\Omega$ which have different lines of sight as axes. If we consider that the number of particles contained within an element of such a cone is $R^2 d\Omega dh$, and that the intensity of light received from each such element varies inversely as R^2, we see that

$$d\Omega \int g f(c, \chi) \, dh$$

will be the intensity required if $f(c, \chi)$ represents the density of scattering matter along a line of sight. Leaving $d\Omega$ out of account, as we only compare intensities received within cones of equal aperture, we obtain, if I_0 is the intensity of light polarised radially and I_1 that polarised tangentially,

$$I_0 - I_1 = c \int_0^{\frac{\pi}{2}} f(c, \chi) \, k^2 \sin^2 \chi \, \Delta\chi \, d\chi, \qquad (13)$$

$$I_0 = \frac{c}{3} \int_0^{\frac{\pi}{2}} f(c, \chi) \left[\frac{4}{\sin^2 \chi} (1 - \Delta\chi) + k^2 \Delta\chi \right] d\chi. \qquad (1\!/$$

We first take the simplest case, that the density of scatte ing matter is constant and everywhere equal to unity.

We obtain, by easy transformations and the usual notatio elliptic integrals,

$$\int_0^{2\pi} d\chi \sin^2 \chi \, \Delta\chi = \frac{1}{3k^2} [k'^2 F_1 + (k^2 - k'^2) E_1],$$

$$\int_0^{2\pi} d\chi \frac{(1 - \Delta\chi)}{\sin^2 \chi} = E_1 - k'^2 F_1,$$

$- k^2.$

Hence we find, remembering that c is the reciprocal of k,

$$3(I_0 - I_1) = kE_1 + \frac{k'^2}{k}(F_1 - E_1),$$

$$3I_0 = 5kE_1 - 4\frac{k'^2}{k}(F_1 - E_1),$$

$$3I_1 = 4kE_1 - 5\frac{k'^2}{k}(F_1 - E_1).$$

For the polarisation we obtain, writing $F_1 - E_1 = L_1$,

$$p = \frac{I_0 - I_1}{I_0 + I_1} = \frac{1}{9}\frac{k^2E_1 + k'^2L_1}{k^2E_1 - k'^2L_1}.$$

This quantity will increase or decrease for varying k with

$$\frac{k'^2L_1}{k^2E_1};$$

but we shall not stop here to prove that this quantity increases with increasing values of c, as the proof will be included under the more general case, which we now propose to treat. Tables II., III., IV. give some numerical values for the above expressions. The arrangement of the Tables will be explained further on.

We shall now suppose that the distribution of matter is only a function of the distance from the Sun's centre. This function is supposed to be expanded in inverse powers of the distance, and may contain a constant term.

In Fig. 2 we have

$$\frac{1}{OP} = \sin\omega = k\sin\chi.$$

We may therefore suppose the expansion to have taken place in powers of $k\sin\chi$. We shall consider first, the case of a distribution varying as the inverse nth power of the distance from the Sun's centre. In equations (13) and (14) we have to put

$$f(c, \chi) = ak^n\sin^n\chi,$$

and we shall choose our unit of density so that $a = 1$.

We obtain in this way

$$I_0 - I_1 = \int_0^{\frac{\pi}{2}} k^{n+1}\sin^{n+2}\chi\,\Delta\chi\,d\chi,$$

$$3I_0 = \int_0^{\frac{\pi}{2}} [4k^{n-1}\sin^{n-2}\chi(1 - \Delta\chi) + k^{n+1}\sin^n\chi\,\Delta\chi]\,d\chi.$$

For any given n, the exponent of $\sin\chi$ can, by partial integration, be depressed, and if n is even, the expressions can be finally reduced to elliptic integrals; if n is odd, the integrals can be expressed by means of circular and logarithmic functions.

We are here only concerned with the variation of the fraction $\frac{I_0-I_1}{I_0}$, and we can easily show that this fraction must increase with increasing c, or with decreasing k.

We adopt the following notation:—

$$P_n=\int_0^{\frac{\pi}{2}}\sin^n\chi\,\Delta\chi\,d\chi,$$

$$Q_n=\int_0^{\frac{\pi}{2}}\frac{\sin^n\chi}{\Delta\chi}\,d\chi.$$

It is easily found that

$$\frac{d.k^m P_n}{dk}=k^{m-1}\left[(m+1)P_n-Q_n\right],$$

and hence

$$\frac{d(I_0-I_1)}{dk}=k^n\left[(n+2)P_{n+2}-Q_{n+2}\right]=\frac{(n-1)}{k}(I_0-I_1)+k^n(3P_{n+2}-Q_{n+2}),$$

$$\frac{dI_0}{dk}=\frac{n-1}{k}I_0+k^n(P_n+Q_n).$$

If we expand

$$\frac{4}{3}\cdot\frac{1-\Delta\chi}{k^2\sin^2\chi}+\frac{\Delta\chi}{3}$$

in powers of k^2, the constant term is found to be 1, the term containing k^2 will disappear, and the term containing k^{2p} will be

$$\frac{1.3\quad.\quad.\quad.\quad2p-3}{2.4\quad.\quad.\quad.\quad2p-2.2p}\cdot\frac{p-1}{p+1}.k^{2p}\sin^{2p}\chi.$$

We find similarly, by expanding $\frac{1}{2}\left(\Delta\chi+\frac{1}{\Delta\chi}\right)$, the first term to be 1 and the second term to disappear; but the term containing k^{2p} will be

$$\frac{1.3\quad.\quad.\quad.\quad2p-3}{2.4\quad.\quad.\quad.\quad2p-2.2p}(p-1)\,k^{2p}\sin^{2p}\chi.$$

As all terms are positive in both cases, we conclude that for every value of k

$$\frac{4}{3}\cdot\frac{1-\Delta\chi}{k^2\sin^2\chi}+\frac{\Delta\chi}{3}<\frac{1}{2}\left(\Delta\chi+\frac{1}{\Delta\chi}\right);$$

or, multiplying both sides with $k^{p+1}\sin^n\chi$, and integrating,

$$I_0<\frac{k^{n+1}}{2}(P_n+Q_n).$$

The increase or decrease of $\frac{I_0-I_1}{I_0}$ will depend on the sign of

$$\frac{1}{I_0-I_1}\cdot\frac{d(I_0-I_1)}{dk}-\frac{1}{I_0}\cdot\frac{dI_0}{dk}. \tag{15}$$

As $\frac{dI_0}{dk}$ is always positive as well as $\frac{1}{I_0}$, this expression will be rendered algebraically larger if we substitute for I_0 the higher value just found. Substituting for $\frac{d(I_0-I_1)}{dc}$, $\frac{dI_0}{dc}$, and I_0-I_1, their values, the expression (15) becomes

$$\frac{1}{k}\cdot\left[1-\frac{Q_{n+2}}{P_{n+2}}\right]=-kQ_{n+4}. \tag{16}$$

We see that $\frac{I_0-I_1}{I_0}$ always increases as k decreases; that is, as the line of sight moves away from the Sun. We have found therefore that: *The polarisation must increase with increasing distances from the Sun if the scattering matter is distributed according to any inverse power of the distance from the Sun.*

Tables II., III., and IV. contain some numerical calculations which I have made for different laws of distribution.

Table II. contains the intensity of the stronger component; Table III. gives the difference, I_0-I_1, between the two components; and Table IV. gives the values of $q=\frac{I_0-I_1}{I_0}$. The argument θ is given by $k=\sin\theta$. At the head of each column, the law according to which the scattering matter is supposed to be distributed is given. The first column gives the values for a constant distribution of matter. It will be seen that, up to the inverse 6th power and most likely beyond, the percentage of polarisation is much the same whatever law we take. In Table V. the corresponding values of θ and $k-1=c$ are given; $k-1$ is very nearly proportional to the angular distance from the Sun's centre. For the edge of the Sun $k=1$.

If the law of distribution of the scattering particles contains more than one term, the polarisation does not necessarily increase throughout with increasing distance from the Sun. Suppose the law to be expressed by

$$\Sigma A_n k^n \sin^n \chi,$$

we have, writing R_n for

$$\int_0^{\frac{\pi}{2}} \left[\frac{4}{3}\sin^{n-2}\chi \,.\, k^{-2}(1-\Delta\chi) + \frac{1}{3}\sin^n\chi\,\Delta\chi\right] d\chi.$$

$$I_0 = \Sigma A_n k^{n+1} R_n, < \frac{1}{2}\Sigma A_n k^{n+1}(P_n + Q_n),$$

$$I_0 - I_1 = \Sigma A_n k^{n+1} P_{n+2},$$

$$\frac{dI_0}{dk} = \Sigma A_n (n-1) k^n R_n + \Sigma A_n k^n (P_n + Q_n),$$

$$\frac{d(I_0 - I_1)}{dk} = \Sigma A_n (n-1) k^n P_{n+2} + \Sigma A_n k^n (3P_{n+2} - Q_{n+2}).$$

We observe that $\frac{d(I_0 - I_1)}{dk}$ is always negative for values of k which are sufficiently near unity; for while P_{n+2} approaches a finite quantity, Q_{n+2} becomes infinitely large if Q is equal to 1.

The expression corresponding to (15) and (16) will be, if we multiply by k,

$$\frac{\Sigma_n A_n k^n P_{n+2}}{\Sigma' A_n k^n P_{n+2}} - \frac{\Sigma_n A_n k^n R_n}{\Sigma A_n k^n R_n} - \frac{\Sigma A_n k^n Q_{n+2}}{\Sigma A_n k^n P_{n+2}} + 1 \qquad (17)$$

If k is nearly 1, the third term becomes very large compared to the others, and hence *the polarisation close to the Sun must always increase with increasing distance from the Sun's centre.* As k becomes smaller, however, the positive terms may have the upper hand, and the polarisation may therefore approach a maximum.

In order to show this, suppose, for instance, that the distribution of matter contains only two terms, with indices n and t respectively. Bringing (17) under the same denominator, and remembering that

$$Q_{n+2} = P_{n+2} + k^2 Q_{n+4},$$

we find for the numerator

$$(t-n) A_n A_t k^{n+t} (R_n P_{t+2} - R_t P_{n+2})$$
$$- k^2 (A_n k^n Q_{n+4} + A_t k^t Q_{t+4})(A_n k^n R_n + A_t k^t R_t). \qquad (18)$$

If k is very small, the sign of the whole expression will depend on the sign of the lowest power of k. The lowest power in the

second term is k^{2n+2}. If $t > n+2$, this will be lower than the lowest power contained in the first term. In that case (18) is always negative for small values of k; that is, the polarisation will increase with increasing distance from the Sun. But if $t = n+1$, the first term contains the lowest powers of k. The constant term in R_n and P_n is

$$\int_0^{\frac{\pi}{2}} \sin^n \chi \, d\chi,$$

for which we write s_n. Thus we find for the lowest power of k

$$(t-n) \, A_n A_t s_n s_t \left[\frac{t+1}{t+2} - \frac{n+1}{n+2}\right].$$

This expression will have the same sign as A_n, A_t, and if, therefore, these two quantities are of the same sign, the differential coefficient of $\frac{I_0 - I_1}{I_0}$ will be positive for small values of k; that is, the polarisation will *decrease* with increasing distance from the Sun.

If A_n and A_t are of opposite sign, the polarisation will finally increase. If $t = n+2$, it may be positive or negative according to the relative value of A_r and A_t, but must always be negative if A_n and A_t are of opposite sign.

If the law of distribution contains more than two terms, then it is clear that for small values of k only the first two need be taken into account, and I arrive therefore at the following law :—

If the distribution of scattering matter is expanded in inverse powers of the distance from the Sun, the increase or decrease of polarisation for large distances from the Sun will depend on the first two terms only; if these are of opposite sign, the polarisation will finally increase with increasing distances from the Sun. If they are of the same sign, and if the exponent of $\frac{1}{r}$ *in the second term differs by more than 2 from the exponent in the first term, the polarisation will finally increase; if it differs by 1 only, the polarisation will decrease; if by 2, the final increase or decrease will depend on the value of the constant quantities.*

I have followed out the case where the expansion contains only two terms; but I shall only give the results of the investigation. If A_r and A_t are of opposite sign, the polarisation will always increase. If they are of the same sign and $t = n+1$, the polarisation decreases for small values of k. Now if either A_r or A_t becomes very small, it is clear that the maximum of polarisation which must take place is very far removed from the centre of the Sun; for we have seen that if either constant is zero, the polarisation must steadily increase. There must therefore be one relative value of A_n and A_t for which the maximum of polarisa-

tion is brought nearest to the Sun. The question is of interest, as in reality the corona shows a maximum of polarisation and we might suppose this maximum to be produced by such a distribution of matter.

As a first approximation, I find that if the distribution of matter takes place according to the law $\frac{A_n}{r^n} + \frac{A_t}{r^t}$, both A_n and A_t being positive, the value of A_t which brings the maximum of polarisation nearest the Sun is given by

$$A_t = A_n k^{n-t} \frac{s_n}{s_t} \sqrt{\frac{n+3}{n+4} \cdot \frac{n+1}{n+2} + \frac{t+3}{t+4} \cdot \frac{t+1}{t+2}}, \tag{19}$$

where s_n is again written for

$$\int_0^{\frac{\pi}{2}} \sin^n \chi \, d\chi.$$

The value of k at which the maximum takes place and which has to be substituted in (19) is given by

$$k^2 = \left(\frac{t+1}{t+2} - \frac{n+1}{n+2}\right) a + \left[(1-a) \frac{n+3}{n+4} \cdot \frac{n+1}{n+2} + (1+a) \frac{t+3}{t+4} \cdot \frac{t+1}{t+2}\right].$$

where $a = \frac{t-n}{2}$.

These equations are true for any value of t larger than n, only if $t > n+2$. We have seen that the polarisation finally increases, and in this case a minimum of polarisation occurs after the maximum has been passed.

If, for instance, $n = 0$, $t = 2$, we find

$$k^2 = \frac{1}{5} \quad \text{and} \quad A_q = A_p \, 50\sqrt{\frac{3}{5}}; \; \theta = 26° \; 34'.$$

The maximum therefore takes place about 20 minutes away from the Sun's limb. Tables II., III., and IV. contain in the last column the calculations for this special case. Table IV. shows the maximum of polarisation at the point indicated. As the difference between t and n gets larger, the maximum moves nearer and nearer the Sun, and finally approaches a value which we obtain by putting $t = \infty$, $n = 0$. We find in this special case,

$$k^2 = 0{\cdot}8, \quad \text{and hence} \quad \theta = 63° \; 6'.$$

In this extreme case, however, it is clear that A_q becomes infinite. An approach to it in the actual corona might be found if the presence of scattering matter was due to two different

causes; one of which produced an even distribution throughout the corona, and the other of which produced a very large but rapidly diminishing amount close to the body of the Sun. In that case we might observe a maximum of polarisation near the point indicated. Outside this maximum the polarisation would diminish, reach a minimum, and finally increase again.

It is, however, clear that the maximum which takes place in the corona is due to a different cause; for if the whole corona was only formed by scattering matter, the polarisation ought to approach a constant value as we move away from the Sun, and this constant value could not be below $q=0·5$. In reality the polarisation rapidly diminishes and very soon a point in the corona is reached at which no polarisation can be observed; the corona must therefore contain some matter which is either self-luminous or too large to polarise the light while scattering it.

4. Light Scattered by an Atmosphere of Small Particles surrounding a Luminous Point.

If, instead of a luminous sphere, we have only a luminous point, the investigation is much simplified. Let the distribution of matter vary as r^{-n}; then the polarisation in a radial plane will be

$$I_0 = A\int_0^{\infty} dh \frac{1}{r^{n+2}},$$

and in a perpendicular plane

$$I_1 = A\int_0^{\infty} \frac{\cos^2\chi}{r^{n+2}}\, dh,$$

where, as before,

$$dh = -\frac{c}{\sin^2\chi}\, d\chi \quad \text{and} \quad r = \frac{c}{\sin\chi}.$$

Hence we have, if $A=1$,

$$I_0 = \frac{1}{c^{n+1}}\int_0^{\frac{\pi}{2}} \sin^n\chi\, d\chi,$$

$$I_0 - I_1 = \frac{1}{c^{n+1}}\,\frac{n+1}{n+2}\int_0^{\frac{\pi}{2}} \sin^n\chi\, d\chi.$$

For the polarisation we obtain

$$q = \frac{I_0 - I_1}{I_0} = \frac{n+1}{n+2}.$$

The polarisation is therefore independent of c. *If the distribution of matter takes place according to any inverse power of*

the distance, the polarisation is constant and independent of the distance from the luminous point.

If the density of scattering matter is constant, $n = 0$; hence

$$q = \frac{1}{2} \quad \text{and} \quad p = \frac{1}{3}.$$

A third part of the whole light in this case will be polarised. Our formulæ are, of course, not applicable to small distances from the luminous point. For $c = 0$ the polarisation must necessarily vanish. Close to the luminous point there will be, therefore, a very rapid increase of polarisation; which, after this increase, will remain constant.

If the distribution of matter takes place according to the law

$$\frac{A_n}{r^n} + \frac{A_t}{r^t},$$

we find

$$I_0 - I_1 = \frac{A_n}{c^{n+1}} s_{n+2} + \frac{A_t}{c^{t+1}} s_{t+2},$$

$$I_0 = \frac{A_n}{c^{n+1}} s_n + \frac{A_t}{c^{t+1}} s_t.$$

Putting $t = n + l$, we find

$$\frac{I_0 - I_1}{I_0} = \frac{A_n c^l s_{n+2} + A_t s_{t+2}}{A_n c^l s_n + A_t s_t}.$$

Differentiating with respect to c, the denominator will contract into

$$lc^{l-1} A_n A_t (s_t s_{n+2} - s_n s_{t+2}), = lc^{l-1} A_n A_t s_n s_t \left(\frac{n+1}{n+2} - \frac{t+1}{t+2}\right).$$

This expression will always have the opposite sign to $A_n A_t$, and can never be zero. Hence,

If the distribution of matter surrounding a luminous point is expressed by the law

$$\frac{A_n}{r^n} + \frac{A_t}{r^t},$$

the polarisation, as looked at from a point outside the scattering atmosphere, will steadily increase if A_t is negative; it will steadily decrease if A_t is positive.

5. Light Scattered from a Partly Self-luminous Atmosphere of Small Particles.

We have hitherto only been considering cases which are very different from the one we have to deal with in the solar

atmosphere. The rapid decrease of polarisation with increasing distances from the Sun, as well as the comparatively small amount of observed polarisation, shows that a large part of the light is not due to scattering particles. This light may either be produced by incandescence, or by particles which are too large to polarise the light in the act of scattering it. If unpolarised light of intensity $2A$ be added to the light due to the scattering of particles, it is easy to see that, by taking A large enough, we shall be able to produce a maximum of polarisation in any place outside a certain circle surrounding the Sun. The intensity of the whole light will be $I_0 + I_1 + 2A$. Hence, the part which is polarised will be

$$\frac{I_0 - I_1}{I_0 + I_1 + 2A}.$$

In order that this should be a maximum, we must have

$$(I_0 + I_1 + 2A)\frac{d(I_0 - I_1)}{dc} = I_0 - I_1 \frac{d(I_0 + I_1 + 2A)}{dc} = 0.$$

Supposing A to be constant first, the above equation reduces to

$$(I_0 + A)\frac{d(I_0 - I_1)}{dc} - (I_0 - I_1)\frac{dI_0}{dc} = 0,$$

or

$$A\frac{d(I_0 - I_1)}{dc} = -I_0^2 \frac{d}{dc}\left(\frac{I_0 - I_1}{I_0}\right).$$

This equation determines A if I_0 and I_1 are given as functions of c; that is, if the law of distribution of the scattering particles is known. We observe first, that close to the Sun the differential coefficients on both sides have been shown to be always positive. In that case A ought to be negative, which has no meaning in our problem. We arrive therefore at the important conclusion:—

Close to the Sun the polarisation always increases, however large the intensity of the unpolarised light may be which is superposed to the light due to the scattering particles.

This law remains still correct if A is variable. If A is a constant, it will begin to be positive when $\frac{d(I_0 - I_1)}{dc}$ begins to be negative. If the distribution varies as the inverse nth power of the distance, this point is given by

$$Q_{(n+2)} = (n+2)\,P_{(n+2)}.$$

Referring to Table III., we find for a constant distribution of matter this value to be near $\theta = 52^\circ{\cdot}5$, or about 4 minutes away from the Sun. As n increases, the point at which a maxi-

mum of polarisation may begin to take place gets nearer to the Sun. In order to show what intensity of natural light would be necessary to bring about a maximum of polarisation at a given point for various laws of distribution, I have calculated in Table VI. the proportion $\frac{I_0+I_1}{I_0+I_1+2A}$ of the scattered to the total light for various values of θ. I have added in each compartment the value of $q=\frac{I_0-I_1}{I_0+A}$ for the same point. In the Table the maximum of polarisation is supposed to take place at the distance from the Sun's centre fixed for each row by the affixed value of θ. The head of each column gives the law of distribution of scattering matter. The number at the top of each compartment fixes the polarisation; the number underneath indicates what must be the proportion of scattered to natural light which produces a maximum of polarisation at that point.

In reality the light due to self-luminosity and other causes varies with the distance of the Sun. The law of variation is one of our unknown quantities. We now turn to treat this general case.

6. The Solar Corona.

We write J for the total intensity and p for the observed polarisation. We have

$$J = I_0 + I_1 + 2A,$$

$$p = \frac{I_0 - I_1}{J};$$

and hence

$$\frac{1}{p}\frac{dp}{dc} = \frac{1}{I_0-I_1}\frac{d(I_0-I_1)}{dc} - \frac{1}{J}\frac{dJ}{dc}. \tag{20}$$

p and $\frac{dp}{dc}$ are quantities which we can observe. $\frac{1}{J}\frac{dJ}{dc}$ is the ratio of the increase of intensity to the total intensity, and can also be observed. Hence we can, by observation alone, find the value of $\frac{1}{I_0-I_1}\frac{d(I_0-I_1)}{dc}$ for different parts of the corona. Assuming the distribution of scattering matter to take place according to a law $a+\frac{b}{r}+\frac{c}{r^2}+\ldots$, we can, by constructing tables corresponding to the different terms in the expansion, find out what values of a, b, c, &c. agree most nearly with observation.

We may, if the observations are sufficiently accurate, for instance, find out whether a is zero or not. For the limiting value of $\frac{1}{I_0-I_1}\frac{d(I_0-I_1)}{dc}$ for small values of k is $-(n+1)k$

where n is the lowest exponent of r which occurs in the expansion. We may thus hope to settle the important point whether the scattering matter is falling into the Sun, or is projected outwards. If it is falling into the Sun, we expect a to be finite; if it is projected outwards, a will be zero. Having found a law which agrees with observed facts, we deduce easily all the unknown quantities; for, knowing I_0 and I_1 for different places in the corona, we find

$$J = \frac{I_0 - I_1}{p},$$

$$2A = \frac{I_0 - I_1}{p} - (I_0 + I_1) = \frac{1-p}{p} I_0 - \frac{1+p}{p} I_1;$$

and, by differentiation,

$$2\frac{dA}{dc} = \frac{1-p}{p}\frac{dI_0}{dc} - \frac{1+p}{p}\frac{dI_1}{dc} - \frac{I_0 - I_1}{p}\frac{dp}{dc}.$$

For the maximum of polarisation the last term is equal to zero, and we also have by (20)

$$\frac{1}{J}\frac{dJ}{dc} = \frac{1}{I_0 - I_1}\frac{d(I_0 - I_1)}{dc}.$$

The only published measurements, as far as I know, of the polarisation of the corona have been taken by Mr. Winter during the eclipse of 1871. He found, at a distance from the Sun's limb which he estimated to be about 10 minutes, $p = \cdot 26$. Mr. Winter did not observe the place of maximum polarisation. It is very improbable that this maximum was further away from the Sun than 10 minutes. Considering that Mr. Winter does not say whether his polarimeter has been properly tested, and did not actually measure the distance from the Sun, it would not be wise to attach too much importance to his numbers. I shall discuss them only to show how even a solitary observation can give us some information. Assuming the maximum of polarisation to have taken place just at the point observed by Mr. Winter, I have calculated the following data for the three laws of distribution given at the head of each column:

$p = 0\cdot 26.$

	r^0	r^{-2}	r^{-4}
$\frac{I_0 + I_1}{J}$	·97	·58	·50
$\frac{dA}{dc}$	+ ·23	− ·09	− ·07
$\frac{1}{J}\frac{dJ}{dc}$	−·37	−1·55	−2·75.

We see that if the density of the scattering matter was constant in the corona, Mr. Winter's observations would show that

over nine-tenths of the light was due to scattering matter. If the density of the scattering matter varied inversely as the square of the distance from the Sun's centre, only about one-half of the light would be due to that cause. If we assume the first law to be true, the light not due to scattering particles would increase at an almost impossible rate as we go away from the Sun, while it would decrease if we assume the second or third law to be true. The values given in the last row might have been observed, and would have told us which of the three laws agreed most nearly with observed facts. At present we can only say that the first law is very improbable, as the natural light can hardly increase with the distance from the Sun, while the density of the scattering particles remains constant. Considering that the maximum of polarisation was very likely nearer the Sun than we have assumed, we shall not be far wrong in concluding that the intensity of the scattered light was not much more than one-half of that of the total light during the eclipse of 1871.

Strictly speaking, the observations ought to be made for all different colours; but this will be hardly possible during the short time available in an eclipse. As, however, the scattered light will have a different composition from the light due to other causes, it would be of the utmost importance to get an idea of the composition of light in the solar corona. An instrument invented by Mr. Diro Kitao, and called by him 'Leukoskop,' will be very useful for this purpose.

7. Numerical Calculations.

In order to calculate the two components and the polarisation, if the distribution takes place according to the inverse nth power of the distance from the Sun's centre, we observe that

$$P_0 = E_1,$$

$$k^2 P_2 = \int_0^{\frac{\pi}{2}} k^2 \sin^2 \chi \, \Delta\chi \, d\chi = \frac{1}{3}[k^2 E_1 + k'^2 (F_1 - E_1)],$$

and for any value of n higher than 2 we find, by partial integration,

$$(n+1) k^2 P_n = (nk^2 + n - 2) P_{n-2} - k^2 (n-3) P_{n-4}.$$

After having calculated in this way successively $k^2 P_2$, $k^4 P_4$, ... up to $k^{n+2} P_{n+2}$, we find

$$I_0 - I_1 = k^{n+1} P_{n+2},$$

$$I_0 = \frac{1}{3}[k^{n+1} P_n - 4k^{n-1} P_{n-2}],$$

and

$$\frac{d(I_0 - I_1)}{d\sigma} = -k^2 \frac{dI_0}{dk} = k^{n+2}[Q_{n+2} - (n+2) P_{n+2}].$$

By successively applying the identity

$$k^2 Q_{(n+2)} = Q_n - P_n,$$

we find

$$\frac{d(I_0 - I_1)}{dc} = (F_1 - E_1) - k^2 P_2 - k^4 P_4 - \dots k^n P_n - (n+2) P_{n+2}.$$

Similarly

$$\frac{dI_0}{dc} = -k^2 \frac{dI_0}{dk} = -k^{n+2}(Q_n + P_n) - (n-1) kI_0$$

$$= -k^2 [(F_1 - E_1) - k^2 P_2 - k^4 P_4 \dots - k^{n-2} P_{n-2} + k^n P_n] - (n-1) kI_0.$$

Table I.

Minutes of Arc.	$2g_0$	$2g_1$	q
16	1·333	1·333	0·000
19	0·742	0·359	0·516
22	0·539	0·176	0·673
25	0·414	0·099	0·761
28	0·328	0·060	0·816
31	0·267	0·039	0·853
34	0·222	0·027	0·880
37	0·187	0·019	0·900
40	0·160	0·014	0·916
52	0·095	0·004	0·951
64	0·062	0·002	0·968

Table II., giving the value of I_0.

θ	r^0	r^{-2}	r^{-4}	r^{-6}	$r^0 - r^{-2}$	$r^{-2} - r^{-4}$	$a = 38·73$ $r^0 + ar^{-2}$
15°0	0·4066	0·0136	0·0007	0·0000	0·3930	0·0129	0·9337
22·5	0·6013	0·0441	0·0048	0·0006	0·5573	0·0392	2·3074
30·0	0·7863	0·0984	0·0185	0·0038	0·6879	0·0799	4·5962
37·5	0·9590	0·1780	0·0495	0·0153	0·7810	0·1285	7·8537
45·0	1·1171	0·2804	0·1053	0·0439	0·8367	0·1751	11·9769
52·5	1·2593	0·3992	0·1891	0·0993	0·8600	0·2102	16·7214
60·0	1·3841	0·5255	0·2973	0·1865	0·8586	0·2282	21·7378
67·5	1·4904	0·6486	0·4190	0·2997	0·8418	0·2296	26·6102
75·0	1·5765	0·7567	0·5368	0·4210	0·8198	0·2199	30·8821
82·5	1·6386	0·8370	0·6289	0·5215	0·8016	0·2080	34·0547
90·0	1·6667	0·8722	0·6694	0·5664	0·7945	0·2028	35·4457

Table III., giving the values of $I_0 - I_1$.

θ	r^0	r^{-2}	r^{-4}	r^{-6}	r^0-r^{-2}	$r^{-2}-r^{-4}$	$a = 38{\cdot}73$ $r^0 + ar^{-2}$
°							
15	0·1981	0·0099	0·0006	0·0000	0·1882	0·0094	0·5823
22·5	0·2835	0·0309	0·0038	0·0005	0·2518	0·0272	1·4814
30·0	0·3537	0·0655	0·0136	0·0030	0·2882	0·0519	2·8901
37·5	0·4055	0·1104	0·0337	0·0109	0·2951	0·0766	4·6793
45·0	0·4370	0·1586	0·0650	0·0281	0·2784	0·0937	6·5796
52·5	0·4482	0·2015	0·1029	0·0558	0·2467	0·0985	8·2507
60·0	0·4406	0·2308	0·1387	0·0887	0·2098	0·0922	9·3802
67·5	0·4179	0·2417	0·1621	0·1164	0·1762	0·0796	9·7781
75·0	0·3857	0·2342	0·1671	0·1285	0·1515	0·0672	9·4562
82·5	0·3526	0·2150	0·1555	0·1221	0·1376	0·0595	8·6795
90·0	0·3333	0·2000	0·1429	0·1111	0·1333	0·0571	8·0793

Table IV., giving value of $\dfrac{I_0 - I_1}{I_0}$.

θ	r^0	r^{-2}	r^{-4}	r^{-6}	r^0-r^{-2}	$r^{-2}-r^{-4}$	$r^0 + ar^{-2}$
°							
15·0	0·4872	0·7292	0·8073	0·8423	0·4788	0·7247	0·6236
22·5	0·4715	0·7022	0·7777	0·8160	0·4518	0·6929	0·6420
30·0	0·4499	0·6657	0·7348	0·7685	0·4190	0·6498	0·6288
37·5	0·4228	0·6199	0·6806	0·7093	0·3779	0·5964	0·5958
45·0	0·3912	0·5656	0·6167	0·6400	0·3327	0·5349	0·5494
52·5	0·3559	0·5046	0·5443	0·5612	0·2868	0·4689	0·4934
60·0	0·3183	0·4392	0·4664	0·4758	0·2443	0·4038	0·4315
67·5	0·2804	0·3726	0·3868	0·3884	0·2093	0·3468	0·3675
75·0	0·2447	0·3095	0·3118	0·3053	0·1848	0·3054	0·3062
82·5	0·2152	0·2569	0·2472	0·2341	0·1716	0·2865	0·2548
90·0	0·2000	0·2293	0·2136	0·1962	0·1678	0·2818	0·2279

Table V.

θ	c	θ	c
15°	3·864	60·0°	1·155
22·5	2·613	67·5	1·082
30·0	2·000	75·0	1·035
37·5	1·643	82·5	1·009
45·0	1·414	90·0	1·000
52·5	1·261		

Table VI.

The numbers at the top of each compartment denote $\frac{I_0 - I_1}{I_0 + A}$.

The numbers at the bottom of each compartment denote $\frac{I_0 + I_1}{I_0 + I_1 + 2A}$.

θ	r^0	r^{-2}	r^{-4}	r^{-6}
°				
15·0	0·4612	0·714	0·807	0·8
	0·9307	0·969	—	—
22·5	0·4122	0·669	6 755	0·78
	0·8416	0·930	0·955	—
30·0	0·3427	0·6053	0·6924	0·739
	0·7125	0·8699	0·9110	0·941
37·5	0·2523	0 5224	0·6115	0·655
	0·5029	0·7872	0·8534	0·888
45·0	0·1410	0·4204	0·5131	0·559
	0·3120	0·6749	0·7742	0·824
52·5	0·0104	0·3000	0·3970	0·4457
	0·0235	0·5229	0·6624	0·7167
60·0		0·1307	0·2207	0·2644
		0·2485	0·4079	0·4879
67 5			0·1202	0·1619
			0·2666	0·3655
75·0				0·0192
				0·0538

A Comparison between the Right Ascensions and North Polar Distances of the Nautical Almanac and the General Cape Catalogue for 1880. By E. J. Stone, M.A., F.R.S.

The following paper contains a comparison between the results of my Cape Catalogue, now passing through the press, and the Right Ascensions and North Polar Distances of the stars given in the *Nautical Almanac* for 1880.

The places of the principal stars given in the *Nautical Almanac* are chiefly based upon the numerous and long-continued Greenwich observations. These places, if not entirely free from systematic errors, must have their errors confined within small limits. It appeared to me, therefore, desirable that the Cape results should be compared with these *Nautical Almanac* places before the Cape Catalogue was finally committed to the press. The

results obtained will, I think, be found sufficiently interesting to justify their publication in the proceedings of the Society.

The comparison here made is, however, by no means a favourable one for the exhibition of the accuracy obtained in the Cape work. It was no part of my plan, at the Cape, to obtain a large number of observations of the principal stars. These *Nautical Almanac* stars have therefore not been observed more frequently than the smallest stars contained in the Cape Catalogue; whilst the position of many of the *Nautical Almanac* stars has been unfavourable for accurate observation at the Cape.

All the results, whether discordant or not, have been given. The differences have been inclosed in parentheses, and not used when the *Nautical Almanac* places may, to some extent, depend upon Cape observations, or when the stars are double stars and the centre of the mass has been observed, or when the proper motions are very large and, perhaps, variable, as in the cases of *Sirius* and *Procyon*.

Comparison between the Results of the Cape Catalogue for 1880 and those of the Nautical Almanac for the same Year.

Name of Star.	*N.A.* R.A. h	m	s	Seconds of Cape R.A. s	Cape − *N.A.* s	*N.A.* N.P.D. °	′	″	Seconds of Cape N.P.D. ″	Cape − *N.A.* ″	No. of Cape Obs. N.P.D.
α Andromedæ	0	2	11·17	11·14	−0·03	61	34	19·72	20·58	+0·86	2
γ Pegasi	0	7	3·43	3·41	−0·02	75	29	1·86	0·92	−0·94	7
ι Ceti	0	13	18·71	18·75	+0·04	99	29	22·61	22·15	−0·46	8
β Hydri	0	19	25·36	25·32	(−0·04)	167	55	49·42	48·68	(−0·74)	73
12 Ceti	0	23	54·82	54·82	0·00	94	37	14·19	14·32	+0·13	9
β Ceti	0	37	33·84	33·85	+0·01	108	38	44·46	44·05	−0·41	3
δ Piscium	0	42	27·33	27·32	−0·01	83	4	6·31	6·02	−0·29	3
ε Piscium	0	56	43·01	42·95	−0·06	82	45	23·50	22·89	−0·61	5
β Andromedæ	1	3	0·97	1·07	+0·10	55	0	57·32	56·25	−1·07	2
θ Ceti	1	18	1·45	1·50	+0·05	98	48	10·75	11·21	+0·46	4
η Piscium	1	25	3·75	3·79	+0·04	75	16	23·89	24·06	+0·17	5
α Eridani	1	33	14·54	14·60	(+0·06)	147	50	48·60	49·01	(+0·41)	72
ν Piscium	1	35	11·14	11·18	+0·04	85	7	13·63	13·14	−0·49	4
ο Piscium	1	39	3·43	3·43	0·00	81	26	48·95	49·16	+0·21	3
β Arietis	1	48	0·64	0·71	+0·07	69	46	45·59	45·45	−0·14	3
α Arietis	2	0	24·59	24·56	−0·03	67	6	21·10	20·35	−0·75	5
67 Ceti	2	10	59·83	59·82	−0·01	96	58	33·88	33·40	−0·48	4
ξ^2 Ceti	2	21	46·73	46·76	+0·03	82	4	43·81	43·31	−0·50	4

Name of Star.	*N.A.* R.A.			Seconds of Cape R.A.	Cape − *N.A.*	*N.A.* N.P.D.			Seconds of Cape N.P.D.	Cape − *N.A.*	No. of Cape Obs. N.P.D.
	h	m	s	s	s	°	′	″	″	″	
γ^2 Ceti	2	37	4·97	4·98	(+0·01)	87	16	16·22	14·14	−2·08	2
σ Arietis	2	44	52·03	52·09	+0·06	75	24	48·23	46·96	−1·27	2
α Ceti	2	56	0·39	0·39	0·00	86	22	55·84	55·37	−0·47	5
δ Arietis	3	4	46·10	46·13	+0·03	70	43	41·92	41·55	−0·37	3
α Persei	3	15	45·66	45·62	−0·04	40	34	3·30	1·68	−1·62	4
o Tauri	3	18	21·43	21·37	−0·06	81	23	40·73	40·69	−0·04	2
ϵ Eridani	3	27	16·61	16·63	+0·02	99	51	56·91	55·39	−1·52	5
η Tauri	3	40	21·12	21·14	+0·02	66	16	2·70	0·53	−2·17	1
γ Hydri	3	49	6·80	6·89	(+0·09)	164	36	22·03	(23·96)	(+1·93)	23
γ^1 Eridani	3	52	25·78	25·79	+0·01	103	51	4·30	3·64	−0·66	4
A Tauri	3	57	36·09	36·01	−0·08	68	14	51·72	51·60	−0·12	4
o^1 Eridani	4	6	0·41	0·46	+0·05	97	9	6·48	6·28	−0·20	4
γ Tauri	4	12	57·90	57·87	−0·03	74	39	49·01	48·74	−0·27	3
ϵ Tauri	4	21	36·56	36·57	+0·01	71	5	14·21	13·49	−0·72	2
α Tauri	4	29	2·13	2·11	−0·02	73	44	0·43	0·29	−0·14	4
μ Eridani	4	39	30·15	30·13	−0·02	93	28	33·07	34·43	+1·36	2
ι Aurigæ	4	49	10·75	10·77	+0·02	57	1	32·54	32·38	−0·16	2
ϵ Leporis	5	0	22·82	22·82	0·00	112	31	61·12	59·62	−1·50	7
α Aurigæ	5	7	49·55	49·54	−0·01	44	7	34·21	33·61	−0·60	10
[illegible]nis	5	8	46·23	46·22	−0·01	98	20	30·56	29·62	−0·94	5
[illegible]	[illegible]	18	42·45	42·39	−0·06	61	29	45·13	43·84	−1·29	3

δ Orionis	5	25	52·58	52·57	−0·01	90	23	22·42	21·88	−0·54	5
α Leporis	5	27	26·27	26·23	−0·04	107	54	34·05	33·91	−0·14	3
ε Orionis	5	30	7·41	7·46	+0·05	91	16	48·28	48·47	+0·19	3
α Columbæ	5	35	18·27	18·22	−0·05	124	8	21·11	20·18	−0·93	7
κ Orionis	5	42	3·87	3·89	+0·02	99	42	49·38	48·47	−0·91	3
α Orionis	5	48	40·51	40·48	−0·03	82	37	1·25	0·15	−1·10	3
ν Orionis	6	0	43·23	43·24	+0·01	75	13	7·66	7·70	+0·04	2
η Geminorum	6	7	38·02	38·12	+0·10	67	27	36·34	36·88	+0·54	2
μ Geminorum	6	15	42·07	42·02	−0·05	67	25	36·29	35·82	−0·47	2
α Argûs	6	21	17·21	17·23	+0·02	142	37	50·24	50·04	(−0·20)	3
γ Geminorum	6	30	46·73	46·73	0·00	73	29	59·70	59·57	−0·13	3
ξ Geminorum	6	38	33·25	33·27	+0·02	76	58	36·27	35·12	−1·15	2
α Canis Majoris	6	39	51·58	51·38	(−0·20)	106	33	9·67	11·29	(+1·62)	6
θ Canis Majoris	6	48	36·86	36·88	+0·02	101	53	22·20	22·28	+0·08	2
ε Canis Majoris	6	53	54·59	54·51	−0·08	118	48	35·61	34·93	−0·68	4
γ Canis Majoris	6	58	19·80	19·75	−0·05	105	27	25·43	25·98	+0·55	3
δ Geminorum	7	12	57·39	57·36	−0·03	67	47	54·23	54·25	+0·02	3
β Canis Minoris	7	20	38·56	38·56	0·00	81	28	12·64	12·34	−0·30	3
α² Geminorum	7	26	56·55	56·54	−0·01	57	50	59·70	59·12	−0·58	2
α Canis Minoris	7	33	1·22	1·12	(−0·10)	84	28	8·93	7·14	(−1·79)	3
β Geminorum	7	37	58·28	58·25	−0·03	61	41	7·72	6·74	−0·98	2
ξ Argûs	7	44	14·87	14·81	−0·06	114	33	33·88	34·81	+0·93	3

Name of Star.	*N.A.* R.A.			Seconds of Cape R.A.	Cape—*N.A.*	*N.A.* N.P.D.			Seconds of Cape N.P.D.	Cape—*N.A.*	No. of Cape Obs. N.P.D.
	h	m	s	s	s	°	′	″	″	″	
6 Cancri	7	56	8·74	8·76	+0·02	61	52	14·94	14·36	−0·58	6
15 Argûs	8	2	26·02	25·96	−0·06	113	57	33·90	33·01	−0·89	6
β Cancri	8	10	0·39	0·42	+0·03	80	26	44·52	44·90	+0·38	2
η Cancri	8	25	46·04	46·06	+0·02	69	9	9·13	8·88	−0·25	3
γ Cancri	8	36	20·37	20·36	−0·01	68	6	3·34	3·19	−0·15	3
ε Hydræ	8	40	25·23	25·18	−0·05	83	8	31·37	30·91	−0·46	3
α Cancri	8	51	55·35	55·41	+0·06	77	40	43·54	43·36	−0·18	2
κ Cancri	9	1	14·81	14·81	0·00	78	50	59·78	58·48	−1·30	2
83 Cancri	9	12	16·88	16·93	+0·05	71	47	13·35	12·13	−1·22	1
ι Argûs	9	13	52·62	(52·68	+0·06)	148	46	19·07	(19·82	+0·75)	3
α Hydræ	9	21	41·37	41·37	0·00	98	8	21·80	21·57	−0·23	6
ο Leonis	9	34	44·67	44·68	+0·01	79	33	44·97	45·52	+0·55	3
ε Leonis	9	39	2·30	2·25	−0·05	65	40	27·01	26·57	−0·44	2
μ Leonis	9	45	56·15	56·10	−0·05	63	25	43·39	42·28	−1·11	2
π Leonis	9	53	52·27	52·28	+0·01	81	22	51·19	50·90	−0·29	5
α Leonis	10	1	58·80	58·78	−0·02	77	26	49·01	48·36	−0·65	3
γ^1 Leonis	10	13	21·28	21·28	0·00	69	33	7·72	7·20	−0·52	2
μ Hydræ	10	20	17·21	17·18	−0·03	106	13	28·21	27·39	−0·82	2
ρ Leonis	10	26	29·54	29·52	−0·02	80	4	36·41	34·94	−1·47	1
η Argûs	10	40	24·46	(24·54	+0·08)	149	3	14·00	(14·05	+0·05)	4
l Leonis	10	42	56·89	56·92	+0·03	78	49	13·13	11·28	−1·85	2

d Leonis	10	54	21·72	21·76	+0·04	85	44	19·48	18·92	−0·56	3
χ Leonis	10	58	49·57	49·58	+0·01	82	0	57·35	54·98	−2·37	3
δ Leonis	11	7	43·54	43·47	−0·07	68	49	9·09	9·36	+0·27	3
δ Crateris	11	13	20·50	20·46	−0·04	104	7	46·04	45·61	−0·43	3
τ Leonis	11	21	45·90	45·97	+0·07	86	28	58·46	59·72	+1·26	3
υ Leonis	11	30	48·22	48·24	+0·02	90	9	41·22	40·73	−0·49	3
β Leonis	11	42	56·26	56·24	−0·02	74	45	25·91	25·63	−0·28	3
π Virginis	11	54	43·41	43·41	0·00	82	42	60·91	58·15	−2·76	2
ε Corvi	12	3	57·29	57·21	−0·08	111	57	8·25	8·06	−0·19	3
β Chamæleontis	12	11	20·53	20·40	(−0·13)	168	38	44·29	45·19	(+0·90)	12
η Virginis	12	13	45·94	45·96	+0·02	89	59	59·43	59·34	−0·09	2
α Crucis (Mean)	12	19	56·35	56·42	(+0·07)	152	26	2·24	3·06	(+0·82)	8
δ^2 Corvi	12	23	39·50	39·39	−0·11	105	50	50·30	49·95	−0·35	2
β Corvi	12	28	4·96	5·01	+0·05	112	43	58·90	58·46	−0·44	3
γ Virginis (Mean)	12	35	34·83	34·75	(−0·08)	90	47	29·24	27·43	(−1·81)	3
δ Virginis	12	49	33·55	33·54	−0·01	85	57	1·36	0·77	−0·59	2
α Canum Venat.	12	50	24·73	24·72	−0·01	51	1	59·51	60·54	+1·03	5
ε Virginis	12	56	12·20	12·19	−0·01	78	23	43·88	43·68	−0·20	1
θ Virginis	13	3	44·21	44·23	+0·02	94	53	52·84	53·01	+0·17	7
α Virginis	13	18	52·27	52·26	−0·01	100	32	4·18	4·47	+0·29	4
ζ Virginis	13	28	34·75	34·72	−0·03	89	58	54·25	54·49	+0·24	4
τ Boötis	13	41	33·58	33·56	−0·02	71	56	40·23	41·97	+1·74	1

Name of Star.	*N.A.* R.A. h	m	s	Seconds of Cape R.A. s	Cape − *N.A.* s	*N.A.* N.P.D. °	′	″	Seconds of Cape N.P.D. ″	Cape − *N.A.* ″	No. of Cape Obs. N.P.D.
η Boötis	13	48	58·28	58·19	−0·09	70	59	60·58	59·91	−0·67	1
β Centauri	13	55	21·99	22·09	(+0·10)	149	47	35·15	35·09	(−0·06)	51
τ Virginis	13	55	32·39	32·38	−0·01	87	52	27·92	26·13	−1·79	1
α Boötis	14	10	11·30	11·23	−0·07	70	11	30·53	30·97	+0·44	4
f Boötis	14	20	52·57	52·48	−0·09	70	13	58·33	58·77	+0·44	1
ρ Boötis	14	26	39·52	39·43	−0·09	59	6	3·90	3·78	−0·12	1
α² Centauri	14	31	27·14	27·26	(+0·12)	150	20	27·99	24·19	(−3·80)	49
α Libræ	14	44	14·47	14·46	−0·01	105	32	31·68	30·97	−0·71	2
ψ Boötis	14	59	18·26	18·19	−0·07	62	35	1·20	2·46	+1·26	1
β Libræ	15	10	32·97	32·99	+0·02	98	56	20·56	21·25	+0·69	3
α Coronæ	15	29	36·46	36·40	−0·06	62	52	49·71	49·40	−0·31	1
α Serpentis	15	38	21·45	21·45	0·00	83	11	45·01	44·26	−0·75	3
ε Serpentis	15	44	50·06	50·11	+0·05	85	9	36·34	36·83	+0·49	2
β¹ Scorpii	15	58	27·61	27·62	+0·01	109	28	32·56	32·55	−0·01	4
δ Ophiuchi	16	8	3·39	3·41	+0·02	93	23	2·23	3·17	+0·94	4
γ Herculis	16	16	37·58	37·59	+0·01	70	33	50·00	50·76	+0·76	1
α Scorpii	16	22	3·05	2·97	−0·08	116	9	50·82	50·59	−0·23	13
ζ Ophiuchi	16	30	33·09	33·08	−0·01	100	19	22·03	20·99	−1·04	1
α Trianguli Aust.	16	35	58·32	58·32	0·00	158	48	15·87	16·08	+0·21	22
κ Ophiuchi	16	51	59·23	59·28	+0·05	80	26	13·58	13·74	+0·16	2
η Ophiuchi	17	3	29·75	29·78	+0·03	105	34	29·03	29·18	+0·15	7

α¹ Herculis	17	9	10·54	10·53	−0·01	75	28	18·40	18·87	+0·47	2
θ Ophiuchi	17	14	38·39	38·35	−0·04	114	52	40·03	40·35	+0·32	4
σ Ophiuchi	17	20	33·63	33·75	+0·12	85	45	14·20	15·79	+1·59	1
α Ophiuchi	17	29	21·79	21·82	+0·03	77	21	4·56	4·24	−0·32	3
β Ophiuchi	17	37	32·62	32·67	+0·05	85	22	52·67	51·72	−0·95	2
μ Herculis	17	41	45·73	45·66	−0·07	62	12	29·67	29·10	−0·57	2
72 Ophiuchi	18	1	39·60	39·63	+0·03	80	27	7·67	6·71	−0·96	2
μ Sagittarii	18	6	35·11	35·15	+0·04	111	5	19·11	18·42	−0·69	3
η Serpentis	18	15	5·99	6·00	+0·01	92	55	41·84	43·43	+1·59	4
λ Sagittarii	18	20	33·86	33·84	−0·02	115	29	12·93	10·89	−2·04	5
σ Octantis	18	24	30·43	29·62	(−0·81)	179	16	30·02	29·92	(−0·10)	170
α Lyræ	18	32	52·52	52·47	−0·05	51	19	38·36	38·81	+0·45	3
β¹ Lyræ	18	45	38·95	38·90	−0·05	56	49	33·07	32·73	−0·34	1
ε Aquilæ	18	54	10·48	10·53	+0·05	75	5	37·11	36·18	−0·93	1
ζ Aquilæ	18	59	53·56	53·62	+0·06	76	18	49·25	49·95	+0·70	2
ω Aquilæ	19	12	10·96	10·99	+0·03	78	37	11·52	11·63	+0·11	3
δ Aquilæ	19	19	26·78	26·82	+0·04	87	7	23·56	23·78	+0·22	5
h² Sagittarii	19	29	24·13	24·16	+0·03	115	8	47·86	48·89	+1·03	3
γ Aquilæ	19	40	33·28	33·25	−0·03	79	40	41·47	40·88	−0·59	4
α Aquilæ	19	44	55·68	55·67	−0·01	81	26	51·19	49·64	−1·55	5
β Aquilæ	19	49	25·09	25·09	0·00	83	53	30·76	31·02	+0·26	5
θ Aquilæ	20	5	6·73	6·77	+0·04	91	10	34·88	34·64	−0·24	3

Name of Star.	N.A. R.A. h	m	s	Seconds of Cape R.A. s	Cape−N.A. s	N.A. N.P.D. °	′	″	Seconds of Cape N.P.D. ″	Cape−N.A. ″	No. of Cape Obs. N.P.D.
α^2 Capricorni	20	11	23·67	23·69	+0·02	102	54	56·35	56·52	+0·17	3
α Pavonis	20	16	8·67	8·77	(+0·10)	147	7	2·85	3·13	(+0·28)	3
ρ Capricorni	20	22	0·77	0·83	+0·06	108	12	32·87	32·69	−0·18	4
ϵ Delphini	20	27	28·74	28·78	+0·04	79	6	12·63	13·33	+0·70	2
α Cygni	20	37	20·43	20·44	+0·01	45	8	52·43	52·20	−0·23	1
ϵ Aquarii	20	41	10·68	10·72	+0·04	99	56	2·51	2·47	−0·04	3
32 Vulpeculæ	20	49	26·73	26·67	−0·06	62	23	53·20	54·03	+0·83	1
θ Capricorni	20	59	11·98	11·96	−0·02	107	42	30·73	31·51	+0·78	3
ζ Cygni	21	7	49·70	49·69	−0·01	60	15	53·06	52·48	−0·58	2
β Aquarii	21	25	14·38	14·42	+0·04	96	5	53·81	53·41	−0·40	7
ϵ Pegasi	21	38	17·56	17·49	−0·07	80	40	28·65	28·52	−0·13	3
16 Pegasi	21	47	36·17	36·11	−0·06	64	38	20·52	18·67	−1·85	3
α Aquarii	21	59	37·11	37·16	+0·05	90	54	8·34	7·75	−0·59	10
α Gruis	22	0	39·75	39·78	(+0·03)	137	32	28·64	27·97	(−0·67)	3
θ Aquarii	22	10	29·98	30·01	+0·03	98	22	49·35	49·04	−0·31	3
γ Aquarii	22	15	27·41	27·43	+0·02	91	59	28·73	28·98	+0·25	4
η Aquarii	22	29	11·30	11·34	+0·04	90	44	8·12	7·98	−0·14	7
ζ Pegasi	22	35	28·54	28·59	+0·05	79	47	40·98	40·96	−0·02	6
λ Aquarii	22	46	21·09	21·17	+0·08	98	13	3·44	3·80	+0·36	3
α Piscis Aust.	22	51	0·93	0·93	0·00	120	15	29·15	28·20	−0·95	6
[illegible] Pegasi	22	58	46·99	46·98	−0·01	75	26	24·26	24·10	−0·16	3

γ Piscium	23	10	56·60	56·60	0·00	87	22	24·54	23·79	−0·75	3
κ Piscium	23	20	46·81	46·83	+0·02	89	24	4·82	4·61	−0·21	9
ι Piscium	23	33	46·67	46·67	0·00	85	1	26·83	26·83	0·00	9
δ Sculptoris	23	42	40·37	40·32	−0·05	118	47	37·32	37·36	+0·04	3
ω Piscium	23	53	8·94	8·95	+0·01	83	48	4·23	3·51	−0·72	7

The large error in the *Nautical Almanac* for *α Centauri* is chiefly due to the proper motion used in the *Nautical Almanac*.

The mean N.P.D's of λ *Sagittarii* from the Greenwich Catalogues 1860, 1864, and 1872 corrected, are 115° 29′ 11″·75, 12″·94, and 10·″48, respectively. The Cape Catalogue for 1860 gives 115° 29′ 11″·55. The large discordance therefore does not appear to be mainly due to error in the Cape observations.

16 *Pegasi*. There is one very discordant observation included in the Cape mean result. If this observation were rejected, the Cape N.P.D. would be 64° 38′ 19″·40, with a difference of − 1″·12 from the *Nautical Almanac*. The Greenwich results are accordant, but I can see no reason for doubting the Cape result, except to the extent mentioned.

χ *Leonis*. The Cape observations appear correct.

π *Virginis*. One of the observations was picked up through cloud. The N.P.D. appears to be about 4″ too small, and should probably be rejected.

The following are the corrections required by the *Nautical Almanac* Right Ascensions in order that they should agree with the Cape observations:—

R.A.

0^h to 6^h	6^h to 12^h	12^h to 18^h	18^h to 24^h
$+0^s\!.001$	$-0^s\!.005$	$-0^s\!.018$	$+0^s\!.010$

The clock-errors at the Cape were determined from Right Ascensions based upon the Greenwich Fundamental Right Ascensions, and the above agreement is therefore no more than might fairly be expected; but these differences agree in sign with those which I obtained, when forming the Cape Catalogue for 1860, from a discussion of the observations made during the years 1856 to 1860. It is probable, therefore, that, although very small, these differences do not arise from mere chance errors of observation.

The refractions used for the reduction of the Cape observations have been computed from the *Tabulæ Regiomontanæ*; but no change has been made in the coefficient in passing through the zenith distance 85°. The barometer used has been the standard one of the Observatory. The thermometer is the one read for the temperature of the air and is exposed in a cage, with free circulation of air, before a southern window of the Transit-Circle room. The thermometer reads too high by $0^\circ\!.55$ F. To avoid mistakes, I thought it better to enter and reduce the observations with the readings uncorrected. The refractions used in the reductions are therefore those of the *Tabulæ Regiomontanæ* reduced in the ratio of

$$0.9988 \text{ to } 1.$$

I have examined, from time to time, the accuracy with which the refractions thus computed represented the Cape observations, and have been satisfied upon the point.*

The residual errors given in the present paper will, however, afford additional information on the point.

The latitude of the Observatory adopted in the formation of the Catalogue is

$$33^\circ\ 56'\ 3''\!.41 \text{ South.}$$

If $33^\circ\ 56'\ 3''\!.41 - x$ is assumed to be the true south latitude and Tab. refra. $(1 + y)$ the true refraction, the following are the values of x which result from a discussion of all the observations above and below the Pole contained in the Catalogue.

* *See* Cape Catalogue, 1860, Introduction; Cape Observations, 1871, 1872, and 1873 and 1874.

Mean N.P.D.	
° °	″
175 to 180	$x = +0{\cdot}09 + 87\,y.$
170	$x = +0{\cdot}16 + 95\,y.$
165	$x = -0{\cdot}05 + 109\,y.$
160	$x = -0{\cdot}24 + 136\,y.$
154	$x = -0{\cdot}34 + 213\,y.$
150	$x = +0{\cdot}05 + 366\,y.$

The corrections which x and y require must therefore be very small.

The refractions of Bessel's *Tabulæ Regiomontanæ* appear to represent the Cape observations in *mean* results with considerable accuracy. So far as these colatitude equations are alone concerned, the agreement between the different values of x would be slightly improved if the correction for the index error of the thermometer had been applied, or if y had been put equal to 0·001. The changes thus introduced would, however, be unimportant, and it would appear, from what follows, very doubtful whether these changes are really required. I have, at all events, not considered it necessary to introduce the changes, and the North Polar Distances of the Cape Catalogue are therefore, as before stated, based on Bessel's mean refractions diminished in the ratio of

0·9988 to 1.

If we divide the corrections to the North Polar Distances of the *Nautical Almanac* into four groups depending upon N.P.D., we have the following corrections:—

	N.P.D. ° ° 40 to 60	N.P.D. ° ° 60 to 90	N.P.D. ° ° 90 to 105	N.P.D. ° ° 105 to 124
Correction =	″ −0·33	″ −0·35	″ −0·15	″ −0·40

These corrections are all small and there does not appear to be any systematic change in them depending upon the N.P.D. of the group. This is a further proof of the substantial accuracy of the refractions adopted, and we are justified in forming a general mean result. We have therefore for the mean result—

$$N.A.\ \text{N.P.D.} = \text{Cape N.P.D.} + 0{\cdot}31''.$$

Next, in order to test whether there is any large outstanding correction depending upon the meteorological elements, I shall give the corrections required by the *Nautical Almanac* North Polar Distances for groups of six hours of Right Ascension.

From	0^h to 6^h	6^h to 12^h	12^h to 18^h	18^h to 24^h
Correction =	$-0''{\cdot}55$	$-0''{\cdot}42$	$+0''{\cdot}08$	$-0''{\cdot}25$

Applying to these corrections the general mean correction $-0''{\cdot}31$, we have

From	0^h to 6^h	6^h to 12^h	12^h to 18^h	18^h to 24^h
Correction =	$-0''{\cdot}24$	$-0''{\cdot}11$	$+0''{\cdot}39$	$+0''{\cdot}06$

The changes in these corrections are systematic, and it appears that the complete reversion of the seasons at the northern and southern Observatories is not quite accurately allowed for in our refraction tables. The observations 0^h to 6^h R.A. were made during the dry seasons at the Cape, and the tabular refractions used are apparently relatively too large. The observations from 12^h to 18^h were made during the wet seasons at the Cape, and the tabular refractions used are apparently relatively too small. But the changes in humidity at the Cape are simultaneous with considerable changes in mean temperature.

There cannot, I should conceive, be any doubt about the systematic character of the small discordance to which I have called attention; but there will probably be differences of opinion respecting the cause.

Some astronomers will probably consider it to arise from an imperfect correction for changes of temperature, whilst others may consider it due to the neglect of any alteration of refraction as dependent upon moisture. My own opinion is that the last explanation is likely to be the true one. I was led to a similar result many years ago, and I hope before long to be able to investigate the question of the effects of humidity upon refraction.

Ephemeris for finding the Positions of the Satellites of Uranus, 1880.

By A. Marth, Esq.

Angle of position, p_0, of the major axes, major and minor semi-axes, a and b, of the apparent ellipses described by the satellites, and latitude of the Earth above the plane of their orbits.

Greenwich Noon. 1880.	p_0 °	*Ariel.* a ″	*Ariel.* b ″	*Umbriel.* a ″	*Umbriel.* b ″	*Titania.* a ″	*Titania.* b ″	*Oberon.* a ″	*Oberon.* b ″	Lat. of Earth. °
Jan. 15	14·03	15·05	1·88	20·96	2·60	34·38	4·25	45·98	5·72	−7·16
25	13·97	15·14	1·97	21·09	2·74	34·60	4·49	46·27	6·01	7·46
Feb. 4	13·91	15·21	2·07	21·19	2·89	34·76	4·74	46·49	6·33	−7·82
14	13·84	15·26	2·18	21·26	3·04	34·87	4·99	46·63	6·67	8·23
24	13·76	15·28	2·30	21·29	3·20	34·92	5·25	46·69	7·03	8·66

Greenwich Noon. 1880.	p_0 °	*Ariel.* a ″	*Ariel.* b ″	*Umbriel.* a ″	*Umbriel.* b ″	*Titania.* a ″	*Titania.* b ″	*Oberon.* a ″	*Oberon.* b ″	Lat. of Earth. °
Mar. 5	13·68	15·27	2·41	21·28	3·36	34·90	5·51	46·67	7·37	−9·09
15	13·61	15·24	2·52	21·23	3·51	34·82	5·75	46·57	7·69	9·51
25	13·54	15·18	2·61	21·15	3·63	34·69	5·96	46·39	7·97	9·89
Apr. 4	13·47	15·10	2·68	21·04	3·73	34·51	6·12	46·14	8·19	−10·22
14	13·42	15·00	2·73	20·90	3·81	34·28	6·24	45·84	8·35	10·49
24	13·38	14·89	2·76	20·74	3·85	34·02	6·31	45·49	8·44	10·69
May 4	13·36	14·76	2·77	20·56	3·85	33·73	6·32	45·11	8·45	−10·80
14	13·36	14·63	2·75	20·38	3·83	33·43	6·28	44·71	8·39	10·82
24	13·37	14·50	2·71	20·20	3·77	33·13	6·18	44·30	8·27	−10·76

Longitudes of the satellites in their orbits, reckoned from the points where they are at their greatest northern elongations.

Greenwich Noon. 1880.	*Ariel.* long. °	*Ariel.* diff. °	*Umbriel.* long. °	*Umbriel.* diff. °	*Titania.* long. °	*Titania.* diff. °	*Oberon.* long. °	*Oberon.* diff. °
Jan. 15	346·86		265·20		247·99		102·53	
		1428·40		868·69		413·48		267·35
25	335·26		53·89		301·47		9·88	
		·37		·67		·47		·35
Feb. 4	323·63		202·56		354·94		277·23	
		·34		·66		·47		·34
14	311·97		351·22		48·41		184·57	
		·31		·64		·46		·34
24	300·28		139·86		101·87		91·91	
		·29		·62		·45		·33
Mar. 5	288·57		288·48		155·32		359·24	
		·26		·61		·44		·33
15	276·83		77·09		208·76		266·57	
		·25		·60		·45		·33
25	265·08		225·69		262·21		173·90	
		·23		·60		·44		·34
Apr. 4	253·31		14·29		315·65		81·24	
		·23		·60		·45		·34
14	241·54		162·89		9·10		348·58	
		·22		·60		·46		·35
24	229·76		311·49		62·56		255·93	
		·22		·60		·47		·36
May 4	217·98		100·09		116·03		163·29	
		·23		·61		·48		·38
14	206·21		248·70		169·51		70·67	
		1428·23		868·62		413·49		267·38
24	194·44		37·32		223·00		338·05	

These values are to be interpolated for the times for which the positions of the satellites are required. The position angles, p, and distances, s, are then found by means of the equations,

$$s \sin (p_0 - p) = b \sin \text{long.}$$
$$s \cos (p_0 - p) = a \cos \text{long.}$$

Observations of Comet Palisa made with the Merz Equatoreal of the Collegio Romano. By Prof. Pietro Tacchini.

Data. 1879.	Tempo medio di Roma. h m s	α☄ − α✱ m s	δ☄ − δ✱ ′ ″	α apparente ☄ h m s	δ apparente ☄ ° ′ ″	Stelle di confronto.	Posizione delle stelle di confronto (Equinozio medio 1° Gennaio 1879). α h m s	δ ° ′ ″	Numero dei confronti.	Osservatore.	Numero di ordine.	Osservazioni sulla cometa.
3 Sett.	8 18 5	−4 44·40	− 0 32·0	11 28 38·59	+45 48 54·4	½ (22088 Lal. + 1813 Groom.).	11 33 21·27	+45 49 37·9	1	T.	1	Nucleo piccolo e debole.
6 ,,	8 13 25	+4 45·26	+ 1 38·1	11 49 2·12	44 21 25·6	165 Hora XI. Piazzi.	11 44 15·12	44 19 58·8	5	T.	2	
7 ,,	8 21 55	−0 10·30	+ 6 5·3	11 55 49·18	43 48 52·2	4057 (C. of B.A.A.).	11 55 57·82	43 42 57·4	7	T.	3	Diametro 119″.
,,	7 42 42	−0 51·18	+ 6 46·4	12 2 21·44	43 15 58·7	22799 Lalande.	12 3 10·93	43 9 23·4	4	T.	4	
9 ,,	7 51 34	+1 3·05	− 9 51·6	12 9 1·05	42 39 47·9	140 H. 12 Weisse.	12 7 56·35	42 49 50·5	3	M.	5	
10 ,,	8 44 37	+0 40·70	− 8 48·1	12 15 56·41	42 0 22·4	294 H. 12 Weisse.	12 15 14·06	42 9 21·5	2	M.	6	La cometa passò colla nebulosità sopra una stella di 10° gr. senza alterarne lo splendore.
12 ,,	8 21 8	−1 9·50	+19 30·6	12 28 48·42	40 40 24·8	½ (613 H. 12 Weisse + 23557 Lalande).	12 29 56·26	40 21 5·2	1	M.	7	
13 ,,	8 36 20	+2 13·80	− 1 24·6	12 35 14·26	39 57 20·7	23646 Lalande.	12 32 58·81	39 58 56·3	2	M.	8	
14 ,,	9 7 44	−2 17·61	+10 19·2	12 41 40·44	39 11 48·0	23930 Lalande.	12 43 56·41	39 1 39·5	3	M.	9	
18 ,,	7 49 14	−3 54·45	− 3 8·5	13 5 40·63	36 1 51·6	143 H. 13 Weisse.	13 9 33·48	36 5 10·7	4	M.	10	
19 ,,	8 6 48	+1 35·83	+20 15·8	13 11 39·76	35 9 33·2	156 H. 13 Weisse.	13 10 2·24	34 49 28·9	4	M.	11	
22 ,,	7 46 43	+0 8·50	+18 11·9	13 28 29·44	32 28 16·5	2360 Z. 32° Arg.	13 28 19·29	32 10 15·1	2	M.	12	
24 ,,	7 44 32	−3 45·50	+ 7 51·6	13 39 18·35	30 35 33·2	886 H. 13 Weisse.	13 43 2·19	30 27 51·0	3	M.	13	
2 Ott.	6 56 35	+0 30·26	−11 54·8	14 19 50·56	22 37 19·7	½ (361 H. 14 Weisse + 4679 H. 14 Rümker).	14 17 48·09	22 49 24·0	7	T.	14	
3 ,,	7 12 16	+2 4·05	− 1 39·2	14 22 51·33	31 36 42·4	2651 Z. 21° Arg.	14 20 45·70	21 38 31·0	6	T.	15	Diametro 109″.
5 ,,	7 9 7	+1 48·70	− 9 20·8	14 31 29·79	19 35 12·6	½ (26620 Lalande + 606 H. 14 Weisse).	14 29 39·27	19 44 42·7	4	T.	16	
5 ,,	7 9 7	+1 37·72	−11 43·5	14 31 29·94	19 35 14·3	26626 Lalande.	14 29 50·40	19 47 7·1	4	T.	17	
[illegible] ,,	7 35 24	−0 36·02	+ 8 40·1	14 35 45·68	18 33 5·5	746 H. 14 Weisse.	14 36 19·62	18 24 33·8	4	T.	18	
	[illegible] 35 24	−1 40·17	− 1 46·8	14 35 46·23	18 33 20·4	772 H. 14 Weisse.	14 37 24·32	18 35 15·6	4	T.	19	
		[illegible]·00	+ 5 13·4	14 39 44·73	17 33 46·8	4873 (C. of B.A.A.).	14 39 35·87	17 28 42·6	5	T.	20	
			[illegible]	14 39 50·09	17 32 33·8	,, ,,	14 39 35·87	17 28 42·6	5	T.	20	Diametro 103″.

8	,,	6 47 58	+1 1·82	− 6 1·2	14 43 47·09	16 32 55·1	26982 Lalande.	14 42 43 39	16 39 5·5	5	T.	21
8	,,	7 6 21	+1 5·34	− 6 45·0	14 43 50·61	16 32 11·3	,, ,,	14 42 43·39	16 39 5·5	5	T.	21
9	,,	6 47 40	+0 28·02	− 5 38·6	14 47 45·03	15 32 2·1	27114 ,,	14 47 15·11	15 37 49·9	4	T.	22
9	,,	6 47 40	−1 36·52	+ 3 59·4	14 47 44·04	15 32 6·4	27167 ,,	14 49 19·06	15 28 16·2	4	T.	23
9	,,	7 30 48	+0 35·15	− 7 9·1	14 47 52·16	15 30 31·6	27114 ,,	14 47 15·11	15 37 49·9	4	T.	22
9	,,	7 30 48	−1 29·87	+ 2 20·9	14 47 51·09	15 30 28·0	27167 ,,	14 49 19·06	15 28 16·2	4	T.	23
10	,,	6 41 30	−0 58·50	+ 0 39·1	14 51 36·25	14 31 56·1	27283 ,,	14 52 32·83	14 31 26·4	5	T.	24
10	,,	7 4 45	−0 54·58	− 0 12·3	14 51 40·17	14 31 4·7	27283 ,,	14 52 32·83	14 31 26·4	5	T.	24
11	,,	7 7 50	−2 45·22	− 3 46·7	14 55 29·92	13 30 56·6	1079 Weisse H. 14	14 58 13·20	13 34 52·9	6	T.	25
12	,,	6 44 18	−0 29·77	+ 7 8·7	14 59 9·74	12 32 31·3	1100 ,, ,,	14 59 37·55	12 25 33·3	3	T.	26
12	,,	6 44 18	−0 35·43	+13 6·2	14 59 9·61	12 32 40·2	1102 ,, ,,	14 59 43·08	12 19 43·7	3	T.	27
12	,,	7 11 20	−0 26·20	+ 5 57·2	14 59 13·31	12 31 20·8	1100 ,, ,,	14 59 37·55	12 25 33·3	3	T.	26
12	,,	7 11 20	−0 31·87	+11 56·1	14 59 13·17	12 31 30·1	1102 ,, ,,	14 59 43·08	12 19 43·7	3	T.	27
13	,,	6 49 54	+1 54·74	+ 1 11·5	15 2 51·03	11 33 3·9	1125 ,, ,,	15 0 54·29	11 32 2·2	4	T.	28
13	,,	6 49 54	+0 27·60	+10 26·2	15 2 50·45	11 33 7·6	1162 ,, ,,	15 2 30·85*	11 22 51·2	4	T.	29
13	,,	7 16 25	+1 58·80	+ 0 10·8	15 2 55·09	11 32 3·2	1125 ,, ,,	15 0 54·29	11 32 2·2	4	T.	28
13	,,	7 16 25	+0 31·82	+ 9 26·4	15 2 54·69	11 32 7·8	1162 ,, ,,	15 2 30·85	11 22 51·2	4	T.	29
14	,,	6 41 25	+0 26·85	− 6 16·8	15 6 27·01	10 33 56·5	27692 Lalande.	15 5 58·17	10 40 23·1	6	T.	30
14	,,	6 41 25	+0 35·97	− 0 51·2	15 6 25·97	10 33 56·9	27684 ,,	15 5 48·01	10 34 57·9	6	T.	31
14	,,	6 41 25	−3 20·60	− 0 22·0	15 6 26·15	10 34 10·0	27814 ,,	15 9 44·76	10 34 41·8	6	T.	32
17	,,	6 33 15	+0 7·72	− 3 32·5	15 16 49·09	7 42 49·0	270 H. 15 Weisse.	15 16 39·33	7 46 30·7	5	T.	33
17	,,	6 54 17	+0 10·54	− 4 9·2	15 16 51·91	7 42 12·3	270 ,, ,,	15 16 39·33	7 46 30·7	5	T.	33
19	,,	6 33 45	+1 1·76	− 7 11·8	15 23 28·91	5 51 13·5	391 ,, ,,	15 22 25·10	5 58 34·3	4	T.	34
19	,,	6 53 57	−0 57·48	+ 9 36·4	15 23 32·05	5 50 15·4	425 ,, ,,	15 24 27·48	5 40 47·6	4	T.	35
[illegible]	,,	6 37 47	−0 55·75	+11 39·2	15 33 15·27	3 8 35·4	624 ,, ,,	15 34 8·94	+2 57 4·6	4	T.	36

* La stella 29 che è la 1162 H. 14 Weisse deve essere diminuita probabilmente di 10ˢ.
Le posizioni della cometa devono essere corrette di parallasse.
La lettera T. indica Tacchini; la lettera M. indica Millosevich; le riduzioni si fecero da quest' ultimo.

[Ro]ma, *Ottobre* 1879

Observations of Comet Hartwig made with the Merz Equatoreal of the Collegio Romano. By Prof. Pietro Tacchini.

Data.	Tempo medio di Roma.	α☄ − α✱	δ☄ − δ✱	α apparente ☄	δ apparente ☄	Stelle di confronto.	Numero dei confronti.	Osservatore.	Numero di ordine.	Osservazioni sulla cometa.
1879.	h m s	m s	′ ″	h m s	° ′ ″					
9 Sett.	8 28 40	−2 33.60	−11 37.5	13 46 50.94	+37 28 27.5	25625 Lalande	3	M.	1	Debolissima; forma ovale.
11 „	8 3 4	+0 13.55	− 2 25.4	13 52 4.11	34 59 1.6	1122 H. 13 Weisse.	4	M.	2	
11 „	8 38 43	+0 17.77	− 4 25.6	13 52 8.33	34 57 1.4	1122 H. 13 Weisse.	2	M.	2	
12 „	9 18 19	−0 31.90	− 9 32.1	13 54 33.09	33 42 48.4	1186 H. 13 Weisse.	1	M.	3	
13 „	7 53 28	−0 4.20	+ 8 8.5	13 56 42.88	32 36 36.9	2422 Z. 32° Arg.	4	M.	4	
14 „	8 28 16	−2 14.42	+ 0 66.0	13 58 51.80	31 26 51.9	4694 (C. of B.A.A.)	4	M.	5	Oggetto difficile ad osservare.

Equinozio medio 1° Gennaio 1879.

Stelle di confronto.

		h	m	s		°	′	″
1	α	13	49	22.70	δ	+37	40	11.0
2			51	48.84		35	1	33.6
3			55	3.26		33	52	27.1
4			56	45.35		32	28	35.1
5			61	4.48		31	25	52.7

Le posizioni della cometa non sono corrette di parallasse. La lettera M significa Millosevich.

Roma, Ottobre 1879.

On the Correction for Personality required by the Observations of the Moon made with the Greenwich Transit-Circle.

By E. Neison, Esq.

During the course of my investigation of the corrections required by the theory embodied in Hansen's Lunar Tables in order to bring the tabular place of the Moon into accord with observation, I have most carefully compared the corrected theory with the observations of the Moon made with the Greenwich Transit-Circle. The result of the comparison is decidedly satisfactory when the error of the Moon's mean longitude is deduced separately for each year, showing that the theory of the terms of short period has been rendered accurate. It remains to compare the errors of mean longitude deduced for each year from the observations, with the view of deducing the requisite corrections to the theory of the terms of long period. This portion of the investigation has not yet been completed, though the discordance between theory and observation has been already materially reduced by the introduction of several terms of long period, whose existence has been discovered by my theoretical researches. During this part of the investigation I have discovered several sudden changes in the error of mean longitude, which were too sudden to arise from any defect in the theory, and too large and too marked to be merely accidental. Lately, by a rigorous examination of the observations, I have convinced myself that these sudden changes do not arise from any imperfections in the theory, but must arise from some cause within the Observatory itself.

The observations of the Moon with the Transit-Circle at Greenwich appear to be, as far as possible, equally divided amongst four of the Assistants, though occasionally observations are made by others. As a rule, however, about five-sixths of the total number of observations are made by four of the Assistants. It might therefore be very fairly assumed that the mean of the observations made by these four observers would be to all intents independent of any systematic error due to personality on their part, as it is known that such personality is not large in any one case. Therefore, so far as I am aware, hitherto everyone who has investigated the theory of the Moon has assumed that the error of mean longitude derived from a year's observation might be held to be independent of any personality on the part of the observers, and to be comparable with the error deduced from the observations of any other year.

Some time back it was suggested to me that the personal error of the Greenwich observers might not be constant, and that therefore the mean error derived from their observations might vary from year to year. From some notes I had of a paper by

Mr. Dunkin, entitled 'On Personality in observing Transits of the Limbs of the Moon" (*Monthly Notices*, 1869, vol. xxix. page 259), I was aware that the personal equations of the lunar observers were tolerably constant and in no case large, so that it did not seem likely that any such variation would occur. To satisfy myself that this view was correct, I compared the mean of all the observers for each year with the mean for the entire period of six years, using the data given in my notes of Mr. Dunkin's paper. The result was entirely satisfactory, the corrections to be applied to each year's observation to reduce them to the same standard being as under:—

$$
\begin{aligned}
1863 &= -0^{s}\!\cdot\!010 \\
1864 &= +0\cdot003 \\
1865 &= -0\cdot010 \\
1866 &= +0\cdot010 \\
1867 &= 0\cdot000 \\
1868 &= +0\cdot008
\end{aligned}
$$

These quantities were smaller than the probable error of their determination, and were themselves so small that they could be neglected.

When, therefore, my investigations showed that these variations, or rather sudden changes, in the error of mean longitude were probably due to some cause within the Observatory itself, it was difficult to imagine what could be the real source of these variations. There had been no change in the method of reduction, so that it could not arise from that cause. Further investigation showed that the observations with the altazimuth were affected by the same sudden changes, though to a modified amount; it was evident, therefore, that the origin of these changes was not in the instrument itself. Everything pointed to its arising from some effect of the personality on the part of the observers. Yet this conclusion seemed negatived by the results already referred to, which I had obtained some years previously. I resolved, however, to investigate anew this question.

I knew that in his paper in the xxix. volume of the *Monthly Notices*, Mr. Dunkin had arrived at the conclusion that observers had a very different personal equation for each limb of the Moon, but that for each limb this personal equation was almost constant. My first step was, therefore, to examine Mr. Dunkin's paper to see what light it might throw on this subject. It appeared that Mr. Dunkin had based his investigation on the assumption (*loc. cit.* page 263) that "the mean tabular errors are assumed to be constant throughout the mean lunation." It is now known that this assumption is not a permissible one, for the tabular

errors are not constant throughout the mean lunation; but that, owing to imperfections in the theory, they are systematically different before and after Full Moon. My own investigations have shown that this error varies during the year to such an extent that the difference in the Moon's tabular error before and after opposition will vary from nearly 8″ in one part of the year to less than 1″ in another part. For this reason, therefore, the conclusions which have been arrived at by Mr. Dunkin in his paper will require material modification.

The data which has served as the basis of my investigation has been the Greenwich Transit-Circle observations of the Moon made during the fourteen years between 1863 and 1876. Each Assistant's observations were separately extracted from the different volumes of *Greenwich Astronomical Results* and discussed by themselves, the observations of the I and II limbs being kept separate. As far as possible without separately calculating the corrections for each observation, the main errors of short period in the theory were eliminated, and it was assumed that the smaller errors would practically destroy each other in the mean result. Next, the results obtained for each observer were referred to one observer, who was taken as the standard. For this purpose Mr. Criswick was selected, for the reason that his observations alone extended throughout the entire period of fourteen years. Fortunately it also happens that his observations are the most numerous.

In this manner an approximate value of the personal equation of each observer was obtained for each year. The results were perfectly analogous to the similar results given by Mr. Dunkin, in his paper already quoted, and they clearly showed that, not only was the personal equation of each observer small, but that it was nearly constant in each case. Much of the deviation from the mean could be traced to the effect of the errors in the theory which had been left to correct themselves. The result for each year was weighted in proportion to the number of observations on which it rested, some slight weight being also given to the equable distribution of the observations. Then, from the different results for each of the fourteen years, the following values were obtained for the mean personal equation of the observers:—

		For I Limb.		For II Limb.	
		s	Weight.	s	Weight.
Dunkin — Criswick	= D − C =	−0·012	4	−0·032	3
Ellis — Criswick	= E − C =	−0·106	9	−0·065	7
J. Carpenter — Criswick	= JC − C =	−0·125	8	−0·038	6
Lynn — Criswick	= L − C =	+0·034	4	+0·058	3
A. Downing — Criswick	= AD − C =	+0·007	2	+0·109	2
Thackeray — Criswick	= T − C =	+0·040	1	+0·093	½

Substituting these values in the original expressions, it was found that they well satisfied them, the residuals being reduced to small quantities, in the greater number of cases less than ^{s}·020.

The next step is to refer all the observations to some standard which will be more independent of the variations due to the imperfect elimination of the errors of the lunar tables and the chance errors of observation than can be the case with the mean of the observations which any selected observer may make in a year. It is evident that, by merely referring the results to the mean of the observations obtained by the selected observer in the year, the entire deviation of that mean from the true value of the personal equation of the observer is thrown on all the values for the personal equations of the other observers as determined from the observations of the year, creating in them merely fictitious variations. If it were possible to determine the absolute personal equation of Mr. Criswick, it would be possible to completely eliminate from the observations the effect of the personalities of the observers. This cannot be done from the Greenwich observations alone. Some standard must be assumed, therefore, and it is immaterial what standard be chosen in so far as the accuracy of the results is concerned. The most convenient standard would be to assume that the mean of all the personal equations of the seven observers is free from error. However, as the materials for determining the personal equation of two of the observers, Messrs. Downing and Thackeray, are very imperfect, it has been assumed that the sum of the personal equations of Messrs. Dunkin, Ellis, Criswick, J. Carpenter, and Lynn will be zero, and the mean of their observations correspond to the true place of the Moon.

The observations of each observer for each year were then referred to the mean error for the year derived from all the observations, so as to eliminate as far as possible the outstanding errors. The total number of observations was divided into four groups, 1863–69; 1870–72; 1873–74; 1875–76; in each of which the main observations of the Moon were made by four Assistants. Each of these groups was marked out from the others by the fact that in it the place of one of the observers of the preceding or following group was taken by another observer It was then assumed that each of these groups needed a cor rection, a_1 a_2 a_3 a_4 respectively, to bring the mean of tb observations in the group into accord with the selected standa· Then these corrections were determined by equating the m results of the three observers who observed in both groups, correcting each for proper personal equation.

In this manner the following values were obtained fo personal equations of the different observers for each limb Moon. The probable errors were obtained from the resid

		I Limb.		II Limb.	
		s	s	s	s
Dunkin	=	+0·025	±0·012	+0·004	±0·014
Ellis	=	−0·060	±0·009	−0·041	±0·010
Criswick	=	+0·045	±0·008	+0·026	±0·009
J. Carpenter	=	−0·074	±0·010	−0·014	±0·012
Lynn	=	+0·075	±0·015	+0·066	±0·020
A. Downing	=	+0·006	±0·022	+0·132	±0·026
Thackeray	=	+0·063	±0·025	+0·183	±0·050

These personal equations are unmistakably real, as is clearly shown by the small probable errors, which show how small are the outstanding residuals which they leave. Where the same four observers have been engaged, and where there can be no uncertainty as to the values of the quantities a_1, a_2, a_3, a_4 to interfere with the accuracy of the results, the differences between the mean of all observers and the mean of each are remarkably constant and uniform, showing clearly that they are due to real personal equations, and not merely to outstanding errors. Moreover, these personal equations are quantities whose variations must be due either to purely accidental errors which may have remained uneliminated, or to a real change in the personal equation of the observer. There seems little indication of the presence of any real change. Therefore the variations are presumably accidental, and of the nature to which may be most safely applied the theory of probable errors.

Owing to the much smaller number of observations, and the shorter period over which they extend, there must be a considerable uncertainty as to the accuracy of the values which have been assigned to the personal equations of Messrs. Downing and Thackeray, and especially of the latter. This will, of course, disappear to all probability when the investigation can be extended to the observations which have been made during the years 1877, 1878, and 1879. I believe, however, that the true values of the personal equations of these observers will be found to be included within the limits of the probable errors assigned to them.

The consideration of these personal equations leads to a very important result. Of the four observers—Messrs. Dunkin, Ellis, J. Carpenter, and Criswick—who observed the Moon from 1863 to the end of 1869, no less than three of them—the three first—were replaced by other observers during the five years 1870–74. But whereas the three retiring observers had personal equations whose mean value was largely negative, they have been succeeded by three observers whose personal equations are all largely positive. This change of observers has therefore introduced a considerable increase in the mean tabular error of the Moon which has no real existence. Each replacement of one observer by

another—a change which has practically come into operation at the beginning of a year—has been accompanied by a sudden increase in the tabular error of the Moon, an increase which has no foundation in fact. Before, therefore, the errors in the Moon's mean longitude can be properly investigated from observation, it is necessary to remove these fictitious increases and reduce the entire series of observations to one systematic standard. From the values which have been determined for the personal equations of the seven observers, it is easy to calculate the correction which should be applied to each year to reduce the errors in the Moon's Right Ascension to a uniform standard. They are—

	I Limb.	II Limb.	Centre.
	s ″	s ″	s ″
1863	+0·001 = +0·015	−0·002 = −0·030	0·000 = 0·000
1864	+0·017 = +0·255	−0·003 = −0·045	+0·009 = +0·135
1865	+0·007 = +0·105	−0·010 = −0·150	0·000 = 0·000
1866	+0·016 = +0·240	−0·006 = −0·090	+0·007 = +0·105
1867	+0·015 = +0·225	−0·009 = −0·135	+0·007 = +0·105
1868	+0·020 = +0·300	−0·008 = −0·120	+0·008 = +0·120
1869	+0·020 = +0·300	−0·006 = −0·090	+0·007 = +0·105
1870	+0·013 = +0·195	−0·012 = −0·180	+0·002 = +0·030
1871	−0·011 = −0·165	−0·020 = −0·300	−0·018 = −0·270
1872	−0·004 = −0·060	−0·017 = −0·255	−0·009 = −0·135
1873	−0·014 = −0·210	−0·040 = −0·600	−0·025 = −0·375
1874	−0·005 = −0·075	−0·054 = −0·810	−0·029 = −0·435
1875	−0·041 = −0·615	−0·081 = −1·215	−0·057 = −0·855
1876	−0·047 = −0·705	−0·079 = −1·185	−0·058 = −0·870

These results clearly show the unexpected effect of the change in the observers at Greenwich, there being a change of 1″ in the mean error of longitude in the short space of five years. The variation is of such unexpected magnitude, that it will be necessary to more rigidly examine the observation, correcting each individual observation for the errors of short period in the lunar theory, and taking into consideration the observations for the years 1877 and 1878. For this purpose, in my discussion of these observations, I shall have to introduce seven more undetermined coefficients to represent the personal equations of the seven Greenwich observers, and to be determined simultaneously with the errors in the mean longitude and coefficients of long period employed by Hansen in his Tables. In the meantime it appears to be important to make known the unexpected result of the present investigation.

Note on the Correction to the Mean Longitude of Hansen's Lunar Tables. By Prof. S. Newcomb.

In vol. xxxix. No. 9, of the *Monthly Notices* Mr. Lynn shows that the correction to the mean longitude of the Moon has apparently diminished during the last two years, and concludes that the mean error is probably still diminishing. I wish to call attention to the fact that the fluctuations in the course of the error may be accounted for by the seventeen-year term due to the action of *Jupiter*, computed by Mr. Neison, namely—

$$\delta l = 2''.2 \sin (2\varpi - 2J),$$

ϖ being the longitude of the Moon's perigee and J the mean longitude of *Jupiter*.* Substituting for ϖ and J their values in terms of the time, the correction becomes

$$\delta l = 2''.2 \sin \{293^\circ + 20^\circ.66 (t - 1870.5)\}.$$

The dates of its maxima and minima are as follows:—

Maximum, of +2″.2	1860.2,	1877.6.
Minimum, of −2.2	1868.9,	1886.3.

It is therefore to be expected that the positive error of the Tables would reach a minimum about 1878.0.

I have taken the corrections to Hansen's mean longitude given in my *Researches on the Motion of the Moon*, p. 268, interpolated them to the middle of each year from 1876 to 1885, and applied Mr. Neison's inequality. The results are:—

Year.	Long Period Correction. ″	Neison's Correction. ″	Sum. ″
1876.5	− 9.4	+2.0	− 7.4
77.5	− 9.9	+2.2	− 7.7
78.5	−10.4	+2.1	− 8.3
79.5	−10.9	+1.7	− 9.2
80.5	−11.5	+1.1	−10.4
81.5	−12.0	+0.3	−11.7
82.5	−12.6	−0.4	−13.0
83.5	−13.2	−1.1	−14.3
84.5	−13.8	−1.7	−15.5
85.5	−14.4	−2.1	−16.5

* See *Monthly Notices*, vol. xxxvii. pp. 359 and 428.

Mr. Lynn's results for 1878 were:—

	″
From Meridian observations	−8·23
„ Altazimuth „	−7·48

They are as nearly accordant with the empirical theory as could be expected. The errors in 1876 are apparently 2″ greater than theory. This may be due in part to inequalities not yet discovered, but is more likely to arise from systematic errors of a personal character in noting the times of transit of the Moon's limb. For several years preceding 1876 the Moon's R.A. as observed at Greenwich and Washington was some 1″ or 2″ greater than that derived from occultations, and from the meridian observations to other Observatories. Hence arose the constant difference between the corrections for 1875·0 found by Captain Tupman from observations at a number of Observatories and by myself from Greenwich and Washington alone. The occultations seem to show that the constant error was in my own results.

On the whole, I do not yet see any good reason to believe that the course of the errors during the next six years will be materially different from that above given. It would be a matter of interest if the few Observatories where the Moon is systematically observed would, after the end of each year, publish the mean correction to the Moon's longitude derived from the observations of the year.

On Recent Changes in the Mean Error of Longitude of Hansen's Lunar Tables.

By W. T. Lynn, B.A.

The critical point apparently reached in the mean error of longitude of the Moon, as assigned by Hansen's Tables, has led me to examine its circumstances a little more minutely than in my previous paper printed in the last *Supplementary Notice*; and, by the permission of the Astronomer Royal, I beg to offer the Society the following results of that examination.

It will have been noticed that the diminution of the error in 1878, to which I called attention as above, is shown by the observations made both with the meridian and altazimuth instruments. But in what follows I have discussed the altazimuth observations only, which are not only more numerous, but distributed through a larger part of each lunation than those made with the Transit-Circle.

My previous paper left it doubtful whether the turning point was in the year 1876 or 1877. I have now compared the mean error of longitude from the altazimuth observations in 1877,

which are 182 in number, and find it to be $+8''\cdot25$. (I may take the opportunity of mentioning that I detected a mistake in the result of the observation as printed (and shortly to be published) on February 26 for that year, when the error in longitude should be $+10''\cdot90$ instead of $+11''\cdot79$.) As this mean error was $+9''\cdot31$ in 1876, it is evident that it had reached a *maximum* in that year.

Now, for the purpose of ascertaining what the amounts of fluctuation of the error in the course of separate years were, I have put together the number of days in 1877 and 1878 in which the error of longitude was contained between each pair of consecutive seconds of its whole range. The following Table exhibits the result of that comparison:—

Error of Longitude.				No. of Days. in 1877.	No. of Days. in 1878.
	$''$		$''$		
Between	−4	and	−3	2	0
,,	−3	,,	−2	0	1
,,	−2	,,	−1	0	3
,,	−1	,,	0	1	2
,,	0	,,	1	2	4
,,	1	,,	2	5	7
,,	2	,,	3	9	3
,,	3	,,	4	7	7
,,	4	,,	5	15	10
,,	5	,,	6	10	14
,,	6	,,	7	18	18
,,	7	,,	8	14	17
,,	8	,,	9	17	20
,,	9	,,	10	19	17
,,	10	,,	11	12	13
,,	11	,,	12	22	9
,,	12	,,	13	11	7
,,	13	,,	14	11	1
,,	14	,,	15	2	3
,,	15	,,	16	2	1
,,	16	,,	17	0	2
,,	17	,,	18	1	1
,,	18	,,	19	1	1
,,	19	,,	20	0	0
,,	20	,,	21	1	0

It results from this comparison that in 1877 the proportion 0·70, and in 1878 the proportion 0·73 of the whole of the errors

were contained between the limits $+4''$ and $+12''$; also that in neither was there more than a single observation, the error from which was included between two seconds of which the smallest was $+17''$.

In 1878, it will have been remarked that the number of altazimuth observations is considerably under the average, and the same will undoubtedly be the case for the current year also, the state of the sky for more than a year past having been very unfavourable for observations. An examination of the observations hitherto made during 1879 (embracing, of course, now by far the largest part of the year) shows* that the mean error in longitude is larger than in 1878, and about the same as in 1877; also that it has been increasing during the year, and that the mean value for the first three quarters (January to September) is about $+8''{\cdot}9$.

I may mention that some time ago I compared the mean error of longitude for several years as resulting from the observations made in the different quarters of each lunation. In no case did I find any appreciable difference in the mean error for the first and second, or for the third and fourth quarters of each of different lunations. There was, indeed, a decided difference between the mean error in the first and second *halves* of the lunation, or the error from observations made before and after the Full Moon. But this may, of course, arise in great measure from the difference in the method of observing the first and the second limb of the Moon respectively; on which there is a well-known paper by Mr. Dunkin in the *Monthly Notices* for 1869, April 9 (vol. xxix. page 259), in which he shows how this varies for different observers.

Recently I have made a fresh determination of the apparent diameter of the Moon as resulting from the altazimuth observations from 1862 to 1878. This too varies very much for different observers; but the mean result seems to show that the correction to the semi-diameter adopted by Hansen, as used in the *Greenwich Observations* since 1863, is somewhat too large, both in azimuth and zenith distance. A change in the former, however, would make but little difference in the mean result; because the number of observations of the first and second limbs respectively in each year is very nearly the same. The semi-diameter used in zenith distance is more effective in the mean result; since, on an average, about three times as many lower as upper limbs of the Moon are observed. But then the mean

* The results for the first three quarters of the year (all now reduced) are as follows:—

		No. of Obs.	Mean Error in Longitude. ″
1879	January to March	28	+8·25
„	April to June	42	+8·69
„	July to September	47	+9·48

error of zenith distance produces a comparatively small effect on the mean error of longitude.

Blackheath,
1879, *Nov.* 24.

Addendum.

By the request of Mr. Neison, I have calculated the mean error in longitude for 1878 in two divisions—up to June 1 and after that date. Before June 1 there were 61 observations made with the altazimuth, of which the resulting mean error of longitude is $+7''.54$; after June 1 there were 100 observations, and the corresponding resulting mean error is $+7''.44$. So that the observations in the earlier and later part of that year give results almost identical with each other.

A question having been raised as to how far the changes in the mean error may have been affected by changes in the observers, and in their personality in observing, I have also put together the results of the observations of Mr. Criswick alone, who has been for several years the standard observer at the Royal Observatory, and whose observations extend regularly through the whole of the period during which Hansen's Tables have been used. The following are the resulting mean errors of longitude from the Transit-Circle observations made by him on the separate years specified:—

Year.	No. of Observations.	Mean Error of Longitude by Transit-Circle. ″
1862	15	−2·70
1866	21	+2·94
1870	21	+4·61
1874	25	+8·80
1878	18	+7·74

1879, *Dec.* 22.

Note on the North Polar Distances of the Greenwich Seven-Year Catalogue for 1860. By A. M. W. Downing.

In a communication to the Society at the November Meeting Mr. Stone has given the results of a comparison between the North Polar Distances of his great Catalogue and those of the *Nautical Almanac*; and has deduced the remarkable result that the adopted refractions are too large for the observations made at the Cape during the dry season in November, December, and January, and too small for the observations made at the opposite season of the year. I have examined my comparison of the

Cape and Greenwich Catalogues for 1860, published in the *Monthly Notices* for 1878, December, in order to ascertain whether any similar effect of the season of the year on the refraction can be detected in the Greenwich observations. For this purpose I have arranged in order of R.A. the stars used in the comparison which are situated between N.P.D. 110° and the horizon of Greenwich, as it is reasonable to suppose that for these stars (which pass the meridian of the Cape within 15° of the zenith) any effect that can be traced to refraction is due to the Greenwich, rather than to the Cape observations.

The quantity $-0''.18$ has been applied to the Greenwich places of all the stars used, this being the systematic difference of the Catalogues between the limits of N.P.D. under consideration.

The result then is:—

Right Ascension	0^h—6^h	6^h—12^h	12^h—18^h	18^h—24^h
Corr. to Greenwich	$-0''.10$	$-0''.04$	$+0''.26$	$-0''.11$

The only one of these mean differences worth attention is probably that for 12^h–18^h, and it appears to show that (if the discordance may be considered as due to refraction), from error in temperature correction, or other causes, the refractions used in the reduction of the Greenwich observations are relatively too small during the months of May, June, and July. It will be remarked that the discordance at the opposite season, included within the limits of R.A. 0^h–6^h, is so small that it may probably be considered as merely accidental.

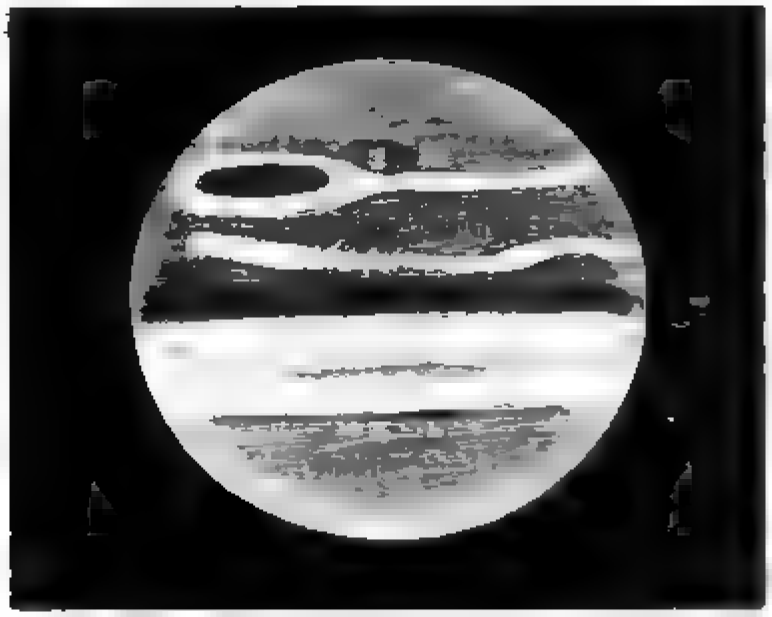
Note on two Sketches of Jupiter.

By Captain William Noble.

The sketches of *Jupiter* which I herewith present to the Society, were made on the nights of October 4 and October 18 of the present year, and to a great extent explain themselves. They show the very remarkable red spot which has attracted the attention of every observer during the present opposition, and the great dark equatoreal belt to the north of it. They further exhibit the curious assimilation of the contour of the surroundings of the red spot to the outline of the spot itself; an assimilation which would seem to indicate that the convulsion to which this most noteworthy marking can only be referable must be of the most stupendous kind.

One point, however, to which I would particularly invite attention is in connection with the white spot which was visible S.F. the red marking on October 4. In a paper read before the Society by Mr. Brett, in June 1876, he directed our notice to the fact that a bright spot sometimes casts a shadow. Now on the night

on which my sketch was made the spot of which I am speaking not only cast *a* shadow (assuming these markings to be such), but *shadows*, as two (as shown in my drawing) were visible, one on



Jupiter, 1879, Oct. 4. 10ʰ 40ᵐ G.M.T.
4·2-inch Ross Achromatic; 61 inches focus; power, 255.

its preceding and the other on its following edge. I confess that no immediate explanation of this occurs to me. In the relative position occupied by *Jupiter* and the Sun at the epoch of my drawing, if we assume the white spot to have consisted of a globular mass of vapour raised high above the surrounding surface of the planet, it is quite evident that it would cast a well-marked shadow behind it. Upon what principle, however, it should simultaneously throw another one in front of it I am wholly at a loss to determine. It may be worthy of remark that the light interval separating the northern and southern equatoreal belts, which at the epochs of my sketches merely appeared as a sinuous but unbroken line on that part of *Jupiter's* disk more immediately north of the red spot, on the opposite side of the planet presented a much more complicated form. It altered in this respect between September and October.

Forest Lodge, Maresfield,
Uckfield, Sussex,
1879, *November* 12.

Note on the Spectrum of the Red Spot on Jupiter.

By Lord Lindsay.

Spectrum of *Jupiter* in Grubb spectroscope on 15-inch. Lowest power best. Probably 100 solar lines seen. The 3 magnesium lines beautifully separated. Slit at right angles to belts. The red equatoreal belt seen as a dark band running through the

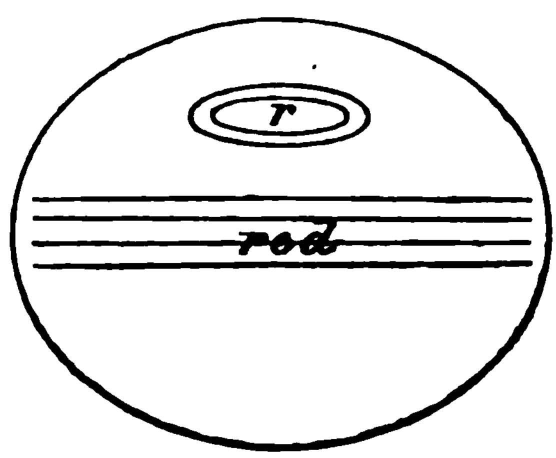
spectrum from extreme red end to between F and G, limit about $38^{r} \cdot 31 = 453^{mmm}$ wave-length, much the darkest from about *b* to F, somewhat darkest of all nearer F.*

Examined satellites 1, 2, and 3; could not make out any lines. The spectrum of 3, when much broadened by a cylindrical lens, was repeatedly seen to be traversed from end to end by a dark band, as if it had belts.

Jupiter.—Slit parallel to belts. Only one very small spot showed the obscuration described above. On examining *Jupiter* with the low power on the micrometer, it was found that the obscuration in this case arose from a detached red spot in the southern hemisphere. The absorption seemed to be more restricted to the region of spectrum near *b* and F than in previous observation.

With the prism of the Vogel spectroscope held in front of eye-piece of the micrometer, obscuration-lines were very distinctly seen when refraction angle of prism was parallel to axis of *Jupiter*. In other positions they were overwhelmed by the light from the remainder of the disk.

The general appearance of *Jupiter* was very roughly somewhat like this at 13^{h}:



The upper red spot was surrounded by a very narrow white margin.

Observers, R. Copeland and J. G. Lohse.

A few weeks ago the Rev. James Virtue, of Dumfries, was so kind as to point out that he saw the large detached red spot on *Jupiter* with the Dun Echt Refractor on the night of June 26, 1878. The date is confirmed by the Diary at Dun Echt, although the observation is not recorded.

The visitor describes the "cloud" as seen "on the upper right quarter of the disk."

1879, *Sept.* 26.

* This phenomenon is fully described by Vogel in his *Untersuchungen über die Spectra der Planeten*, pp. 25, *et seq.*

The Nebula in the Pleiades. By Maxwell Hall, Esq.

It is now about three years since I called attention to this nebula in the pages of *Nature*. At that time it had not been seen through Lord Rosse's telescopes, and as it was very distinct in my own 4-inch Refractor aided by splendid skies, I sought for an explanation.

One was given by the Editor, who said that the light from *Merope* would extinguish the nebula in large telescopes, if I understand him aright.

But as the ratio of the light of the star to that of the nebula remains the same whatever the aperture may be, this explanation did not seem very conclusive; and indeed the nebula has been seen repeatedly since that time at the Birr Castle Observatory.

Again, it has been seen through Mr. Newall's 26-inch Refractor; and his verbal description of the nebula corresponded so closely to what I have always seen in Jamaica, that my sketch made in March 1877, after about a year's attention to the nebula, may prove valuable.

When the annexed drawing of the *Pleiades* was made, the stars with Flamstead's numbers from 16 to 28 were taken from the British Association Catalogue and brought up to date; these stars formed the groundwork, and the chart was filled in from observation at the telescope by the eye alone, so that the drawing is a mere sketch of that group of stars. But *the wreathing of the stars* is well brought out, and great care was taken with the stars in the neighbourhood of the nebula. The limits of the nebula were obtained by moving the telescope rapidly in order to obtain contrast, and great pains were taken in the shading of the drawing of the nebula in order to show the contrast between light and darkness. A power of 100 was generally employed.

It was with great surprise that I saw very recently the drawing of the nebula in Dr. Engelmann's chart of the *Pleiades* (see p. 304, vol. i., *Abhandlungen von F. W. Bessel*, herausgegeben von R. Engelmann, Leipzig, 1875), where it is represented as a very small circular patch of light in no way connected with *Merope*, and situated about 11′ from that star along the axis of figure as represented in my drawing. It is perhaps as well to add that this nebula was discovered by Tempel in 1859, twenty years after the first series of observations made by Bessel.

Tempel also employed a 4-inch telescope, but he found it much smaller than my drawing; it appeared to him of an elliptical form, the greatest and least diameters being about 35′ and 20′; while my estimated diameters are 45′ and 30′.

For the evidence respecting the variability of the light of this nebula I must refer to an excellent summary given by Mr. Webb, in the *Intellectual Observer*, vol. iv., p. 449; where we read, "Schönfeld at Mannheim doubts the fact of variation, and he thinks that this and other suspected nebulæ, being very feeble, large, and diffused, are influenced in visibility by magnifying

power, varying transparency of the air, and practice of the eye, so that aperture is less concerned in their case than in that of minute stars. Auwers, of Göttingen, argues on the same side. It has often, this observer says, been remarked—Encke's comet being an instance of it—that large, ill-defined, faint objects are best seen with small instruments."

Again, in the third edition of Mr. Chambers' *Descriptive Astronomy*, the following remarks were taken from the *Ast. Nach.*, vol. 86, No. 2045 :—"Schiaparelli, at Milan, trying a new telescope on February 25, 1875, saw this nebula very clearly, and was much surprised at its size. He noted it to extend from the star *Merope*, beyond *Electra* and as far as *Celæno.*"

That is to say, he saw it extending in a direction at right-angles to the axis of my figure, seen the year following his observation, though not drawn until later.

It is to be hoped that further observations will be made of this most interesting nebula by possessors of telescopes with both large and small apertures; indeed, there seems to be quite a new field of research opened out for smaller telescopes. At the present moment I am inclined to think that the state of the air has great effect upon the visibility of this nebula and similar objects; for, with my four-inch Refractor on the mountains of Jamaica, I have seen the third or *gauze* ring of *Saturn* as clearly and as fully extended as in the drawings taken with the largest and most perfect instruments.

Discovery of a Gaseous Nebula in Cygnus.

By the Rev. T. W. Webb, F.R.A.S.

On the night of November 14, as I was sweeping in *Cygnus* with a power of probably about 50 on my 9·38-inch silvered mirror, by With, I came across an object resembling a bluish 9-magnitude star, which, however, on closer examination, did not entirely resemble other stars of that size, and which was soon proved by change of eye-piece to be of an entirely different nature. Under powers of 212, 375, and 450 it appeared as a nebulous disk, sur rounded perhaps by a feeble glow, and about 4″ in diameter. I was readily identified by Lord Lindsay and Mr. Knott as Nc 4004 in Argelander, +41°, with a place for 1880 of R.A. 21^h 2^m 3· D.N. 41° 45′·3, and both observers recognised its monochrom light.

Through the kindness of Dr. Copeland, by whom it has l carefully examined, under peculiarly favourable circumstance Lord Lindsay's Observatory, I am enabled to add the follc interesting particulars.

It is not round, and has a sharp nucleus near the *n.p.* with a faint effusion of light in the opposite direction. The vı three very measurable bright lines was given respectively

sets as 500·1, 495·7, 487·0, and 500·1, 495·6, 486·0, and these accord so closely with the results deduced by D'Arrest, 500·4, 495·7, and 486·1, for lines of this nature, that there can be no question as to the character of the object. The relative strength of the lines appeared to Dr. Copeland to be about 8, 5, and 1, reckoned from the least refrangible end.

Hardwick Vicarage,
Dec. 1, 1879.

Note on the Gaseous Nebula in Cygnus. By G. Knott, Esq.

In a letter dated November 22, the Rev. T. W. Webb called my attention to a small object in *Cygnus* which he suspected might prove to be a gaseous nebula, a suspicion which I was able to confirm by observation on the 25th, when, with a McClean Star Spectroscope on my 7⅓-inch Refractor, I found its spectrum to consist apparently of one bright line of considerable intensity. The nebula is found in the *Bonner Durchmusterung*, where it is Zone +41° No. 4004, and with a small aperture and low magnifying power has the appearance of a hazy star of the 8·5 magnitude, which is the magnitude assigned by Argelander. With the full aperture it presents the appearance of a bright bluish white nebulosity slightly elongated *n.p. s.f.*, and I have the impression that it is brightest at its north preceding extremity.

Knowles Lodge, Cuckfield,
Dec. 11, 1879.

Note on the Rev. T. W. Webb's New Nebula.
By Lord Lindsay.

At 6 P.M. on November 22 Rev. Mr. Webb's planetary nebula was seen approximately monochromatic with a prism in front of the micrometer eye-piece. This observation was confirmed on the following evening, and late in the night Mr. Lohse saw a spectrum of three bright lines with the Grubb spectroscope.

On November 26, with the same spectroscope, the lines were found to have the following wave-lengths:—

	1	2	3	
	500·1	495·7	487·0	Observer, R. Copeland.
	500·1	495·6	486·0	„ J. G. Lohse.
Brightness	8	5	1	

The comparison spectrum was a hydrogen tube of Professor Piazzi Smyth's construction.

The nebula under a power of 307 shows a sharp nucleus near the *n.p.* edge, while the opposite side fades away like a very short tail or wisp.

The Nebula in Cygnus. By Prof. Winnecke.

(*Extracts from letters to Lord Lindsay.*)

I beg to express my best thanks for the kind announcement of the interesting discovery of Mr. Webb.

November 28, the Moon being full, I could look at the new nebula. With the Orbitsweeper (aperture 6 Paris inches, power 260) it looked like a star of the 8th magnitude out of focus, and the object-glass not centred. It was oblong in the direction 136°·1, with a lucid point like a star 10–11 mag. in the preceding part; greatest diameter, 5″·5; least, 4″·9. The nearest star which I could see in the bright moonlight was 11 mag. By four measures I found:—

Neb. – ✱ 11 mag.

Dist. = 135″·7. Pos. angle = 282°·0.

On applying a stellar spectroscope the spectrum appeared to be continuous, but rather knotty (perhaps bright lines?). It was about 3′ long, and quite different from the nearly monochromatic gaseous spectrum of H. iv. 18, which was looked at immediately afterwards.

I should feel much obliged if you would communicate these remarks to the next meeting of the Royal Astronomical Society in London.

Strasburg, 1879, *Nov.* 30.

I wrote to you on Mr. Webb's new planetary nebula, on November 30. Afterwards I saw it (December 2), but the night was very foggy, so that the small star near it was not visible. The nebula had the appearance of a very small comet, the nucleus being near the preceding edge; it was elongated in $p = 134°·3$ (1), and with the wire-micrometer the length of the major axis was found (with power 365) = 6″·2 (1). On applying the small Merz star spectroscope, I was very much surprised to find the light nearly monochromatic! Yesterday evening it was very clear, and I could look, for the first time, at the nebula in a dark sky. The spectrum was decidedly of the gaseous character; the first very bright image of the nebula was crossed by a long faint ray, which is to be ascribed, probably, to the spectrum of the small nucleus, which was afterwards very prominent with power 365. The small Merz star spectroscope (without slit) showed yesterday the spectrum of the star 9·1 ma[illegible] near it (B.D., 41°, No. 4001) about 10′ long. I have theref[illegible] no doubt that the new nebula is a gas nebula; still I can by means understand the observation of November 28; perhap[illegible] clue may be found in the circumstance that the nebula was

visible that day in the finder, the moon being too bright. Still the clock was driving perfectly the Orbitsweeper, as it did yesterday evening with -13° C. I should be very glad if another astronomer had observed the nebula with a spectroscope at the same time. The position of the small star preceding the nebula was found:—

Dec. 7 Dist. = $136''\!.9$ (4) Pos. angle = $281^\circ\!.9$ (4)

Pos. angle of elongation of the nebula $p = 132^\circ\!.3$ (2)

Length of major axis = $5''\!.7$ (1)

Strasburg, 1879, *Dec.* 8.

Observation of General Catalogue (Supplement) No. 6,000.

By Lord Lindsay.

This object is marked " Planetary ? " in the Harvard Zones (*Annals*, vol. i. p. 67). Assuming that this query indicated that the object was seen as a planetary nebula by W. C. Bond, D'Arrest observed it at Copenhagen, but found it quite stellar in appearance. At the suggestion of Mr. Dreyer, it was observed prismatically at Dun Echt on December 10 by Ralph Copeland. The spectrum is quite continuous and offers no peculiarity; the object is therefore not a nebula at all. It is identical with D.M. $+0^\circ$ No. 4741, 8·8 mag.

Note on Mimas and Hyperion. By A. A. Common, Esq.

Attempts were made in 1877 and 1878, with an 18-inch Reflector, to observe *Mimas*, but without success; on two nights it was thought to have been seen. *Hyperion* was seen on several nights.

With the 36-inch telescope *Mimas* can be fairly well seen when some $3''$ or $4''$ from end of ring; one attempt to follow it up to conjunction has failed, but under better circumstances it may perhaps be done. The more certain observations (which seem to indicate that the Ephemeris published by Mr. Marth in the *Ast. Nach.* No. 2273 is some one and a half or two hours late) are given below, with some measures of *Hyperion*.

Moonlight that had no effect on *Mimas* utterly obliterated *Hyperion*. This was particularly noticed on the night of November 22.

Sept. 25. At 9.15 G.M.T. *Mimas* was seen E. and expected to be going out from ring; it was, however, found to be approaching, and at $10^h\ 50^m$ appeared to be much closer. The *n.f.* conjunction was estimated to take place at $11^h\ 40^m$, as it could not be followed after $10^h\ 50^m$.

October 10. At $10^h\ 0^m$ to $10^h\ 15^m$ *Mimas* was seen E. a little S. of end of ring; a deposit of dew on the small mirror preventing further observation.

Nov. 13. At $9^h\ 15^m$ *Mimas* was seen E. and apparently half-way between *Tethys* and end of ring. At $9^h\ 24^m$ it was well seen below a line joining *Tethys* and end of ring.

Nov. 22. At $8^h\ 44^m\ 40^s$ *Mimas* was seen W., estimated to be exactly under *Tethys* and $5''{\cdot}96$ from end of ring. At $9^h\ 25^m$ *Tethys* had moved to a place about half-way between *Mimas* and end of ring; and at $9^h\ 40^m$ *Tethys* was nearly up to conjunction and *Mimas* some $3''$ or $4''$ distant from ring.

HYPERION 1879.

	G.M.T.	Pos.	Dist.	Value.	Remarks.
	h m s	° ′	″		
Oct. 21	10 58	271 46	252·77	2	This was measured, as decided motion was observed from a star near.
	11 0	102 45	87·18	1	Noted as something like *Hyperion*.
Nov. 11	8 22	270 54		2	Night fine, but windy.
	8 23	270 48		2	
	8 27		253·55	2	
	8 32	270 40		2	
	10 16	271 14		2	
Nov. 13	7 20	267 8		1	Fine night.
	7 23 30		203·3	1	
	7 25 5	267 1		1	
	9 7	268 25		3	
	9 8 15	268 13		2	
	9 10 30	268 7		2	
Nov. 15	8 26	256 44	102·52	1	Very fine night.
	8 31	258 15	103·62	2	
	8 32	256 16		2	
	8 44	257 42		3	
	8 49		99·38	4	
	8 50	259 28		4	
	8 49 30		99·85	4	
	10 34	252 50		1	
	10 36 30		92·47	1	

	G.M.T. h m s	Pos. ° ′	Dist. ″	Value	Remarks
Nov. 15	10 40		95·45	2	
Nov. 18	8 9 5	105 7	108·49	2	Night very fine, with heavy dew.
	8 11 50	103 48		3	
	8 13 25	103 33		4	
	8 15		106·28	2	
	8 43 50	102 24	110·99	3	
	8 47 15	103 33	109·9	3	

Nov. 1879.

Observations of the Satellites of Mars. By A. A. Common, Esq.

The following observations of the satellites of *Mars* were made by me at Ealing, near London, with an Equatoreal Newtonian Reflector of 36 inches aperture. In all cases the full aperture was used with powers of 220, 240, or 380. The great disparity in light between the objects to be measured necessitated some modification of the ordinary micrometer which it would perhaps be better to describe.

Instead of using a bar in the field, behind which the planet could be placed by moving the whole micrometer, as was done by me at the last opposition, this was done :—The spider webs were taken out of an ordinary double parallel-wire micrometer and the springs taken away from one frame. This frame was then free to slip easily to and fro, and as far in as the nut was adjusted to. Attached to this frame, somewhere near where the wire would be, was a light arm, carrying at the end a small disk of steel just large enough to hide the planet, and so placed as to slide central over and along the position wires up to the intersection of these by the fixed wire supplied to replace the wire taken from the frame spoken of. These position wires were of silk fibre, two in number, and placed at a distance apart equal to about 12″ of arc. The other frame carried a similar wire, and was movable in the ordinary way. The idea with this arrangement was to place the intersection of the fixed wire with the position wires central on the planet and then bring up the steel disk to hide it, adjust the position wires at an equal distance on each side of the satellite, then bring down the movable wire, and so get a measure of position and distance at one operation. But in practice it was found that the fixed wire interfered somewhat with the proper placing of the others on the planet, and it was not always done in this way.

Measuring positions in this way with double wires, where the objects are comparatively close (considering the width of the parallel wires), although suggested or recommended to be done by Professor Hall, in his account of the discovery of the satellites,

seems to me to be inferior to the use of one wire, or of three wires —the two outer ones being placed so as to cut off very small segments of the planet, the central one bisecting the satellite, and the measurements being made in pairs by rotating the position-circle through 180°, unless the middle wire could be placed exactly central—thus getting rid of the uncertainty that exists in properly placing the satellite with respect to the two wires.

Better results may be obtained by using a piece of dark glass (in place of the steel disk) of the proper shade to diminish the light of the planet and allow the wires to be seen through. A micrometer fitted in this way, with the glass behind the wires, answers very well indeed here and on *Saturn*. It was not used in these measures.

As to the brightness of the satellites about opposition (November 2), *Deimos* was considered to be about equal to *Enceladus*; and *Phobos* a little brighter than *Tethys*, not taking into account the glare, but estimating by imagining them isolated. The character of the light was, however, different, being sparkling and starlike, not of the quiet aspect of any of *Saturn's* satellites. This may be due to the absence of any apparent disk, or to contrast with the dull brightness of *Mars*. The colour of *Deimos* was slightly bluish, *Phobos* quite white. At the last opposition *Deimos*, the only satellite well seen, was noted as of a similar colour to *Mars* with a higher power on a smaller telescope.

On November 2 the Moon was very bright, but it had no appreciable ill effect on the visibility of the satellites; if anything, it seemed an advantage.

A weight is attached to each measure on a scale extending from 1 to 5.

All measures of position are subject to the uncertainty of estimation spoken of, but are of course independent of each other, and, with the exception of some few distances, none have been rejected.

The apparent movement of the stars near *Mars* was noted, but nothing like a third satellite was seen.

DEIMOS.

Date.	G.M.T.	Pos.	Dist.	Value.	Remarks.
1879	h m s	° ′	″		
Sept. 21	15 3	240 36		1	A fine night; Satellite seen at once.
	15 5	240 0		1	
	15 19	235 45		1	
	15 37	237 0		2	
	15 41	237 35		3	
	15 43	238 25		1	
	15 45	237 31		3	
	15 48	236 43		3	
	16 17	234 45		2	

Date.	G.M.T.	Pos.	Dist.	Value.	Remarks.
1879	h m s	° ′	″		
Sept. 25	11 54	232 53		1	Rather misty night.
	12 0	234 4		1	
	12 2	233 1		1	
	12 5	233 55		1	
Oct. 15	11 53	249 20		1	Night very windy.
	11 58	247 30		1	
	12 2	246 58		2	
	12 3	247 33		2	
	12 5	247 10		2	
	12 7	245 35		2	
	12 20	245 32		2	
	12 22	243 14		2	
	12 27	244 40		3	
	12 36	245 30		2	
	12 55	243 50		2	
21	9 15	52 20		1	
	9 25		48·8	1	From limb.
	9 34 30	53 36		1	A star brighter than satellite about 20″ beyond. A beautiful night.
	9 36	52 8		2	
	9 37	53 6		3	
	10 16 30	49 22		3	
	10 23 30	49 56		2	
	10 25	49 22		2	
	10 27		45·21	1	From limb.
	10 32	49 41		1	
	11 24	44 31		2	Satellite now faint.
	11 27 30	47 30		2	
	11 30 30	44 30		2	
	11 31 30	46 4		2	
	11 37	46 37		2	
Nov. 2	8 9	231 22		1	Windy night.
	8 11		67·54	2	Definition very fine.
	8 15	233 20		2	Distances taken from centre of disk of planet.
	8 16	232 40	67·55	2	
	8 20	232 25		2	
	8 52	229 53		2	
	8 56	229 14		2	
	8 58	228 55		3	
	8 59	229 34		4	

Date.	G.M.T.	Pos.	Dist.	Value.	Remarks.
1879	h m s	° '	"		
Nov. 2	9 6 30	230 15		3	
	9 8	230 4		3	
	9 10		66·09	3	
	9 18		63·89	3	
	9 19 30	230 8		3	
	10 19		61·39	3	Just visible with Mars in field.
	10 20		61·70	3	
	10 21	227 45		4	
	10 22 30	227 30		4	
5	8 44 30	62 26	56·83	2	Rather hazy.
	8 47 30	61 37	54·32	3	
	8 56	60 21	54·17	3	
	9 4	59 42	57·24	3	
	9 10	60 56		3	
11	8 43 30	210 49		3	Fine night.
	8 48	212 14		2	
	8 52 30	212 5		2	
	8 54 20		36·73	2	
	8 57		39·40	1	
15	9 31	73 2		3	Not at all easy to measure.
	9 40	72 36		3	
	9 42		39·85	1	

PHOBOS.

Date.	G.M.T.	Pos.	Dist.	Value.	Remarks.
Nov. 2	8 31	220 58		1	First seen.
	11 2	55 30		2	
	11 7	57 30		1	Bad seeing.
	11 20	51 1	27·16	3	
5	8 11	51 20	25·12	2	Not very clear night.
	8 15	51 58		2	
	8 19 30	48 40	25·27	2	
	8 23	49 15	26·09	2	
	8 26	50 8		1	
	8 27 30	48 4		3	
11	9 4 30	62 24		2	Fine night, but windy: satellite very bright.
	9 6	60 25		3	
	9 17 45	55 12		4	

Date.	G.M.T.	Pos.	Dist.	Value.	Remarks.
1879	h m s	° ′	″		
Nov. 11	9 20	55 8		3	
	9 22	56 25		2	Under 1 wire.
	9 25	52 10		2	Over 1 wire.
	9 26 15	53 18		3	
	9 28	53 2		4	
	9 49	52 19			
	9 52		26·69	2	
	9 52 30		26·84	3	
	9 59 30	47 19		3	
	10 1	48 14		4	
	10 3	48 16		2	
	10 6 20	46 47		4	
13	7 44 10	51 25		2	
	7 46	49 30		2	
	7 47	50 5		2	
	7 48 30	51 27		1•	
	7 51 40	50 19		3	
	10 55				Seen *s.p.* very faint.
15	9 8 15	235 59		5	Satellite seen with planet; very fine night.
	9 10 30		26·60	5	
	9 12	236 35		5	
	9 13 45	235 28	25·42	4	
	9 15 30	236 4		5	
	9 46	228 46		3	
	9 47 30		23·86	3	
	9 49 15	229 50		3	
	9 52	228 55		4	

November 1879.

New Double Stars. By S. W. Burnham, Esq.

Since the preparation of my last Catalogue and measures (*Memoirs of the Royal Astronomical Society*, vol. xliv.) the work has been continued at the Dearborn Observatory whenever circumstances permitted; and I desire, in advance of the publication of another Catalogue, to call attention to some of the double stars discovered during the present year, in order to give

observers who have the necessary instruments an early opportunity of measuring them.

The following, selected from a list of considerable length, are all naked-eye stars, and most of them important objects. I give the mean result of my measures, usually about three nights for each star.

No.	Star.	Pos. °	Dist. "	Mags.	Epoch.	
1	β Scorpii	89·4	0·79	2 ...10	1879·53	A and B.
2	31 Virginis	28·7	3·56	6 ...12	·33	
3	48 Virginis	229·4	0·48	6 ... 6	·40	
4	86 Virginis	298·4	1·61	6 ...10·5	·37	A and B.
		274·2	1·72	11·5...13	·40	C and D.
		164·7	26·94	...	·33	A and C.
5	46 Eridani	57·0	1·47	6 ...10·5	·05	
6	26 Draconis	149·1	1·36	5·5...10·5	·28	
7	52 Hydræ	276·8	4·00	5 ...11	·42	
8	54 Herculis	175·4	2·56	5 ...12·5	·37	
9	η Cygni	209·0	7·20	4·5...13	·44	
10	65 Aurigæ	8·3	10·36	5 ...12·5	·00	A and B.
		26·8	36·10	...13	·00	A and C.
11	B.A.C. 4389	109·2	2·68	6 ...12	·28	
12	B.A.C. 5248	152·0	1·31	5 ...11	·28	
13	B.A.C. 6966	153·6	0·80	6 ...10	·56	
14	Rad. 6180	244·2	0·88	6·5... 8·5	·46	
15	Virginis 550	81·2	0·47	6 ... 6·5	·39	A and B.
		156·5	23·88	12·5	·37	AB and C.

1. *β Scorpii.*—This star has been known as a wide double for more than a century. After measuring the close pair of *ν Scorpii*, and while the highest micrometer eye-piece (900) was attached, I examined β. The conditions were very favourable for a star so far south, and the duplicity of the principal star was at once detected. I have only been able to get three measures of it during the present season. It is a very difficult pair, and far beyond any close pair hitherto discovered in the inequality of the components. I know of nothing among the large stars that is comparable with it, except *η Piscium*, discovered with this instrument last year. That is generally similar, but the principal star is of the fourth magnitude. Doubles of this description are the best possible tests for the quality of a telescope. In the southern hemisphere a Clark Refractor of 12 inches would probably show it. No second-class instrument, however large, will do it satisfactorily, if at all. There is not much doubt of this proving a physical system.

2. 31 *Virginis.*—This is comparatively an easy pair, and ought not to have been missed by Struve and others.

3. 48 *Virginis.*—A fine close pair, and well separated with the power used in measuring. The components seem to be exactly equal in magnitude.

4. 86 *Virginis.*—(A and C = Σ 1780 *rej.*) This star with a distant faint companion was noted by Struve, and inserted in his first Catalogue; but rejected as too wide and unimportant in the *Mensuræ Micrometricæ.* The 18½-inch shows both of these stars double. The attendant to the large star is very easily seen, and I think not beyond the reach of my 6-inch, but the companion to Struve's star is very difficult, and requires a large aperture. In fact, I did not detect it until after measuring AB two or three times. The components taken together form the closest quadruple system known. Sir John Herschel observed the Struve stars with the 20-foot Reflector at the Cape of Good Hope in 1836, and found the angle, 160°·3, but missed the close stars, as in many other similar instances.

5. 46 *Eridani.*—A fine easy pair.

6. 26 *Draconis.*—Very much like the last in distance and magnitudes.

7. 52 *Hydræ.*—Companion smaller than the two preceding, but more distant. This was found independently with the 6-inch, on Mount Hamilton, and very easily seen.

8. 54 *Herculis.*—The companion is quite faint, and requires a larger aperture than the three preceding pairs.

9. *η Cygni.*—Herschel, with the 20-foot, Reflector noted two distant stars which he called 18 magnitude (= H. 1455). His estimated places are—

	°		″
A and C	170 ±	:	20 ±
A and D	332·0	:	30 ±

They are very much brighter than the new star, and probably have, at least, twice the light. I have called them each 11·5 magnitude. My measures give the following places:—

	°	″	
A and D	P = 325·3	D = 46·17	1879·47
A and C	170·0	49·52	1879·47

The star in the *n.p.* quadrant is really the nearest. The new star is a very minute point, but has been readily seen by Edgecomb with his 9·4-inch Clark Refractor without any intimation of the direction from the principal star.

11. B.A.C. 4389.—The place of this star is, R.A. $13^h\ 0^m$: $+45°\ 54'$. It is a fine unequal pair.

12. B.A.C. 5248.—The place is, R.A. $15^h\ 45^m$: $+55°\ 45'$.

13. B.A.C. 6966.—Both close and unequal. R.A. $20^h\ 10^m$: $+25°\ 14'$.

14. *Radcliffe* 6180.—Heis gives this as a naked-eye star. R.A. $23^h\ 42^m$: $+46°\ 10'$.

15. *Virginis* 550.—A variable star discovered by Schmidt in 1866, and supposed to be variable from 5 to 8 magnitude. It is a fine close pair with a minute distant companion. This is B.A.C. 4531, R.A. $13^h\ 28^m$: $-12°\ 36'$.

In addition to the foregoing, I have divided the principal components of the following known pairs:—

Σ 157	Σ 888
Σ 258	Σ 2005 rej.
Σ 439	S. 752
Σ 707	H. 2661

Chicago, Nov. 15, 1879.

Observations of the Outer Satellite of Mars made at Dun Echt Observatory. By Lord Lindsay.

Date. 1879.	Dun Echt Mean Time.	Pos. Angle.	Cor. for Refraction and Phase.	Dist.	Cor. for Refraction and Phase.	Comparisons.	Observer.
	h m	°	°	″	″		
Nov. 12	11 20·1	226·37	−·01	62·33	+·02	3	R. C.
„	11 44·7	228·02	−·01	59·72	+·02	1	„
„	12 16·1	223·46	−·01	59·44	+·02	3	„
„	12 34·8	222·92	−·01	59·07	+·02	1	„
Nov. 14	11 24·0	40·77	·00	43·48	+·02	1	„
„	11 33·2	39·57	·00	46·78	+·02	1	„
„	11 53·4	38·27	·00	47·46	+·02	1	„
Nov. 17	10 34·6	233·77	−·01	64·14	·00	2	„
„	10 46·3	233·16	−·01	64·78	·00	2	„
„	10 54·6	232·57	−·01	62·57	·00	2	„
„	12 9·1	229·77	−·02	61·65	+·01	1	J. G. L.
„	12 23·2	229·47	−·02	64·07	+·01	1	„
„	12 33·1	227·70	−·02	61·35	+·01	1	„
„	12 47·7	227·33	−·02	62·06	+·01	1	„
* „	13 13·7	225·90	−·02	57·13	+·01	1	„
* „	13 33·7	225·50	−·02	63·58	+·01	1	„

* The last two observations are probably vitiated from the fact of *Deimos* passing very close to a star equal to itself in brightness. In fact, during these two observations they appeared as a single object, the centre of which may not have coincided with that of the satellite.

The observations were made with a power of 229 on the Refractor of 15·06 inches aperture. The field of the eye-piece was contracted to 2′ by a diaphragm in order that *Mars* might be shut out from view by moving the slipping piece while the illuminated wire was placed on the satellite.

It may be remarked that when the sky was quite clear the micrometer wires could not be seen without artificial illumination, when *Mars* was out of the field. The satellite was never seen together with more than the diffracted edge of *Mars*.

The initials are those of the observers, R. Copeland and J. G. Lohse.

Observations of Mr. Baxendell's Star in Canis Minor. By Lord Lindsay.

Mr. Lohse has determined the place as follows:—

1879.	Bax. ✱ − B.W. Δα	Δδ	α 1879·0.	δ 1879·0.	Weight.	Comparison-stars.
	s	′ ″	h m s	° ′ ″		
Nov. 27	− 4·29	− 2 57·2	7 34 45·62	+ 8 39 40·2	3	B.W. vii., 1029
	+ 31·90	− 51·2	7 34 45·69	+ 8 39 37·8	1	B.W. vii., 1014
			7 34 45·67	+ 8 39 39·6		

Adopted places of Comparison-stars for 1879·0.

	δ	Reduction.	δ	Reduction.
	h m s	s	° ′ ″	″
B.W. vii., 1029	7 34 50·91	+ 4·89	+ 8 42 37·5	− 6·4
B.W. vii., 1014	7 34 14·78	+ 4·90	+ 8 40 28·9	− 6·2

Examined with the spectroscope, the star shows no peculiarity. No particular colour predominating, although to the eye the colour is sensibly faint red or purple. Comparisons of brightness from November 27 to December 8 indicated a decrease of half a magnitude; but on December 10, under very favourable circumstances, the star appeared about as bright as on November 27, say 8·9 mag. of Argelander's scale.

Note on a New Star in Canis Minor. By G. Knott, Esq.

A circular was issued from Lord Lindsay's Observatory, Dun Echt, on November 25, announcing the discovery by Mr. Baxendell of a new orange-coloured star of the 8·8 magnitude in *Canis Minor*, near the star B. D. Zone +8° No. 1848. My first view

of the star was obtained on December 1, when it appeared to be some half magnitude less than its neighbour 1848, and about equal to a star some little distance preceding, B. D. Zone +8° No. 1846, the magnitudes of which two stars I determined on December 6 and 10 to be 8·3 and 8·9 respectively. On December 2 I thought the star to be perhaps rather brighter, and on the 6th, 10th, and 11th to be perhaps rather fainter than 1846; but its tint, which is a decided ruddy yellow or orange, combined with varying atmospheric circumstances, may have affected the estimates. On December 6 I determined its differential coordinates with respect to 1848 to be

$$\Delta\alpha = -4^{s}\!\cdot\!35, \quad \Delta\delta = -3'\!\cdot\!0,$$

which would give for its approximate mean place for 1880·0

$$\alpha = 7^{h}\ 34^{m}\ 49^{s}, \quad \delta = +8^{\circ}\ 39'\!\cdot\!5.$$

Knowles Lodge, Cuckfield,
Dec. 12, 1879.

ERRATUM.

P. 5, last line should be

$$15'\ 33''\!\cdot\!37 + 4''\!\cdot\!10 \div (1 + 0''\!\cdot\!70 \times \text{aperture in inches}).$$

viz. the sign of division was omitted.

MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

Vol. XL. JANUARY 9, 1880. No. 3.

Lord Lindsay, M.P., F.R.S., President in the Chair.

Ernest Augustus John Crossley, Esq., 26 Blenheim Road, N.W.;
William Sadler Franks, Esq., 1 High Street, Leicester;
Arthur Riches, Esq., Brunswick School, Leamington;
John F. Sloman, Esq., B.A., Auckland College, New Zealand;
Rev. William Smith, Bradwell, near Sheffield; and
Señor D. Rafael Rorg y Torres, 28 Calle de Fontanella, Barcelona;

were balloted for and duly elected Fellows of the Society.

The Babylonian Astronomy. By R. H. M. Bosanquet, Esq., and Prof. A. H. Sayce.

No. 2.

The history of the Cuneiform Inscriptions may be divided into three periods, which correspond roughly with the three millennia before our era. The earliest of these periods, preceding 2000 B.C., is one of which we know definitely but little. The habit of copying ancient documents seems, however, to have preserved to us through the later periods some not inconsiderable remains of the earlier one. These are recognisable partly by their being in the Accadian language, and partly by the statements of the scribes who copied them, to the effect that they were copies of ancient tablets. The rules of the calendar, the consequences of which were deduced in our last communication, appear to be relics of this early period.

The astronomical documents which appear to belong to this period are for the most part connected with portents, or divination. When the astronomer observed any phenomenon, he was not, as a rule, particular about recording the astronomical circumstances attending it, but connected it with the events of the day. Large numbers of observations of this character have come down to us; and these are, of course, useless for any purpose of interpretation.

There is also a certain number of documents existing, which appear from their language and surroundings to descend from this early period, and yet afford some data towards interpretation. But, so far as we know, none of these documents bear anything of the nature of a year date.

The next period, comprising roughly the second millennium B.C., saw the replacement of the Accadian by a Semitic language. We know little more in an accurate manner of this period than of the first, as regular year dates were not in use in the astronomical records till just at its close.* So far as our subject is concerned, we are disposed to regard this period as the period of copying, of collection, and of commentary on the ancient observations. We shall have occasion to analyse a series of observations of *Venus*, portions of which are undoubtedly of high antiquity; and we shall in that case be able to detect the remarkable result of the passage of an ancient document through the hands of transcribers or commentators, themselves apparently unfamiliar with the phenomena dealt with.

The third period, the millennium preceding our era, may be called the Assyrian period. It is, as compared with the other two, a period of modern history. The Assyrian Eponym Canon, which has been given to the public by the late George Smith, covers a portion of the earlier part of this period, and affords year dates throughout its extent. Certain eclipses falling within these limits are sufficiently defined with respect to the history for their astronomical verification to have been treated already as a matter of importance. The language of the inscriptions of this date is also much clearer, and the observations are more complete than is the case with inscriptions of the earlier periods.

The observations have not year dates generally attached to them even in this period, though it is occasionally the case that the date can be determined within a few years.

We shall see that observations of the equinox exist which belong to the third period. These are incompatible with the continuation up to that time of the ancient Babylonian Calendar sketched in our former communication. The change of calendar will therefore present a distinct problem. We shall not be able to solve it completely without more evidence than is at present at our disposal.

* Dates reckoned from particular events, or by the years of a king's reign, were employed in the public records.

We shall in future abstain from making use of any special hypothesis as to the nature of the position denoted by the words "parallel to *Icu.*" We shall assume that this position is sufficiently represented by the circle of longitude through *Capella.* We thus disregard the small hypothetical correction made use of at the end of our former communication. Such a correction can be applied when desired to the more general values.

To compare the equinoxes under the ancient rule, cited in our former communication, with those of the two equinox inscriptions of Assurbanipal, supposed to be dated about 650 B.C.

Using the rule at the end of our former communication, we find that the difference between the falling of the equinoxes would be something over 20 days, supposing the calendar to go on unaltered. The cases would be as follows:—

Early period (2120 B.C.; longitude of *Capella*, 24° 37′).

1 Nisan.	Sun's longitude in normal year	=	$24\frac{1}{2}^{\circ} - 15^{\circ}$	=	$9\frac{1}{2}^{\circ}$
„	„ mean year	=			$-5\frac{1}{2}$
„	„ latest year	=			$-19\frac{1}{2}$

Whence,

Earliest equinox	=	$-9\frac{1}{2}$	Nisan
	=	$20\frac{1}{2}$	Adar
Mean „	=	$5\frac{1}{2}$	Nisan
Latest „	=	$19\frac{1}{2}$	„

At the time of Assurbanipal the equinox was $20\frac{1}{2}$ days earlier, or thereabouts; so that, supposing the ancient calendar to continue, the equinoxes would fall as follows:—

Later period (650 B.C.).

Earliest equinox	=	0	Adar
Mean equinox	=	15	„
Latest equinox	=	29	„

and the equinox could never fall in Nisan at all, according to the ancient rule.

The equinox inscriptions of Assurbanipal are as follows. There are no dates; the approximate date is supposed to be known from the character and style of the inscriptions, and the dated tablets with which they are associated. (*Western Asiatic Inscriptions** (published by British Museum), iii., 51, 1; *Trans.*

* This publication is generally referred to as *W.A.I.*, which abbreviation is used in what follows.

Soc. Biblical Archæology, iii., 1, 1874, pp. 229, 230.) "The sixth day of Nisan the day and the night were balanced. There were six kaspu of day and six kaspu of night . . . ." (*W.A.I.*, iii., 51, 2.) "The fifteenth day of Nisan the day and the night were balanced. There were six kaspu of day and six kaspu of night."

It is therefore clear, if we suppose that the Assyrians were then able to measure time with any accuracy, that the old rule must have been modified, if the date of these inscriptions is correctly referred to the later Assyrian period. These equinoxes would have agreed well enough with the working of the rule at the ancient period. The most probable view as to the nature of the change seems to be that at the later period there was direct dependence on the equinox.

We get from the above the definition of "kaspu" as a measure of time—the double-hour. This corresponds to the primary division of the heavens into 12 parts, of which we find abundant traces.

Division of the Circle.—Reckoning of Longitude.

The divisions of the circle which we find employed are those into 8, 12, 120, 240, and 480 parts. It has been assumed that the division of the circle into 360 parts was commonly practised by this ancient people. There is, however, no authority in the inscriptions for this assumption. It seems to have been derived originally from Achilles Tatius; and the preconceived idea thus introduced appears to have caused even those most conversant with the inscriptions to see the division of the circle into 360 in matters which do not involve it. An example of this is the sexagesimal reckoning of numbers, which is one of the common methods used in the inscriptions. It is hardly doubtful that the division of the circle as practised by Ptolemy, and in modern times, was an outgrowth of the sexagesimal method of the inscriptions. But the latter does not contain the former.

The numeration of the inscriptions is by two methods, sexagesimal and decimal. The decimal method is in all respects comparable with our own, and was used by preference in the Assyrian period. In it words and signs were used which were precisely equivalent to our "hundreds" and "thousands."

In the sexagesimal method the reckoning was the same as in the decimal up to 60. 60 was 1 soss. The counting went on by multiples of 60 + number over, up to 1 ner = 600. Then by ners + sosses + number over, up to 1 saru = 3,600. The numbers used are always taken in this way. There is no instance of counting by 60, 360, 3,600. The formation of the number 360 was not therefore a natural step in the sexagesimal arithmetic of the inscriptions.

The division of the circle into 480 parts is illustrated by a tablet in the British Museum, written in Accadian, which treats

of the Moon's position during a month. The numbers are, many of them, unintelligible or corrupt; this being, no doubt, partly due to the fact that the tablet is a copy of an ancient one, probably of date before 2000 B.C. But there is amply sufficient left to show that there was a real division into 480 parts, the Moon's mean daily motion being 16°, as it should be roughly, throughout the more intelligible portions.

The numbers of the tablet are as follow :—

1	The 1st day the Moon advances 5° *	
2	10	
3	20	
4	40	
5	80	
6	96	
7	112	
8	128	
9	144	
10	160	
11	176	
12	192	
13	208	
14	224	
15	240	
16	The 16th day for 224° of advance it {becomes obscure† / retrogrades}	16°
17	208	32
18	192	48
19	176	64
20	160	80
21	144	96
22	128	112
23	112	128
24	96	144
25	80	30
26	32	56
27	23	12
28	15	26
29	$5\frac{1}{20}$	$4\frac{2}{3}$
30	The 30th day the Moon is the god Anu.	

* The mark of degrees is here used to represent the units of the division of 480.

† This is the literal rendering of the Accadian word.

The numbers from the 1st to the 5th day are unintelligible. At the 25th day the right-hand column, and at the 26th the middle column, also become so. The final line is explained by the following passage (*Trans. Soc. Bibl. Arch.*, iii. 1, 209):—

"From the 1st day to the 5th day (the Moon is) Anu; for five days (it is) Hea; to the 15th day the orbit during the day is Bel" .... Showing that the new Moon was said to be Anu for five days, Hea for five days, and Bel for five days.

From the middle of the above numerical list it is sufficiently clear that the Moon's synodic revolution was supposed to be divided into 480 parts. The entry on the 16th day is intelligible if we read:—

"The 16th day it is 224° from the Sun, and 16° advanced on its waning course." We can ascribe to the words no other meaning which makes sense. According to this, the measure was taken the shortest way round the circle.

Fragmentary Planisphere, S. 162, British Museum.—Divisions of Circle into 120° and 240°.—Origin of Longitude.

Fig. 1. represents a small fragment of a planisphere. It contains two compartments, each of which is characterised by the name of a month. The month Marchesvan is the 8th, and Cislev the 9th. The arcs have, at their left-hand corners, the numbers shown.

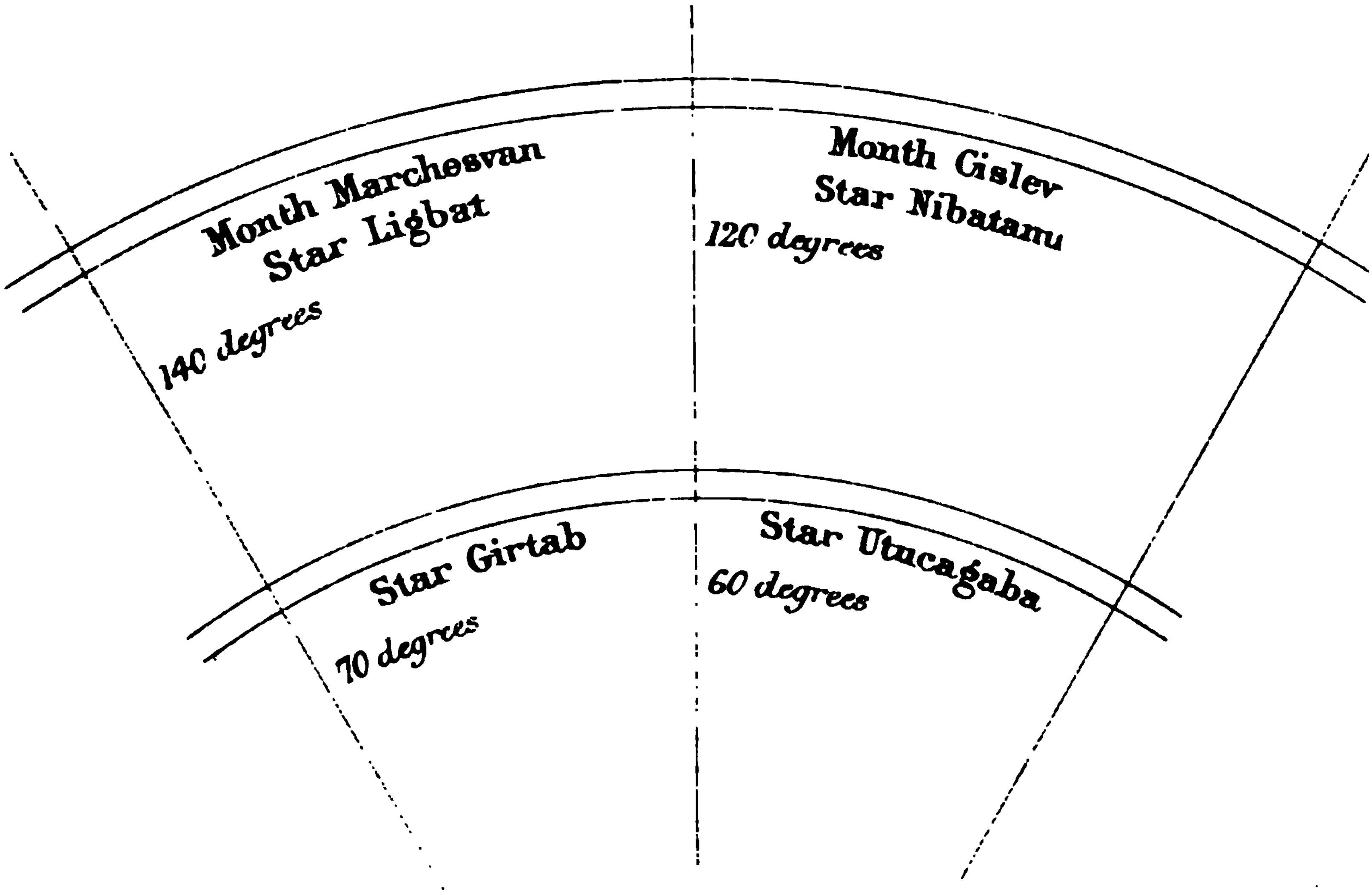
This remarkable fragment is sufficient to determine the following table, in which the year is supposed to be divided into twelve mean months:—

Planisphere S. 162.

	No. of Month.	Outer Circle.	Inner Circle.
		40	20
	1		
		20	10
	2		
		240	120
	3		
		220	110
	4		
		200	100
	5		
		180	90
	6		
		160	80
	7		
		140	70
Fragment	8		
		120	60
Fragment	9		
		100	50
	10		
		80	40
	11		
		60	30
	12		

Fig. 1. S. 162.

OBVERSE



REVERSE

1 . . . . . . of the god Tartakhu, the . . . . . of the god Tartakhu
2 . . . . . . is established; it lifts them up also 81 . . . .
3 . . . . . . the moon in the presence of the stars is established.
4 . . . . . . (the star) . . . . . li is established.
5 . . . . . . (the star) Kaksidi is established.
6 . . . . the star of the Yoke (Niru) is established.
7 . . . the star of the Scorpion (Girtab) is established.
8 . . . the star Khabatsiranu is established.
9 . . . . they seize.
10 . . . . distant.

---

11 . . . . (Hea) king of the upper deep, the great gods
12 . . . . not visible they are come forth.
13 . . . . Merodach-mubasa the scribe.

Spottiswoode & Co. Lith.

The late G. Smith proposed to read 150 for 140, and 75 for 70 (see *Assyrian Discoveries*). There is no foundation for this, except the preconceived idea that the circle ought to be divided into 360 degrees. The numbers are imprinted on the clay with great clearness according to the sexagesimal notation.*

In this fragment the numbers are counted round in the negative direction of the signs; a most curious point. There is no doubt about it, as one of each pair of numbers given is greater than the half-circle in each case; so that the reverse counting cannot arise from reckoning round the nearest way, as in the case of the lunar longitudes above mentioned.

Further, it is to be noticed that the origin of these longitudes does not coincide with the beginning of the year. Of this arrangement we shall obtain an explanation.

We proceed to notice a point of a remarkable character which arises from this fragment, and furnishes a firm basis for two identifications of fundamental importance.

The name "Star Girtab" occurs in the inner circle of Marchesvan (8th month); this has the number 70 adjoining, and apparently characterising the line which would separate it from the compartment of the 7th month. Now there are various reasons which cause us to identify the 8th month with the sign of the Scorpion. In the first place, there is a general correspondence between the months and the signs of our Zodiac, which long ago led to this result. In fact, G. Smith already indicated that this compartment corresponded with the sign of the Scorpion (*Assyrian Discoveries*, p. 407.† We shall return to the general question of the signs of the Zodiac). But since this was pointed out by Smith, evidence of an independent nature has been found by Dr. Oppert, which makes it certain that the word Girtab means "scorpion." There is therefore a high probability that, by assuming the constellation of the Scorpion to be situated in this compartment, and completing the list of stars in the neighbourhood of the ecliptic according to the longitudes thus indicated, we shall learn what it was about the correspondence of the stars with the months that the makers of this planisphere desired to teach.

To compare the longitudes of the planisphere with our own, we have the following Table, taking the numbers of the inner circle, i.e. the division of 120:—

* The planisphere, as well as Planisphere A hereafter mentioned, is of the nature of a projection of the sky on a transparent plane held between it and the eye. To compare its positions with those of the sky it must be looked at while held overhead, face downwards, as if looking through it at the sky. This is easily seen from the sequence of the months on the fragment. The planispheres are, shortly, "sky aspect." The "globe aspect" is got by tracing a copy of the planisphere through on the back of the paper.

† The true phonetic reading of "Girtab" was not known to Smith. He writes the word "Addil."

		Inner Circle.	Longitudes from same zero, with 360°, reckoned positively.
			°
		20	300
Nisan	1		
		10	330
Iyyar	2		
		120	360
Sivan	3		
		110	30
Tammuz	4		
		100	60
Ab	5		
		90	90
Elul	6		
		80	120
Tisri	7		
		70	150
Marchesvan	8		
		60	180
Cislev	9		
		50	210
Tebet	10		
		40	240
Sebat	11		
		30	270
Adar	12		
		20	300

N.B. The months of this Table are mean months, and do not strictly correspond to the lunar months. But it is in this way that the planisphere must have been made out.

We will now show that, if *Scorpio* be placed in the middle of Marchesvan (between the longitudes 150° and 180° of the 360° set), *Capella* is very nearly in the zero of longitude.

Longitudes from *Regulus.*

	°	′	″	
α Scorpii	99	55	15	
β Scorpii	93	20	48	
	193	16	3	
Mean	96	38	1	
	360			
	456	38	1	
Capella	292	0	40	
	164	37	21	= mean longitude of *α*, *β Scorpii* from *Capella.*
	165			= middle of division enclosed by 150° and 180°.

Hence we draw the remarkable conclusion: For the purposes of this planisphere we may take the longitude of *Capella* as the zero of longitude.

This conclusion is entirely independent of our identification of *Capella* as the guiding star of the Babylonian Calendar.

With respect to the other names on the fragment nothing certain is as yet known. We may have to come back to them; and we shall then know that Ligbat may have some reference to the neighbourhood of the Scorpion, and Nibatanu (supposed to be a name of *Mars*) and Utucagabba may have some reference to the neighbourhood of *Sagittarius.*

We have now to see what may be learned from this fragment as to the correspondence between the stars and the months.

We have assumed so far tacitly that the numbers refer to longitudes. It is difficult ts see what other mode of measurement is to be preferred as representing the course of the stars with reference to the months; and we now assume explicitly that the numbers refer to distances measured in the manner of longitudes.

Having the origin of longitude, we can of course lay down the positions of the stars with respect to this origin. The following list shows the positions of a few stars not very far from the ecliptic as they would have been on the completed planisphere. The longitudes with respect to *Capella* are founded partly on Delambre's longitudes from *Regulus,** checked by various means. The printing of this part of Delambre's work is very inaccurate; and some caution was found necessary in accepting his values. A few of the numbers are taken from Flamsteed's Catalogue in the *Historia Cœlestis.*

* *Hist. Astr. Anc.*, ii., 291.

LONGITUDES OF STARS ARRANGED IN MANNER OF FRAGMENT OF PLANISPHERE.

	Inner Circle.	Long. from same origin in degrees of 360°.	Stars.				
	°	°	°	′	″		
	20	300					
Nisan			307	30	42	α Piscium	
			315	48	11	α Arietis	
	10	330					
						Pleiades	
			340	14	4	α Persei	
Iyyar			347	55	53	Aldebaran	
			[355	0	25	β Orionis]	
			[359	6		γ Orionis]	
	120	360	[			*CAPELLA*]	
			[6	54	6	α Orionis]	
Sivan			17	14	50	γ Geminorum	
			[22	16	23	Sirius]	
			28	23	41	α Geminorum	
	110	30					
Tammuz			31	44	6	β Geminorum	
			[33	58	20	Procyon]	
	100	60					
Ab			67	59	20	Regulus	
			89	27	1	β Leonis	
	90	90					
Elul							
	80	120					
			121	59	6	α Virginis (Spica)	
Tisri			[122	22	54	Arcturus]	
			143	14	0	α Libræ	
			147	30	12	β Libræ	
	70	150					
Marchesvan			161	20	8	β Scorpii	
			167	54	35	α Scorpii	Fragment
	60	180					
Cislev			[203	26	45	α Lyræ]	
	50	210					
Tebet			[219	53	9	α Aquilæ]	
	40	240					
Sebat			251	30	11	α Aquarii	
			[251	58	40	Fomalhaut]	
	30	270	266	44	15	β Piscium	
Adar							
	20	300					

The above list includes only a few principal stars, mostly on or near the ecliptic. As questions arise regarding the stars at a distance from the ecliptic, we have mentioned a few chief ones and marked them with brackets—[ ].

We can now obtain the explanation of the curious arrangement which places the origin of longitude between the 2nd and 3rd months. It arises from the north latitude of *Capella*, which, as we have already seen, causes it to rise heliacally before the foot of its longitude circle. So that, although this star is in this first instance prominently connected with the beginning of the year, yet the system of celestial measurement of which it is the origin would bring it into the list later.

This difference does not, however, account for the whole two months. If in the figure of the morning position of *Capella* in our previous communication we calculate the distance along the ecliptic, from the foot of the longitude circle through *Capella* to the horizon, we have :—

Inclination of ecliptic to vertical	$= 55^{\circ}$ nearly.
Latitude of *Capella*	$= 23$ „

Whence, distance along ecliptic $= 37^{\circ}\ 19'$ nearly; and this is approximately the difference in the times of rising of *Capella* and the corresponding point on the ecliptic—*i.e.* about a mean month and a quarter. There remain, therefore, about three weeks to be accounted for, if we suppose *Capella* to have risen strictly at the beginning of the year. This, however, was certainly not the case.

The oscillation of date arising from the intercalary months would prevent the existence of any such fixed rule; and it is enough to satisfy the rules of the inscriptions that *Capella* should rise in Nisan, which this arrangement secures.

Further, we cannot certainly tell to what month the writers of the planisphere would have ascribed *Capella*, or the star *Icu*, with which we identify it. But since this star is spoken of elsewhere as characterising the beginning of the year, or the month Nisan, we infer generally that it would probably not appear in the place where it is inserted in brackets in the list, but rather in connection with Nisan.

At the same time there is no difficulty about *Scorpio*, which is on the ecliptic, being placed in correspondence with its longitude.

The only possible explanation of the planisphere is then, that the stars of a month are those which rise heliacally at a period depending on the month.

We can obtain some information as to the period of the mean month at which the rising of stars of the month took place, by examining the case of *Scorpio*, which has been identified on the planisphere, and may be taken to be on the ecliptic.

We have seen that the mean longitude of α, β *Scorpio* from *Capella* is approximately $164\frac{1}{2}°$.

The longitude of *Capella* at the early date (Babylonian, B.C. 2120) is $24\frac{1}{2}°$.

Therefore,

Longitude of *Scorpio*, early date, from equinox = 189° nearly.

Therefore,

$$\left.\begin{array}{r}\text{Allowing } 15° \text{ of ecliptic for heliacal rising,}\\ \text{position of Sun at rising of } \textit{Scorpio}\end{array}\right\} = 204° \text{ long. from eq.}$$

$$= 6 \text{ signs } 24°.$$

$$\text{Longitude of Sun, 1 Nisan mean year} = \quad -5\tfrac{1}{2}°$$

$$6 \text{ signs } 29\tfrac{1}{2}°$$

Or the mean distance traversed by the Sun from the beginning of the year to the rising of *Scorpio* is 7 signs nearly. But *Scorpio* is ascribed to the 8th month.

Hence, if we assume the early date for the planisphere, the meaning of "star of a month," in the case of *Scorpio* at least, would be, that the star would rise at the beginning of the month in the mean year; or that the star would begin to be prominent before daylight during the month—a very natural and intelligible way for the stars to be connected with the months in the first instance.

We can hardly doubt that the stars must have been connected with the months in the period of the early astronomy; and this connection once established is likely to have been handed down, as, in fact, we know that it has been in later times, with a varied meaning; the original phenomena ceasing to appear in the same way, through the influence of precession.

It will not therefore impair the probability of this explanation if doubts arise as to the date of the planisphere in question.

It is not improbable, judging from the character of some of the writing on the planisphere, that it is of a later date than the early period mentioned; and that the archaisms, which strike one at first as evidences of antiquity, were introduced purposely, as is not unusual in the compilation of the astronomical documents. The names both of the months and of the stars on this planisphere are Accadian. There is a quantity of writing on the back of the fragment which is too incomplete to be of any use, but appears to have been a list of correspondences of stars with the Moon. This is Semitic. The colophon refers to "Merodach-mubasa the scribe." The son of Merodach-mubasa was Nebo-zukup-yucin, mentioned in *Trans. Bibl. Arch.*, iii., page 315 (where the translation should be: "According to

the papyri of the old* tablets of Babylon. By Nebo-zukup-yucin the son of Merodach-mubasa the astronomer"). It seems most probable that these two persons lived in the 7th century B.C.; but this is uncertain, and they may have belonged to an earlier period.

If we select the latest date admissible, and refer the planisphere to the time of Assurbanipal, we have to interpret somewhat differently the association between the stars and the months.

We have seen that the equinox still fell about the beginning of Nisan; and we shall assume that the mean position of the Sun on 1 Nisan is unaltered. This is in the absence of more definite information than is given us by the two known equinox inscriptions. The longitudes of the stars were all increased by about $20\frac{1}{2}°$, which has to be added to the previous result. So that the mean distance traversed by the Sun from the beginning of the year to the rising of *Scorpio* (later date, 650 B.C.) is 7 signs 20° nearly; and the rising of a star belonging to a month would generally take place during the month itself.

It is easy to see how the vague notion of "Stars of a month" would admit of the transformation from the one meaning to the other. And whatever date we assign to the planisphere within the historical limits, it does not fail to illustrate an intelligible notion lying between these two extremes.

In this discussion we have thought it unnecessary to take into account the ellipticity of the Earth's orbit, as the terms introduced thereby are not sufficiently large to be of importance in the present stage of the subject.

One more point may be conveniently spoken of here. The Accadian name of the month Marchesvan means "Opposite to the Foundation."

If we suppose the planisphere reconstructed, Marchesvan would be opposite to Iyyar, the second month. There is reason to think that, at the remote period when the Accadian names came into use, Iyyar may have been the beginning of the year. Iyyar was the month of the Bull. The name as ordinarily written is "The Bull the Director," and it is very commonly indicated by the ideograph of "Bull," written alone. There is also an expression "the path of the Sun," which occurs several times in the old tablets. It is impossible to interpret this otherwise than as the ecliptic. So that, although doubtless the ecliptic would first be recognised as the path of the Moon and planets, yet it was, even in very early times, identified with the path of the Sun. And this expression, the path of the Sun, was in many ways identified with "Bull."

For instance, the name commonly ascribed to *Jupiter* is Lubat Guttav. Lubat is planet, literally "old sheep." Guttav is

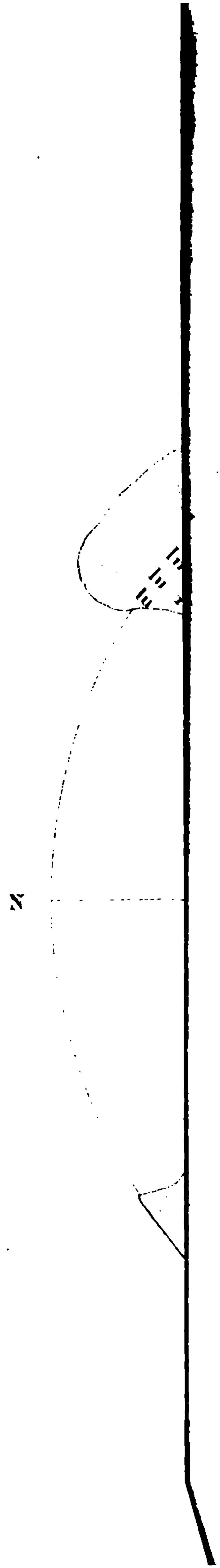
* The word here translated "old" has that meaning if Semitic; but it is more probably Accadian. In that case it means "in parallel columns," "bilingual."

literally "Bull of the Sun," or "Bull of heaven." (
is also explained in an Assyrian gloss by words interpre
Dr. Oppert to mean "furrow of heaven." Thus the
planet was "the old sheep of the furrow of heaven,"
being put for the furrow it ploughed. Again we have
Bull of Heaven . . . rising . . . in the path of the Sun." (
Brit. Arch., iii. 13.) Thus the idea of the ecliptic was inti
connected with "Bull." When therefore we find the
Iyyar called "The Bull the Director," and commonly den
the symbol "Bull" (which also refers to the stars of that
according to the name they still bear), it is natural to co
that the month may have been connected with the beginn
the yearly course of the ecliptic; more especially as the t
certainly correspond at a period more remote than any of
we have direct traces. The month Iyyar being thus
ancient Foundation, or beginning of the year, the Ac
name of Marchesvan, "opposite to the Foundation,"
be explained. (See the use of the word "Foundatio
Planisphere A.)

If we assumed that the numerical reckoning of the plani
was itself of ancient date, it might be possible to force an
pretation of Marchesvan as "opposite to the Foundatio
making reference to our zero of longitude. But it is,
whole, improbable that the numerical reckoning is of very a
date; and it is more reasonable to suppose that the word "
dation," in its employment with reference to Capella, the
longitude and the starting-point of the calendar, was ori
derived from an origin connected with the beginning
year in the most ancient Accadian period.

The agricultural metaphors here alluded to run t
much of the Babylonian astronomy, and we shall find
traces of them.

	Accadian Value.	Assyrian Value.	Meanings.
α	an	Ilu	god, star.
β	nitakh	zisu	man.
γ	en	bel	bond (?), lord, sky.
δ	bat		opening, omen, corpse, hol[illegible] channel.
ε	kas		two, double, road.
ζ	aiu, or zu (uncertain)		city or zodiacal sign.
η	gi		god, knee, multitude.
θ	me		sign for 100.
ι	mat, kur, sat		country, mountain, conquer, ascendant.
	lu		cabin.
	ana		number.



& C° Lith London.

Accadian Value.	Meanings.
gar, sa	to make, four.
khar	circle, centre, body.
X	setting (of the Sun), a foot, &c. &c.
du	to go, advance (as in table of lunar longitudes).
bur	heap.
pa	wing, stalk, dawn.
tar	to cut, place, descend, canal, before.
na	sky, front, chieftain.

Planisphere A.*

This planisphere is a circle divided by radial lines into eight equal parts. Unfortunately large portions of it are missing. We have attempted to represent it in Fig. 2. It is covered with cuneiform characters. We have indicated the translation of some of these, placing the words as nearly as possible in the position of the originals; others we have transliterated; others, which recur in a manner which suggests measurement, we have for the most part represented by Greek characters which refer to notes. We are not able to indicate with certainty the meaning of any of the characters of the last two classes as they occur on the planisphere, though they are mostly characters of ordinary employment in other senses; but see P.S. They appear here to bear some technical meaning. Some inferences can be drawn from the collocation.

We have come to the conclusion that, wherever stars are represented in this planisphere, it is by their names only; not by points or diagrams. The points and diagrams, many of which exist on the planisphere, we believe to possess only interpretational significance. We shall endeavour to show that one figure represents the mechanism by which the lunar beginning of the year was worked out with reference to *Icu*, or *Dilgan* (*Capella*). The other figures we are unable to interpret; but think it extremely probable, for many reasons, that they may have been of an astrological character.

The name of the star *Dilgan*, written in Accadian along the side of one of the figures, gives its locality in the sky. The name, "Star of the Foundation," is written under the name *Dilgan*, the main line of the figure passing between. We interpret "Star of Foundation" as an epithet applied to *Dilgan*, the meaning of which is obvious from our point of view.

The figure, whose principal line passes between these two

* Exact facsimiles of this and the preceding fragment are about to be published by the Society of Biblical Archæology. This planisphere is in Collection K, British Museum; it is not at present numbered.

names, is terminated by two triangular points; the one pointing to the centre of the planisphere, the other pointing outwards. The principal line is not in the straight line joining the vertices of the triangles, but is a broken line, lying mostly to one side. Nearly in the direction which joins the vertices of the two triangles there is a line of seven small circles. On the side of these circles, opposite to the words and line already mentioned, there is written, "Bel that goes before the star." The language of this is Semitic. The head of the triangle which points to the centre of the planisphere is injured, but close to the point four more small circles are clear.

There can be no doubt that this enigmatical figure is susceptible of a simple explanation, on the assumption that it represents the working of the Moon rule for the beginning of the calendar, which was explained in our former communication.

The star *Dilgan* is the star of the rule, whether it be *Capella* or not; so that this application does not depend on our fundamental identification. (*Dilgan* is identified with the "star of stars" of the rule by a passage in an unpublished tablet, which makes "the star of stars" equivalent to "the star *Dilgan* of Babylon.")

The figure having two triangles for its extremities is like a scale, or a rule; the triangles remind one of the arrowheads still frequently employed to mark the terminal points of a scale.

The small circles appear to be the marks on the scale. There are eleven small circles altogether. We cannot fail to notice that there are eleven principal cases of the position of the Moon regarded as parallel to *Dilgan* on 3 Nisan in the working of the rule before explained. If we look at the table of the cases in our previous communication (*Monthly Notices*, xxxix., p. 458), we find the cases in question marked with the letters I. D. (intercalary due), and we verify that there are generally eleven such cases, according to the approximate rule employed. This point is so far one of elementary arithmetic, that we cannot regard it as out of the reach of the makers of the planisphere; we know, indeed, from their tables of squares and cubes, that their command of arithmetic was considerable.

The cases marked I. D. in the table have distances of the Moon from the starting-point, ranging from 3° below to 7° above. This corresponds very fairly with the figure, in which the line o the seven lowest circles is nearly coterminal with the word "Star *Dilgan*," so that the middle of the words corresponds t about the fourth circle from the bottom. The four remainir circles are placed together higher up. From the considerati alluded to at the end of our former paper, and others, it is obvi that any attempt at great accuracy here must be futile. But general success of the explanation hardly admits of doubt.

We have so far assumed that the planisphere repre *Dilgan* in the western sky at the beginning of the yea account of the correspondence of the figure with the ca

rule. But it admits of direct demonstration, by means of the star *Sibzianna*, which is on the planisphere, that the planisphere represents the configuration with *Dilgan* in the western sky, and not in the eastern. Also the sentence, "Bel that goes before the star," can only be interpreted, "The middle of the month before the beginning of the year," since "Bel" was the Moon from the tenth to the fifteenth day.

Sibzianna is mentioned in several passages. It is identified with *Regulus* by Dr. Oppert. We do not admit this identification, but postpone the complete discussion of it. The following passage is sufficient for our present purpose. Inscription of supposed date of Assurbanipal (B.C. 650)—*W. A. I.*, iii., 51, 9, lines 13–19; *Soc. Bibl. Arch.*, iii., 235—"During the month Tammuz current . . . . During the period when the Moon is Anu, in the orbit (region) of *Sibzianna*, it is seen declining." ("Anu" means the first five days of the month. *Vide supra.*)

Examining the consequences of this, we have the following numbers. (λ = longitude.)

Beginning of mean year. (See *Monthly Notices*, xxxix., 459.)

$$\odot\text{'s } \lambda \text{ from } Capella, \text{ 1 Nisan} = -30^\circ \text{ (early period).}$$

Assume that at the later period the Sun's position with respect to the equinox is substantially the same (*Equinox Inscriptions, vide supra*). But at the later period we have

Increase of λ of *Capella* from equinox	=	20° (later period).
$\odot$'s λ from *Capella*, 1 Nisan	=	−50 (later period).
3 months $\odot$'s journey in λ	=	90
$\odot$'s λ from *Capella* 1 Tammuz	=	40 (later period).
New Moon's λ from Sun, in mean	=	15
New Moon's λ from *Capella*, 1 Tammuz	=	55
Moon's journey, 4 days	=	52
Moon's λ from *Capella*, 5 Tammuz	=	107

This would give a range of longitude from 55° to 107° for the region of *Sibzianna*, in which the Moon was. But to this must be added the extension of limits in each direction due to variations from the mean values we have employed. The oscillation due to the intercalary months is fifteen days each way—say 15° in the Sun's longitude, and the uncertainty from the appearing of the new Moon may amount almost to a day each way of the

Moon's journey, or 13°. The combined effect of these extends the limit of possible cases each way by 28°; so that the region is defined by the extreme limits of longitude from *Capella*, 27°–135°.

Now, if we assume *Dilgan* in the planisphere to be *Capella* in the western sky, a region 27°–135° in longitude from it will stretch over to the opposite side of the planisphere, and include the place where *Sibzianna* is marked. But if we assume *Dilgan* to be *Capella* in the eastern sky, the region 27°–135° in longitude from it should lie entirely, or almost entirely, below the horizon; and if any part of the region were on the planisphere, it would be close to the edge, between the edge and "*Dilgan*." This does not correspond at all with the actual position of *Sibzianna*. We infer that the configuration is that of *Dilgan* or *Capella* in the western sky, at the beginning of the year. (*See* P.S.)

The position of the planisphere with reference to the sky is now determined. Holding it overhead, the radial line between *Dilgan* and the Tammuz figure is due west. The equinoxes of the earlier period are near the rim of the planisphere; and the summer solstice is near the centre of the planisphere.

It is at once seen that *Sibzianna* cannot be *Regulus*; for *Regulus* would be nearly in the centre, or a little to the south, whereas *Sibzianna* is considerably to the north.

Comparing the planisphere thus held with the globe on which the stars have been laid down referred to the equator and ecliptic of the globe as equator and ecliptic of the period, we have inferred that *Sibzianna* on the planisphere corresponds with *Arcturus*. The position of *Arcturus* with respect to *Capella* is:—

	Long. from *Capella*.	Lat.
	°	°
Arcturus	122½	31 N.

The longitude lies within the limits of the observation above discussed. As to the planisphere, assuming the position of the sky above indicated, the longitude of *Arcturus* from the equinox is $24\frac{1}{2}° + 122\frac{1}{2}° = 147°$; or, the distance of *Arcturus* from the summer solstice is about 57° in longitude. Both this and the latitude correspond fairly with the position of *Sibzianna* on the planisphere; but, owing to the effect of the projection, and the vagueness of the indication of position along the length of the words, the longitude indication of the planisphere is rather indefinite, while the indication of latitude is pretty clear and is decisive.

The meaning of the word "*Sibzianna*" is "Shepherd of the flock of heaven." It is impossible to avoid comparing it with βοώτης.

There is considerable difficulty about the meaning of the

lines of the planisphere. We think that the line between *Dilgan* and the Tammuz figure is intended for the ecliptic. The marks all along the line look like the marks of a scale. Those on the west (on the side of *Dilgan*) are imperfect; but on the other side we find the series of twelve marks in three sets of four, which we have denoted by four of each of the letters δ, ϵ, ζ. These would, if continued round, correspond to a division of the circle into 48 parts, and so look as if each letter represented 10 degrees of the 480 division.

But the planisphere has strongly the appearance of referring primarily to the zenith as centre and horizon as circumference. If this were so, we should be driven to admit that the makers of the planisphere disregarded the difference between the circle E. zenith W. and the ecliptic at sunset at the beginning of the year. At Babylon, at the period in question, the two are inclined at an angle of about 9°. This would only be admissible on the supposition that the planisphere was made for astrological purposes, or that it was a very early and inaccurate work. The point is not, however, of prime importance, and we shall continue to assume that the E. W. diameter of the planisphere represents the ecliptic, the centre being the summer solstice.

The Tammuz figure corresponds to the position of *Orion*, as is seen by comparing the globe. We have a list of stars of the month Tammuz, in which the star *Gula* appears. Comparing the globe, we identify it with some probability as *α Orionis* (long. from *Capella* 6° 54′ 6″, lat. 16° 3′ 31″ S.). The three marks κ, one of which is nearly obliterated, but is pretty certain, are just where the three stars of the belt should be. We have no idea of the meaning of the rest of the figure.

Star *Bartabba* means double star. There are three different *Bartabbas*, one of which we shall subsequently identify with *Gemini*. The "*Bartabba* which in the hour of *Sibzianna* is fixed" we take provisionally to be the two stars of the Balance, which are seen on the globe to correspond fairly to this description.

In a further communication we propose to examine a tablet containing observations of *Venus*; to treat of the identification of stars; and deal generally with the evidence as to the astronomical knowledge of the ancient inhabitants of Babylonia.

P.S.—On further consideration the meaning of the singular figure with a pointer, a little N. of E. in planisphere A, is apparent. The mark that indicates sunset is written several times within the figure, and the pointer indicates that the sunset is over at the opposite side, where we have placed the letter W. This is entirely consistent with the preceding exposition.

Meteor Showers. By W. F. Denning, Esq.

Several meteor streams observed at Bristol within the last two years appear to be of such marked intensity as to merit special description, and the following notes may be of some use to those who desire to reobserve the annual returns of the showers referred to. I have selected five positions from the large number of radiant points determined since 1877, as under :—

		α δ	
I	July 30—Aug. 1	$32^\circ + 53^\circ$	Max. July 31, 1878.
II	July 27–30	$341 - 13$	Max. July 27–28, 1878–9.
III	August 21–25	$291 + 60$	Max. August 21–23, 1879.
IV	October 14–20	$31 + 9$	Max. October 15, 1879.
V	August 8–11	$44 + 25$	Max. August 10?

I. At the middle of July 1877 a few meteors were traced from a radiant point at $36^\circ + 47^\circ$, and on projecting a large number of meteor tracks registered in foreign catalogues for the period July 25–31, I found the same shower amply manifested from 25 paths, though the radiant was 5° higher in declination. A succession of clear nights occurred from July 26 to August 2 in 1878, and I obtained some lengthy observations. In about 22 hours of watching more than 400 shooting stars were seen in the eastern sky, chiefly amongst the constellations of *Perseus*, *Cassiopeia*, and *Andromeda*. I saw many swift meteors leaving short streaks and otherwise exhibiting much uniformity in their appearances and directions. The radiant was evidently in *Perseus*, and at first I mistook them for early members of the closely approaching Perseïds of August 10, but soon found from a number of foreshortened paths that the radiant point was not reconcilable with that of the well-known annual shower. It was sharply defined about 3° S. of the group χ *Persei* and the maximum of the shower was witnessed on the last night of July, when 21 meteors were noted diverging from the same point below χ *Persei*. In all I saw 63 meteors conforming to this stream; they were short and quick, always with streaks of 3° or 4° in the latter portion of their flights. I looked for the shower again in 1879 and recovered it both on July 28 and 29. It appears to be identical with a radiant given by Schmidt at $31^\circ + 55^\circ$, August 3–12. The diagram shows the observed paths of 86 ↓• (in the region of the radiant point) recorded at Bristol in the years 1877–8 and includes some tracks in the catalogues of foreign observers. Several of the meteors are apparently directed from a point slightly north of the position observed in 1878 and conform to a centre at χ *Persei* precisely; but I am fully satisfied that the true radiant lies quite 3° below that cluster. There is evidence of a prominent shower at about $32^\circ + 57^\circ$ on

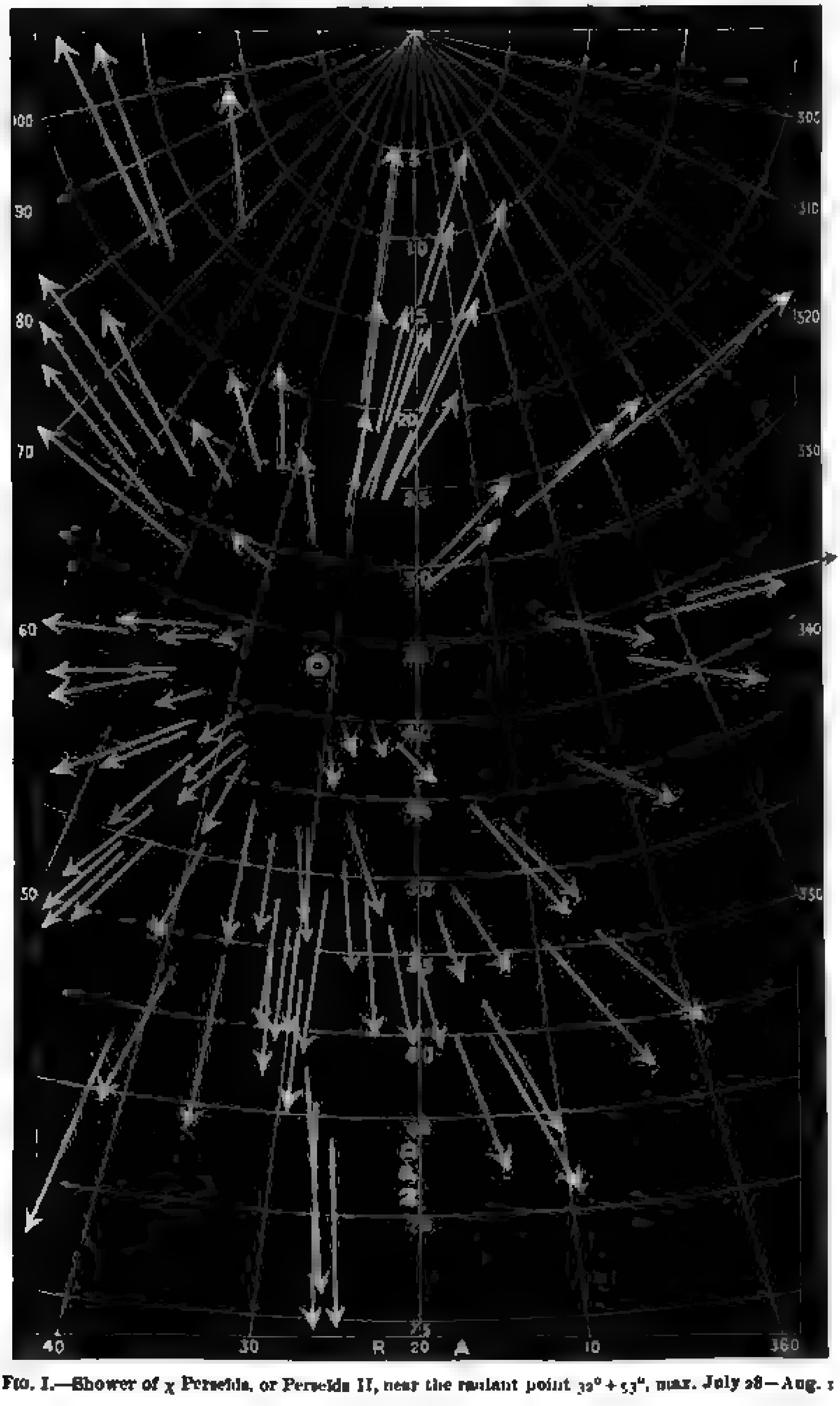


FIG. I.—Shower of χ Perseids, or Perseids II, near the radiant point $32^\circ + 53^\circ$, max. July 28—Aug. 1

August 10; but that is probably a different stream and one of the principal radiants of the major Perseïds.

II. Contemporary with these χ Perseïds, or Perseïds II, my observations in July 1878 led me to recognise a large number of slow meteors with long paths (averaging 17°) and occasionally leaving faint trails of sparks, ascending amongst the stars of *Pegasus*, *Andromeda*, &c., which, by their parallelism of motion, obviously proceeded from a common radiant point presumably situated at a low altitude and considerably southwards of *Pegasus*. I watched them narrowly, and determined the radiant with fair precision as near δ *Aquarii*. The best display of this fine shower was on July 27, when 22 of its meteors were recorded, but they were also numerous on the few ensuing nights. In the three-quarters of an hour preceding midnight on July 30, eight of these Aquariads were visible, though the radiant was close to the horizon; but on the following night the shower appeared to

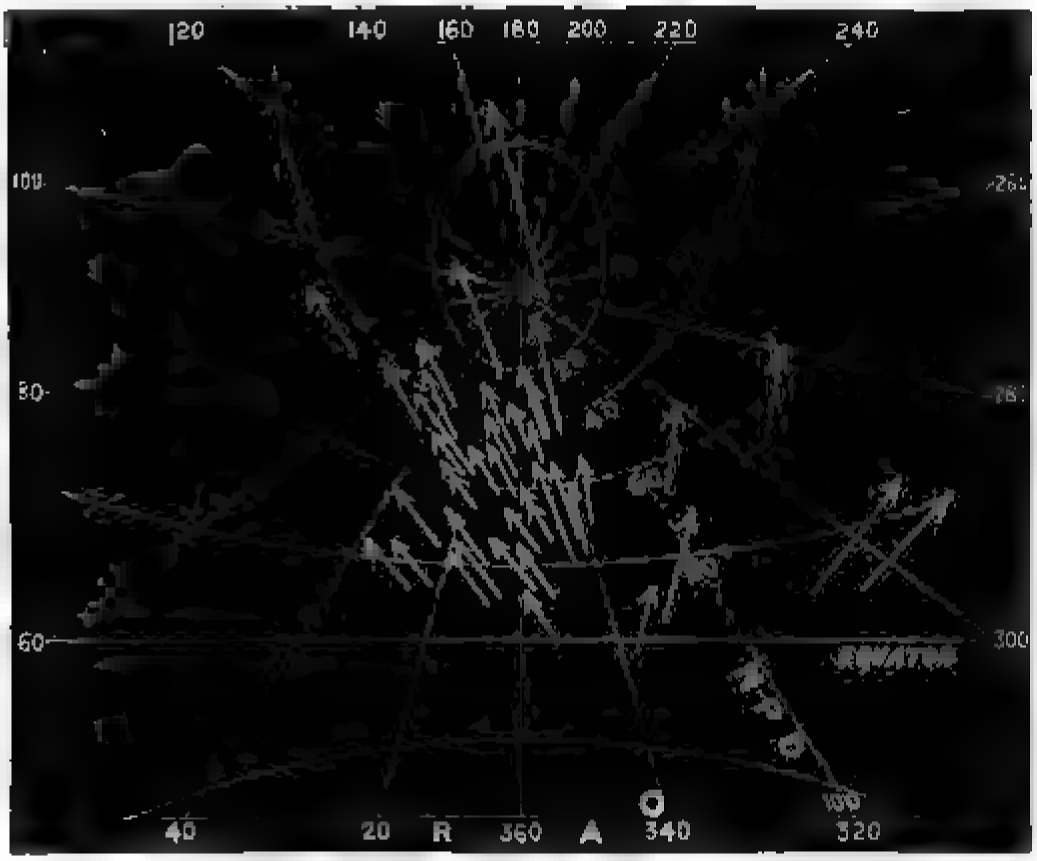


FIG. II.—Shower of Aquariads (341° − 13°), max. July 27–30.

have become extinct. In 1879, July 28, it reappeared, giving very long gradual meteors similar to those seen the year before. In a watch of two hours ($11^h\ 45^m$ to $13^h\ 45^m$) on the night mentioned 45 shooting stars were counted, of which 14 were Aquariads diverging from the point 338° − 14°. Just before midnight the shower was in active progress, supplying several fine meteors brighter than the stars. The continuation of this radiant in August is rendered extremely probable by my observations in 1877, when I detected a good centre at 342° − 12° from twelve meteors,

August 3–16, and by a bright stationary meteor, also recorded at Bristol, on August 9, 1879, at 342°−16°. An observation very similar to the latter was obtained by Sawyer at Cambridgeport, Mass., on August 7, 1879, of a 2nd mag. stationary meteor at 344−°13°, and further proof of the shower's sustenance in August is afforded by several positions of Schmidt and Neumayer:—

	° °	
August 1–12	345− 7	Schmidt.
August 1–31	341−11	Schmidt.
August 1–31	337−10	Neumayer.

Schmidt had also traced the shower in July, for he gives two positions for July 20–31 with a mean = 337°−11°; but Major Tupman had obtained the best previous observations of the shower. On July 27, 1870, and several ensuing nights, he saw many meteors from an accurate radiant at 340°−14°, and the position and epoch of the shower, as he determined it, agree perfectly with its apparition in 1878–79 recorded at Bristol. Prof. Herschel had also, as early as July 28, 1865, traced a few of its meteors. Later showers (of very slow meteors) from the same region of *Aquarius* appear in September and October, and are possibly prolonged into November; for on November 7, 1877, the Rev. S. J. Johnson, at Crediton, saw a 1st mag. meteor stationary at 338°−15°. The diagram shows 41 tracks registered at the end of July, chiefly in 1878, at Bristol.

III. Beginning to observe at about $9^h\ 30^m$ on August 21, 1879, I immediately found that a very active shower of slow trained meteors was proceeding from a point in *Draco*. I noted 9 of them in the $\frac{3}{4}^h$ before $10^h\ 15^m$, though afterwards the shower became less decided, and of the 68 meteors which I counted up to $13^h\ 30^m$ (when clouds overspread the sky) 21 belonged to this system. The two ensuing nights were clear, and I saw 143 shooting stars, including 31 additional paths conforming to the special shower in *Draco*, the exact position of which I determined at 291°+60° (near *o Draconis*). The night of August 24 was overcast; but on the 25th I watched the sky at intervals between large breaks in the clouds and noted several fine meteors from the same radiant point. The total number directed from this stream was 56 out of an aggregate of 225 meteors seen on August 21, 22, 23, and 25. The Draconids were generally brilliant, with short paths and spark-trails; motions rather slow. Ten exceeded or equalled the 1st mag. and 15 were estimated = 2nd mag. The same shower was found by Tupman on August 20–25, 1871, at 280°+58°, and Corder saw it at 286°+61° (10 ↓^{s}) on about August 16, 1879. It is evidently identical with the Draconids II (No. 78) of Greg's *Catalogue of Meteor Showers* (1876), in which the position is averaged at 282°+60° and the whole duration extends from June 28 to August 25 ? from 12 observations. There are other conspicuous showers from the same region near *o Dra-*

conis in April–May, and again in November and early in December, (*see* Greg's Nos. 64 and 166). The diagram gives 41 paths, nearly all of which were traced on August 21–23, 1879. These dates nearly coincide with a fireball epoch (August 19–22), and

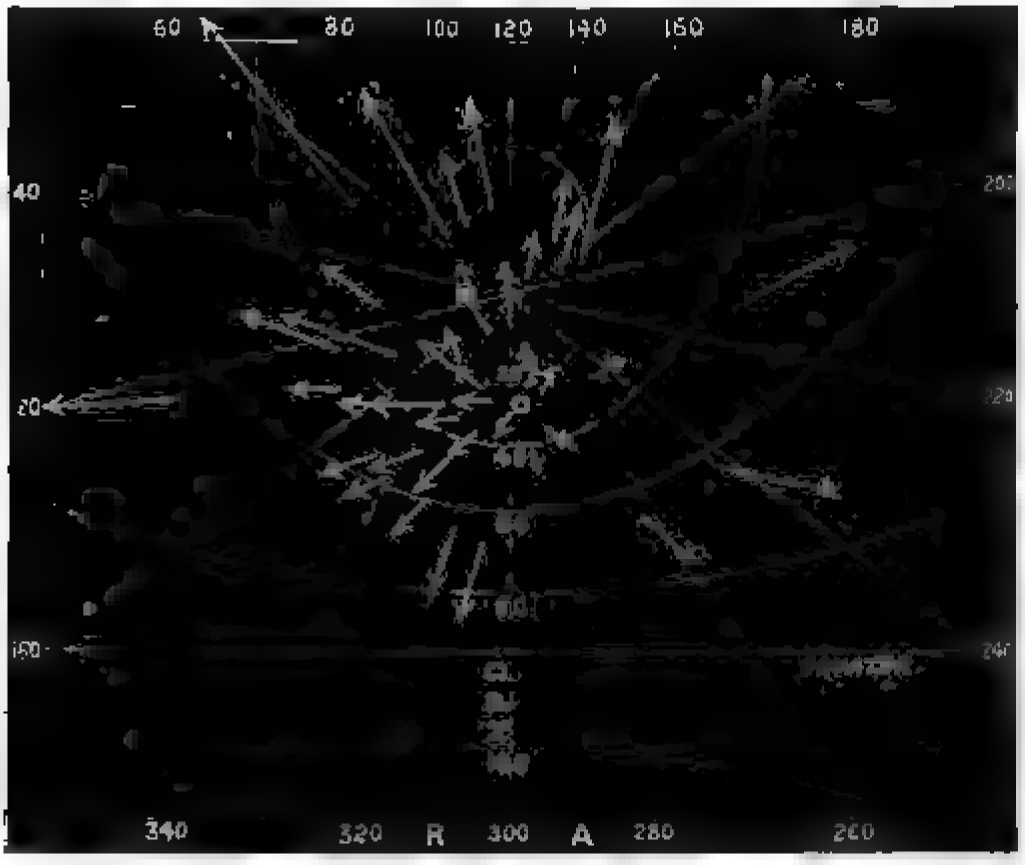


Fig. III.—Shower of Draconids (291° + 60°), max. Aug. 21–23.

there is little doubt that the rich shower of Draconids referred to has furnished us (in conjunction with a shower of Cepheïds from near *β Cephei*) with a large proportion of the brilliant meteors observed at this season of the year.

IV. On October 15, 1879, the sky was watched for 11 hours ($6^h\ 30^m$ to $17^h\ 30^m$), and of the 127 shooting stars seen during that long observation, 21 were slow meteors from a radiant point at 31° + 9°, but the position was not well defined. I had seen several meteors from the same region on the previous night, and on the 20th, when the sky was again favourable, I recorded 10 others, making 37 in all from this shower in the south of *Aries*. They were generally faint, with rather short paths and decidedly slow. The same radiant was seen by Major Tupman in 1869 October 13, at 28° + 10°, and Mr. Corder has distinguished ı series of positions in *Aries* and *Pisces*. He remarks that "ther are several radiants of small meteors about this place in Octob which are difficult to separate." At the middle of October ı he determined an active shower at 25° + 8° (18 ↓*), and, tow the end of the same month, saw a radiant at 32° + 11° (ı Mr. Corder had also witnessed the two preceding appariti

the shower, for in 1878, October, he tracked 17 paths from the point $37°+10°$, and in 1877, October, found a position at $30°+10°$ (9 ↓*). These several observations unquestionably refer to one and the same stream, situated approximately between *α Arietis* and *α Piscium*, and nearer the latter star; but there may be a few branch radiants closely adjoining, as suspected by Mr. Corder, and this would explain the apparently diffuse radiation of the

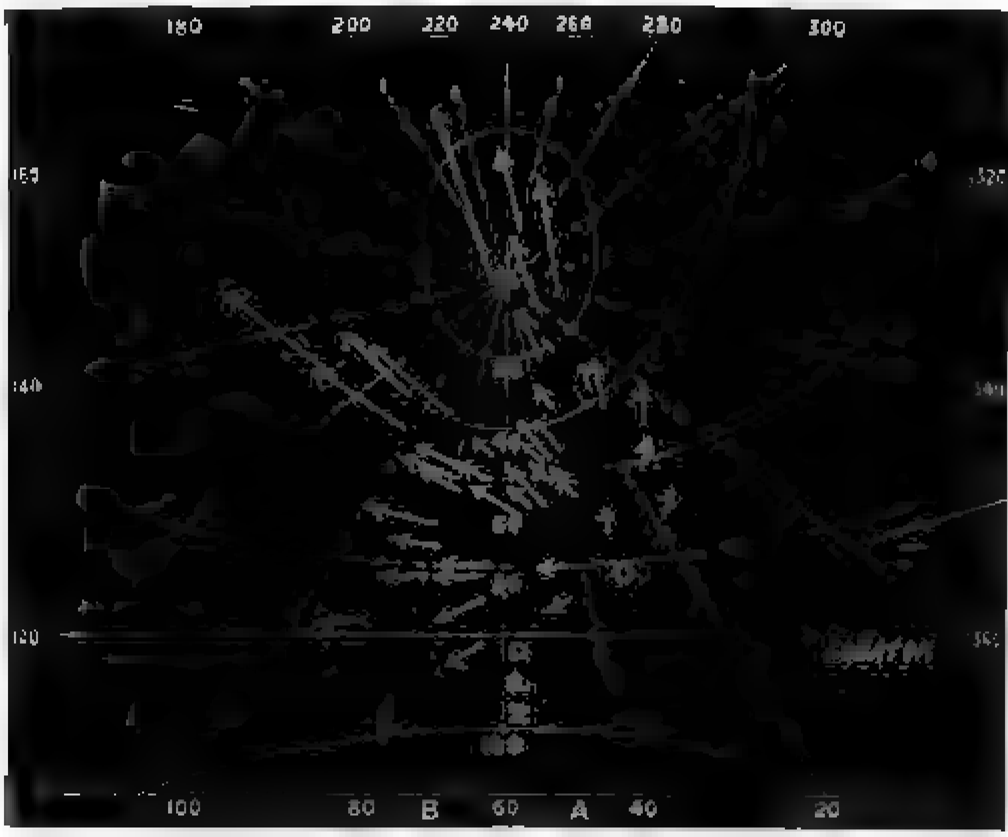


FIG. IV.—Shower of meteors from a radiant point at $31°+9°$, Oct. 14–20 (max. Oct. 15).

meteors seen in 1879. The diagram (IV) includes 42 paths observed by me in the years 1876–79, but chiefly in 1879. The meteors were mostly seen amongst the stars of *Perseus* and *Taurus*, which I was watching at the time of the shower's appearance.

V. At about the middle of August 1877, a few rapid meteors were traced from a radiant in *Musca* at $40°+28°$, and in the following year, while noting the progress of the Perseïds, I recorded several fine meteors, leaving streaks and with paths averaging 40°. The radiant was evidently on the horizon, and the directions of the meteors, which in several instances were very exactly observed and mapped, indicated the point $44°+25°$ as the diverging focus of the shower. With the object of further investigating it, I looked through the observations made at the epoch of the Perseids, by the Italians in 1872 and by Zezioli in 1867–70, and found many meteors conforming to this same shower in *Musca*, which had already been detected by Weiss in 1869, August 11, $46\frac{1}{2}°+23\frac{1}{2}°$, and August 12, $41\frac{1}{2}°+24°$. Denza also appears to have seen it in 1869, August 10, at $48°+19°$.

In this region, between *Musca* and the E. extremity of *Aries*, there are many successive showers during the four months from August to November. I have seen one in September (after the 20th), and again on October 8 and 15, October 31–November 4; and in the present year, on November 12–14, while observations were directed towards the stars of *Leo*, a large number of slow, bright meteors were seen both by myself and Mr. Corder from two very decided radiants of Taurids and Muscids. Early in August, when the first shower from *Musca* is perceptible, the meteors are very swift, with unusually long paths, and seldom without streaks; but in October and November the motions are

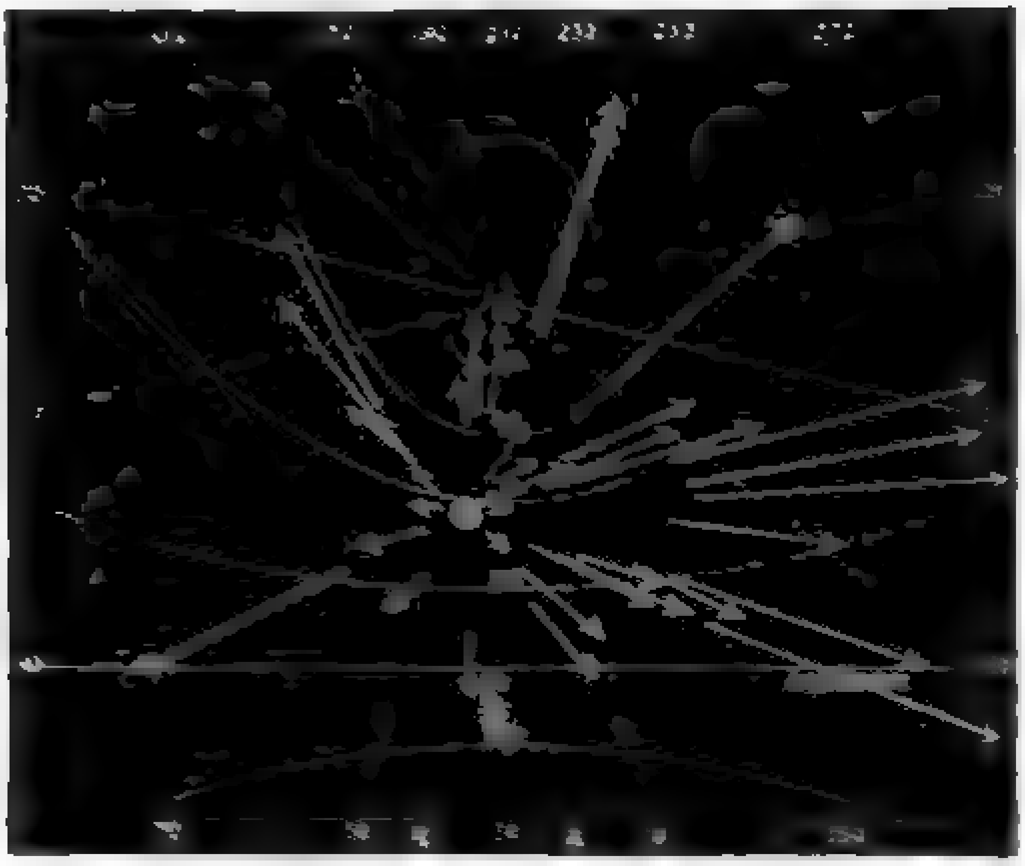


Fig. V.—Shower of Muscids [illegible] Aug. [illegible]

generally slow, and the phosphorescent streaks, forming so persistent a feature of the earlier displays, have given way to occasional trains of [illegible] sparks. The August shower above referred to is not perhaps of special intensity, but it merits description as of contemporary occurrence with the Perseids, and as supplying some fine long meteors in the mornings of August. Thirty-eight paths are shown in the diagram, several of which are notable on account of great length.

The several showers of ζ Perseids, Aquarids, &c. here mentioned, being apparently of little less importance than the [illegible] Geminids, Taurids, &c. will no doubt be frequently seen in future years, and it seems desirable to select them from the mass of feeble systems now ascertained, as affording displays of

than ordinary richness. Many other similar examples of fairly active streams will be found as observations increase, and it will be necessary to single these out for special investigation.

Amongst the radiant points seen at Bristol during the last two years* a few others (in addition to those already specified) have exhibited signs of importance, and the dates and positions of these were as follows:—

	α	δ	↑°	
	°	°		
July 31—August 1	332	+50	14	Lacertids.
July 31—August 1	12	+70	16	
July 31—August 1	321	+31	10	ζ Cygnids.
August 21–23	319	+30	12	
August 1–2	291	+70	14	δ Draconids.
July 26—August 1	28	+36	12	Triangulids.
September 14–25	30	+36	16	
July 25–26	332	+37	11	
August 21–23	46	+47	9	α Perseïds.
September 14–25	99	+43	15	Lyncids.
October 14–20	95	+46	11	
September 21	31	+19	10	Arietids.
September 14–25	76	+32	17	Aurigids.
October 14–15	77	+57	19	
October 15	106	+23	11	Gemellids.
October 4–20	316	+59	17	α Cepheïds.

In each case the radiant may be regarded as accurate.

At the middle of September 1879 a shower was seen from *α Aurigæ*, and in July, September, October, and November many meteors have fallen from an exact centre 3° S. of *α Cassiopeiæ*.

Meteor Showers, 1870–1879. By Henry Corder, Esq.

(*Communicated by W. F. Denning, Esq.*)

The accompanying meteor radiant points have been culled from a list of about 230, chiefly observed between the years 1876–1879, the total number of meteors registered since 1870 being about 5,800, but of this time only the last three years have been made of much real use.

In mapping the meteors, rough star charts copied from an

* The results obtained up to the end of 1877 have already appeared in the *Monthly Notices*. See vol. xxxvi., pp. 283–89; vol. xxxvii., pp. 105–15; and vol. xxxviii., pp. 303–14.

atlas have been used out of doors, and the tracks afterwards laid down on tracing paper over the star charts of the British Association.

In observing the direction of a meteor's path the eye unconsciously prolongs it backwards or forwards to some bright star, and it is therefore found that a ruler held up as near as possible parallel with the track, especially if a streak is left, is a great help in mapping.

The time of watching has generally been from dark till 12 or 1, but occasionally morning watches have been made. Meteors are then more numerous, and the swift streak-leaving class in the best position for observation.

It has lately been found advisable to register all shooting stars in three classes:—

Class 1. The swift streak-leaving meteors.

Class 2. Slow meteors with trains.

Class 3. Small, quick meteors, with short paths, occasionally streak-leaving.

As types, may be selected:—

Class 1. Perseïds, Leonids, Lyrids, Orionids.

Class 2. Andromedes, Taurids I.

Class 3. Geminids.

Out of this year's total of 1,463 meteors (1879), the swift class took 43 per cent., the slow 17 per cent., and the remainder (40 per cent.) were of the small quick class.

The difference in the classes is exemplified by the number of streaks and trains.

Class 1.	63 streaks,	0 trains	per cent.
Class 2.	2 ,,	30 ,,	,,
Class 3.	2 ,,	0 ,,	,,

The fireballs in either class may differ rather from the above. Swift streak-leaving fireballs may have a short train of sparks; and occasionally slow-trained meteors, Taurids for instance, leave narrow streaks; but as a rule these divisions hold good, and it is rarely difficult to tell in which a meteor should be placed. In mapping also they are kept distinct to prevent the chance of adding swift meteors to a radiant of slow ones, or *vice versâ*. Not unfrequently there are two radiants close together, and both active at about the same date. A good example was afforded in December 1877. The Geminids, of Class 3, were preceded by, and partly contemporaneous with, a shower of fine swift meteors,

Class I. This latter radiant (No. 93 in the list) was extremely well defined when its long streak-leaving meteors were sifted out from the short insignificant ones radiating from a few degrees north.

As to colour in meteors, hardly enough have been seen yet to classify them sufficiently well to be of much interest.

About 10 per cent. of all the shooting stars show a distinct colour, the most usual being orange or red. The Taurids and other slow-moving meteors *seem* rarely to get warmed above a red heat; the large ones, or those going a long way, often turn from orange to bluish white like burning magnesium; sometimes the change is very sudden and startling.

Green is a tolerably common colour, especially in slow-moving fireballs about equal to *Venus* in lustre; they generally have a short train of red sparks.

A purple or mauve tint, like that given by copper, is occasionally noticeable.

The following notes have been made on some of the principal showers:—

Lyrids.—Usually white; some pale yellow. A pale green fireball probably also belonged.

Perseïds.—Characterised by deep orange in meteors equal to *Sirius* or *Jupiter*; in smaller 1st mags. yellow; in larger ones a pale green, the meteor suddenly bursting out into a flash like lightning, and leaving a reddish streak. About 20 per cent. show colour.

Orionids.—Not 10 per cent. coloured. First mags. are very rare, and are yellow. The rest white.

Leonids.—Streaks green, meteors yellow; but very few have been seen by the writer.

Andromedes.—A large number orange; also seldom observed.

Taurids.—Over 20 per cent. coloured, usually orange and red, but 1st mags. pale yellow-green, with faint red streaks.

Geminids.—Only 5 per cent. coloured. First mags. rare, generally primrose or white, but the larger ones emerald. One in 1879, deep orange red, was possibly a Geminid.

ι Geminorum.—The shower mentioned above as being almost contemporaneous with the Geminids. Two large meteors in 1877 were purple-mauve in colour.

Writtle, near Chelmsford,
1879, *December.*

List of Radiant Points of Meteors observed in 1876-9 by H. Corder, Writtle, Essex.

Class 1 = swift, streak-leaving meteors.

Class 2 = slow, trained meteors.

Class 3 = commonplace, short meteors, which rarely have streaks.

No.	Date.	R.A. Dec.	Position or Name.	Remarks.	Class.	No. of ☆s
1	Jan., middle	92 + 43	Auriga	A few seen each year.	3	15
2	"	55 + 35	Perseus	Fine meteors; not many seen 1877.	2	
3	Dec.–Jan.	130 + 53	θ ι Ursæ Maj.	Three positions averaged.	1	
4	Jan.	70 + 26	Taurids III		2	3
5	Jan.–Feb.	128 + 27	ι Cancri	Regular; but feeble.	1	10
6	Feb.–March	172 + 62	Ursa Major	In 1877 only.	3	8
7	"	156 + 26	Leo Minor	" "	3 ?	10
8	Feb.–April ?	98 + 37	θ Geminorum	Orange slow-trained meteors.	2	8
9	March	262 0	Ophiuchus	Fine long streaks.	1	3
10	"	280 + 65	Draco	Ill-defined position.	3	10
11	"	247 + 1	Serpens	Principal position out of several adjoining.	3	10
12	"	232 + 26	Corona	Small.	3	12
13	March–April	233 + 11	Serpens		3	12
14	"	200 + 55	ζ Ursæ	Small, short meteors.	3	10
15	"	210 + 23	Arcturus		3	6
16	March	180 − 2	Virgo		2 ?	7
17	April	210 − 7	"	Long continued in 1877.	2	10

18	,,	245 + 55	Quadrans	Badly observed.	1	—
19	,,	312 + 21	Cygnus		2	6
20	April 18–21	275 + 36	Lyrids	Generally white, fairly long meteors; larger ones yellow.	1	50
21	April	174 + 7	Virgo	Nearly all 4th mag.; long continued.	2 or 3	26
22	,,	190 + 20	Coma		3	5
23	April—May	334 − 5	Aquariads	Fine long meteors at daybreak.	1	6
24	,,	300 + 20	Sagitta		1	7
25	May	221 + 1	Virgo		2	5
26	,,	227 − 8	,,	Possibly the same shower.	2	9
27	,,	273 + 55	Draco		3	12
28	June	197 + 45	Canum	Very short; seen at same spot (192 + 38) in March.	3	10
29	,,	240 + 73	γ Ursæ Minoris		3	8
30	,,	320 + 56	Cepheïds	Well seen in 1877, but not much since.	1	14
31	,,	260 − 4	Ophiuchus		2	?8
32	May—Aug.	19 + 57 20 + 60	Cassiopeïds	Almost the same position, but must be distinct showers; always seen in August.	1	40
33	July &c.	333 + 12	Pegasids	Very diffuse; probably two positions.	1	13
34	,,	332 + 48	Lacertids	,, ,, ,,	3	8
35	,,	300 + 8	Aquilids	No defined position seen.	3	12?
36	,,	6 + 40	Andromeda		1	4
37	July—August	295 + 55	Cygnids?	Two green meteors = *Jupiter* &c	2	4
38	,,	337 − 13	Aquariads	Fine long meteors.	2	5

No.	Date.	R.A. °	Dec. °	Position or Name.	Remarks.	Class.	No. of $\downarrow^s$
39	August	44	+ 56	Perseïds	Largest meteors pale green; others orange &c.	1	910
40	July—August	39	+ 52	,,	Several positions before the true Perseïds.	1	
41	August 17 &c.	307	+ 65	Cepheus.	Regular shower.	1	18
42	,,	304	+ 58	,,	Slow meteors.	2	7
43	,,	298	+ 55	,,	Small quick meteors.	3	7
44	,,	335	+ 69	,,		3	7
45	,,	320	+ 22	*f* Pegasi		1	9
46	,,	23	+ 72	Custos	Also 30° + 64° and 45° + 70°, doubtful.	3	11
47	August	62	+ 48	μ Persei	Many other indistinct positions seen every year about the end of August.	1	46
48	,, end	53	+ 44	ν Persei		1	
49	August—Sept.	3	+ 7	Pegasus	Seen in two years.	3	12
50	,,	332	0	Aquariads	Slow, orange mets.; difficult position to distinguish.	2	12
51	,,	351	+ 4	Piscids		2	34
	,,	3	− 2	,,		2	
52	Sept.	30	+ 40	Triangulum	All seen in 1877, and scarcely seen since; all of the streak-leaving class.	1	20
53	,, middle	45	+ 50	Perseus		1	20
54	,,	72	+ 18	Taurus		1	6
55	,,	92	+ 34	Auriga		1	6
56	,,	80	+ 30	,,		1	7
57	,,	50	+ 20	Musca		1	10
58	,,	5	+ 50	Cassiopeia		3	9

59	,,		294+23	Cygnus		3	7
60	,,		297+50	,,	Much finer meteors than 59.	2	7
61	,,		331+48	Lacertids	Short meteors.	3	14
62	,,		15+16	η Piscium		3	13
63	,,		45+34	β Persei	Fine meteors on one evening (7th 1879).	1	6
64	,,		340+86	Polaris		3	12
65	Oct.		334+15	α Pegasi		3	7
66	,,		320+80	κ–γ Cephei	Fine meteors, slow, but with streaks.	1	9
67	,,		75+45	Aurigids		1	13
68	,,		11+22	Pegasus		3	7
69	,,		20+25	η Andromedæ	Perhaps connected with 68.	3	8
70	,,		30+20	Aries	Fine meteors.	1 ?	6
71	,,		25+ 8	o Piscium	There are several radiants of small meteors about this place and date difficult to distinguish.	2	60
72	,,		32+11	ξ Ceti		3	
73	,,		23+ 8	o Piscium		3	
74	,,		110+14	Canis Minor		1	9
75	,,		127+54	ι Ursæ Majoris		1	17
76	,,		106+27	Gemellids		1	8
77	,,		32+34	Triangulum		3	13
78	,,	16–22	90+14	Orionids	Fine shower, but augmented by the following two. Very few large ones, but many streaks; possibly all connected.	1	164
* 79	,,	,,	88+24	η Geminorum		1	
80	,,	,,	93+19	ν Geminorum		1	

No.	Date.	R.A. ° Dec. °	Position or Name.	Remarks.	Class.	No. of ↓*
81	Oct. 27	102 + 19	ζ–γ Geminorum	Very swift; many streaks.	1	16
82	„ 9–22	95 + 34	θ Geminorum	Chiefly on Oct. 9, but more on 22nd? (distinct).	1	10
83	„	306 + 64	Cepheus	Poorly marked.	3	10
84	Oct.—Nov.	43 + 15	*f* Tauri	Small meteors preceding true Taurids.	3	34
85	„	45 + 47	Perseus	White, small meteors, long continued.	3	34
86	„	41 + 13	Aries		3	13
87	„	51 + 7	*o* Tauri		1	14
88	Nov.	59 + 20	Taurids	Always interesting; slow meteors.	2	195
89	„	148 + 25	Leonids	Not properly mapped till 1879.	1	30
90	„ 27 &c.	23 + 44	Andromedes	Only a few seen.	2	30
91	„	140 + 40	Ursites	Average of a number of vague positions.	1	30
92	„	205 + 68	α Draconis		3	9
93	Dec. 9 &c.	108 + 28	ι Geminorum	Fine long meteors; very different to 94.	1	20
94	„ 10–13	107 + 35	Geminids	Larger ones, pale green; very few streaks.	3	206
95	„	78 + 23	Taurids II		2	20

The November Meteors. By the Rev. S. J. Perry.

The utility of acquiring as complete a knowledge as possible of every detail respecting the important meteor stream that radiates in November from the Lion has induced me to keep up an annual watch from 10 P.M. till sunrise on every night that offers any chance of observation from the 12th to the 16th. The weather of late years has been generally unfavourable; but this year the sky was perfectly cloudless, and this, with the absence of the Moon, presented an opportunity not often met with at this season of the year. On the morning of the 13th only one observer was employed; but during the remainder of the time two observers were stationed facing the N.E., and the watch was uninterrupted from 10 P.M. till sunrise on the 13th, 14th, and 15th.

	Total No. of Meteors.	Mag. greater than 1st.	1st and 2nd Mags.	3rd and 4th Mags.	5th and 6th Mags.
Nov. 13	67	2	25	31	9
„ 14	144	4	39	45	51
„ 15	98	4	31	28	27

The paths of nearly all the meteors could be accurately noted, and the following was the proportion that radiated from the Lion :—

	Number.	Mag. greater than 1st.	1st and 2nd.	3rd and 4th.	5th and 6th.
Nov. 13	16	1	2	10	2
„ 14	53	1	11	10	30
„ 15	35	2	14	12	12

The increase on the morning of the 14th was principally due to meteors of the 5th and 6th magnitude, which constituted considerably more than half the total; on the 15th the magnitudes were very equally distributed.

It may be useful to give the paths of the most brilliant meteors for the sake of comparison with other stations.

	G.M.T. h m	From	Towards
Nov. 12	10 36 p.m.	3° below *β Arietis*	*η Ceti.*
„ 13	3 38 a.m.	*δ Leonis*	*Spica.*
	11 8 p.m.	3° left of *Bellatrix*	Midway between *Sirius* and *Procyon.*
„ 14	4 33 a.m.	*λ Leonis*	Head of *Hydra.*
	5 49	*α Hydræ*	S. Horizon.
„ 15	2 24 a.m.	*ξ Hydræ*	5° W. of *α Hydræ.*
	3 39	*μ Orionis*	*β Eridani.*
	4 9	½ dist. from λ to *κ Leonis*	*γ Canis Minoris.*
	4 34	*κ Leonis*	3° N. of *β Canis Min.*

A first magnitude star was observed on the 15th at $1^h\ 34^m$ A.M. perfectly stationary at R.A. 157° 18′, N.P.D. 52° 30′.

The time of the maximum for this year will be best presented in a tabular form:

		Midnight to 1 a.m.	1 to 2.	2 to 3.	3 to 4.	4 to 5.	5 to 6.
Nov. 13	Total meteors	9	9	7	3	6	3
	Leonids	2	4	3	2	4	1
" 14	Total meteors	12	19	22	17	23	19
	Leonids	7	9	6	7	15	9
" 15	Total meteors	10	13	18	23	17	
	Leonids	3	7	5	9	8	

This Table shows that the Leonids were fairly distributed throughout the morning of the 15th, and were generally as numerous as on the 14th. The only approach to a maximum was between 4 and 5 A.M. on the 14th. The principal radiant was slightly below the line joining ε and ζ *Leonis*, and rather nearer the former star, or at R.A. 147° 20′, N.P.D. 67° 10′. The positions of the secondary radiants were R.A. 166°, N.P.D. 67° 40′ between δ and 72 *Leonis*, and R.A. 155° 12′, N.P.D. 54° 54′ between 31 and 37 *Leonis Minoris*. Few of the meteors had trains, and none were visible for more than 4^s.

A large number of meteors radiated towards a point near *Cor Caroli*.

On the 27th the sky was fairly clear from sunset to midnight, but the Moon very bright. No meteors were seen in the direction of *Andromeda*.

Stonyhurst Observatory,
1879, *Dec.* 6.

Occultation of 64 *Aquarii by Jupiter, observed at the Melbourne Observatory*, September 14, 1879. By R. L. J. Ellery, Esq., Director of the Observatory.

Observed by Mr. Ellery.

Telescope, 8 inches aperture; power 300.

Sky clear and objects steady some time before first contact.

The smallness and faint light of the star as compared with the satellites first attracted attention. The grand colouring of *Jupiter's* belts—one quite purple (with streaks of brick-red in the temperate zones), and the other greenish grey and broad—was remarkable. The star first appeared to touch the planet's limb at $10^h\ 5^m\ 19^s$ M.M.T., and was visible in that position for nearly two minutes, when, while still making a projection on the planet's outline, it all at once appeared as if seen through a mist or haze, *and entirely projected on the planet's limb*. This faded away in about ten seconds, leaving still a decided nipple-like projection on the

edge of the planet, as if the planet itself bulged out, without any signs of the true light of the star; and at $10^h\ 7^m\ 43^s.8$ this disappeared, leaving a clean outline to the disk.

The light of the star, although it appeared faint compared to that of the satellites, appeared as a bright speck on the planet's limb when actually in contact with it.

Observed by Mr. E. J. White, with the North Equatoreal, $4\frac{1}{2}$ inches aperture, 5 feet focal length: power 200. Used the full aperture, and a positive eye-piece magnifying 200 times. The time was taken with the sidereal chronometer, Barraud $\frac{2}{861}$, which was compared with the Transit Clock a few minutes after the observation of disappearance, and found to be $1^m\ 4^s.0$ slow of Melbourne sidereal time.

The first impression on looking through the telescope was surprise at the small quantity of light of the star as compared with that of the satellites.

Watched the star, which became nearly overpowered by *Jupiter* as it got near the limb, so that the observation was very difficult. Although the definition was very good in a cloudless sky, yet slight oscillations of the planet occurred which made the star momentarily disappear. At $21^h\ 37^m\ 46^s$ by chronometer I thought the star had really disappeared; but on looking again I saw it projecting as a bright nipple on *Jupiter*, which seemed to gradually lessen in size till $21^h\ 39^m\ 13^s$, when I finally lost sight of it. Soon after this the sky became rather hazy, which quite prevented any observation of the reappearance. The above chronometer times correspond to $10^h\ 6^m\ 23^s.7$, and $10^h\ 7^m\ 40^s.4$ Melbourne mean time.

Observed by Mr. J. Turner, with the Great Melbourne Telescope: power 350.

As the star approached close to the limb of *Jupiter*, the definition (hitherto very unsatisfactory) so improved as to be all that could be desired; the limb of the planet being sharp and steady, thus enabling the contact to be well observed.

At the moment of contact the star did not instantly disappear, but seemed to possess a visible disk, the limb of *Jupiter* seeming to advance gradually upon it, the star by-and-by appearing to be bisected and then gradually disappearing altogether.

The time of final disappearance was $10^h\ 7^m\ 47^s.6$ M.M.T., at which instant the circle of *Jupiter's* limb appeared perfect; previous to this the star appeared as a small protuberance upon the limb, which *gradually* got smaller until final disappearance.

The time of first contact was not noted, but I estimated the interval between contact and disappearance at about 35 seconds—certainly not less; it might be more.

For about 10 seconds after disappearance the star could be seen *through Jupiter's* atmosphere as a speck of light seen through ground glass. This also disappeared *gradually*.

There was no fitful disappearance and reappearance, but gradual disappearance throughout.

Reappearance.

About an hour after disappearance clouds set in with occasional narrow breaks.

At $12^h\ 26^m\ 1^s$ got a clear view of planet, but no trace of star visible,

At $12^h\ 34^m\ 47^s$ got another clear view for about 5 seconds, and could clearly see a minute protuberance where the star was expected to appear. This protuberance exactly resembled the one half of the disk-like appearance of the star at disappearance.

At $12^h\ 37^m\ 57^s$ got another view for about 10 seconds, when the star was seen well separated from *Jupiter*. The small protuberance noted 3 minutes previously was thus proved to be the reappearance of the star.

Heavy clouds now set in, compelling closing up.

The times above noted are Melbourne mean time.

Occultations of Stars by the Moon, seen at the Stonyhurst Observatory.

By the Rev. S. J. Perry.

1879.	Phenomenon.	Moon's Limb.	Power.	G.M.T. h	m	s	Observer.	Remarks.
October 5	Disapp. of *k Tauri*	Bright	303	11	18	43·1	S.J.P.	Well seen up to limb.
	Reapp. „	Dark	„	12	23	49·0	„	Very good.
November 23	Disapp. of 45 *Piscium*	Dark	350	10	49	9·0	W.C.	Instantaneous; sky rather hazy.
„ 27	Disapp. of 9 *Tauri*	Dark, but close to bright	„	11	36	10·0	S.J.P.	Very good.
„ 30	Disapp. of 5 *Geminorum*	Bright	„	9	21	41·7	„	Star indistinct when close to limb.
	Reapp. „	Dark	„	10	17	30·4	„	Good.

On December 1 44 *Geminorum* was not occulted, although Stonyhurst (lat. 53° 50′ 40″) is just within the required limits.

The above observations were all taken with the 8-inch Equatoreal, and generally a translucent screen was used in the eye-piece, covering rather less than half the field of view.

Observations of Jupiter's Satellites, made at the Stonyhurst Observatory. By the Rev. S. J. Perry.

1879.	Satellite.	Phenomenon.	G.M.T. h	m	s	In excess of *N.A.* m	s	Observer.	Remarks.
June 4	I	Ec. D. last seen	14	28	57·5	+0	17·5	W.C.	Fair, daylight, thin clouds passing.
Sept. 28	„	Tr. I. external contact	11	55	47·6			„	Good.
		bisection	11	58	9·7				
		int. contact	12	0	36·6				
	„	Sh I. int. contact	12	42	17·1			„	Very poor, faint. Very unsteady.
„ 29	„	Oc. D. ext. contact	9	12	19·7			„	Poor, tremulous, ill defined.
		bisection	9	14	5·7				
		last seen	9	16	22·2				
	„	Ec. R. first seen	12	13	59·4	−1	8·6	„	Slightly unsteady, but definition very good.
		full light	12	16	29·4				
Oct. 14	„	Tr. I. bisection	10	0	21·5			„	Difficult, thin clouds, very tremulous.
		Sh. I. bisection	11	0	41·3			„	Poor, very tremulous.
		internal contact	11	1	56·4				
		Tr. E. bisection	12	18	52·4			„	
		external contact	12	21	31·1				
„ 15	„	Oc. D. external contact	7	11	39·6			„	
		bisection	7	14	42·6				
		last seen	7	17	9·7				

	I	Ec. R.	first seen	10	34	59·7	−0	8·3	S.J.P.	Very good.
			nearly full light	10	36	9·6				
„ 20	III	Tr. E.	bisection	11	2	54·6			„	Steady.
			external contact	11	7	20·6				
	„	Sh. I.	bisection	12	1	3·6			„	
			internal contact	12	5	26·1			W.C.	
„ 25	II	Tr. I.	external contact	5	56	54·8			W. McK.	Unsteady.
			bisection	5	59	41·8				
			internal contact	6	2	39·3				
	„	Sh. I.	bisection	8	24	22·4			W.C.	Good; cloudy, tremulous.
			internal contact	8	25	43·7				
	„	Tr. E.	internal contact	8	47	56·9			W. McK.	
			bisection	8	52	11·7				
			external contact	8	54	26·4				
	IV	Ec. D.	half light	9	23	57·8			S.J.P.	Sky very clear; compared with Satellite II.
			quarter light	9	26	11·6				
			last seen	9	29	38·8	−4	50·2		
	II	Sh. E.	bisection	11	11	39·5			W. McK.	Rather indistinct; approximate.
„ 27	„	Ec. R.	first seen	5	23	16·2	−0	32·8	S.J.P.	Compared with Satellite I.
			half light	5	25	8·2				
			full light	5	27	17·7				

1879.	Satellite.	Phenomenon.	G.M.T. h	m	s	In excess of *N.A.* m	s	Observer.	Remarks.
	III	Tr. I. external contact	11	3	21·5			W. McK.	External contact perhaps too early; poor definition; very unsteady.
		bisection	11	8	26·0				
		internal contact	11	14	19·5				
Nov. 1	II	Tr. I. external contact	8	29	15·6			S.J.P.	Rather tremulous.
		bisection	8	32	19·5				
		internal contact	8	34	46·7				
„ 2	IV	Tr. I. external contact	8	15	58·0			„	External contact possibly a trifle late; definition good.
		bisection	8	19	21·0				
		internal contact	8	23	17·0				
„ 7	I	Oc. D. external contact	7	14	1·8			W.C.	Cloud; very poor.
		bisection	7	14	59·3				
„ 10	II	Oc. D. external contact	5	9	27·2			S.J.P.	Very steady; definition good. Between bisection and disappearance satellite apparently projected on to limb.
		bisection	5	12	37·5				
		last seen	5	15	36·2				
	„	Ec. R. first seen	10	36	49·2	−0	26·8	„	Slightly hazy.
		half light	10	39	37·5				
		full light	10	41	40·7				
„ 14	III	Oc. D. external contact	8	31	48·5			„	
		bisection	8	35	26·0				

			last seen	8	39	59·0			
	I	Oc. D.	external contact	9	7	0·0		„	
			bisection	9	10	13·0			
			last seen	9	12	41·0			
„ 23	„	Oc. D.	external contact	5	28	14·8		W.C.	Very unsteady.
			bisection	5	30	50·3			
			last seen	5	33	0·3			
	„	Ec. R.	first seen	9	11	31·1	+0 12·1	„	Slightly hazy; good. Compared with Sat. II.
			half light	9	12	16·3			
			full light	9	13	4·1			
„ 27	IV	Oc. D.	external contact	8	51	41·4		S.J.P.	Clear, but unsteady, passing clouds; approx.
			bisection	9	0	23·9			
„ 30	I	Oc. D.	external contact	7	25	56·2		W.McK.	Unsteady.
			bisection	7	28	21·7			
			last seen	7	30	40·2			
Dec. 1	„	Sh. I.	internal contact	6	2	34·5		W.C.	Good; shadow very dark.
	„	Tr. E.	internal contact	6	54	28·0		„	Satellite very faint.
			bisection	6	56	21·5			
			external contact	6	59	56·5			
	„	Sh. E.	internal contact	8	14	3·7		„	Very poor.
			bisection	8	15	50·2			
			last seen	8	17	34·2			

1879.	Satellite.	Phenomenon.	G.M.T. h m s	In excess of *N.A.* m s	Observer.	Remarks.
Dec. 2	,,	Ec. R. first seen	5 36 13·8	+0 22·8	S.J.P.	Very good, sky not very clear; compared with Satellite III.
		full light	5 38 57·7			
	III	Tr. I. external contact	6 44 3·0		,,	Misty; definition very bad.
		bisection	6 51 25·5			
,, 3	III	Tr. I. external contact	8 5 34·9		,,	Definition very poor; hazy.
		bisection	8 9 3·1			
		internal contact	8 12 59·0			
,, 6	III	Ec. R. first seen	5 28 0·0		,,	
		half light	5 29 56·3			
		full light	5 32 8·6			
,, 8	I	Tr. I. bisection	6 36 4·1		W.McK.	
		internal contact	6 39 36·2			
	,,	Tr. E. bisection	8 57 58·8		S.J.P.	Misty, definition bad. After transit Satellite I. was only visible by glimpses, although the other satellites were well seen at the time. Cut off glare of planet, and yet Satellite I. only just visible for more than half an hour after transit. Used powers from 100 to 350.
		external contact	9 0 34·8			

Observations of Occultations of Stars by the Moon, and of Phenomena of Jupiter's Satellites, made at the Royal Observatory, Greenwich, in the Year 1879.

(*Communicated by the Astronomer Royal.*)

Occultations of Stars by the Moon.

Day of Obs.	Phenomenon.		Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation.	Ob-server.
1879.						h m s	
Mar. 30 (*a*)	Disapp. of	Piazzi vi. 165	E. Eq.	310	Dark	7 16 3·8	AD
Apr. 30 (*b*)	„	Bradley 1393	„	140	„	9 56 13·6	R
May 3	„	*q* Virginis	Altaz.	100	„	9 19 32·6	GP
July 28 (*c*)	„	Antares	S.E. Eq.	220	„	9 37 38·5	WC
„ (*d*)	„	„	Simms No. 2	60	„	9 37 38·8	W
„ (*e*)	„	„	Altaz.	100	„	9 37 38·9	BD
„ (*f*)	„	„	Simms No. 1	145	„	9 37 39·2	M
„ (*g*)	„	„	E. Eq.	140	„	9 37 39·2	GP
„ (*h*)	Reapp.	„	Altaz.	100	Bright	10 6 44·6	BD
„	„	„	E. Eq.	140	„	10 6 46·0	GP
Aug. 25 (*i*)	Disapp.	Bradley 2174	„	„	Dark	8 5 38·9	AD
Sept. 26	„	λ Capricorni	Altaz.	100	„	10 40 20·1	T
Oct. 4	Reapp.	36 Tauri	E. Eq.	140	„	11 39 35·8	TP
Nov. 18 (*k*)	Disapp.	σ Capricorni	„	„	„	6 35 41·0	C
„ (*l*)	„	„	Altaz.	100	„	6 35 42·4	AD
Nov. 22	„	16 Piscium	E. Eq.	140	„	7 55 43·7	C
Dec. 22 (*m*)	„	101 Piscium	Altaz.	100	„	5 6 55·4	T
„ (*n*)	„	„	E. Eq.	140	„	5 6 57·4	C

Notes.

(*a*) Instantaneous; registered on the chronograph.

(*b*) Instantaneous.

(*c*) The *comes* disappeared gradually about 12 seconds before *Antares*. It was very distinctly seen just clear of the rays from *Antares*, and was of a deep blue colour. *Antares* disappeared instantaneously.

(*d*) The star's image blurred; disappearance instantaneous.

(*e*) The star seemed to disappear instantaneously, but was a little diffused about a second before disappearance.

(*f*) Instantaneous; observed with the Airy eye-piece. The companion was seen clearly till about two minutes before the disappearance, but the star then became very tremulous, and the companion was lost in its rays.

(*g*) Disappeared instantaneously.

(*h*) The limb of the Moon was rather tremulous, but the observed time is probably within half a second. No particular change of colour was noticed.

(*i*) Instantaneous; registered on the chronograph.

(*k*) Observation considered very good.

(*l*) Observation accurate; the observer could see the dark limb of the Moon; registered on the chronograph.

(*m*) Star faint; observation uncertain.

(*n*) Good.

Phenomena of Jupiter's Satellites.

Day of Obs.	Satellite.	Phenomenon.	Telescope.	Power.	Mean Solar Time of Observation.	Mean Solar Time from *N.A.*	Observer.
1879.					h m s	h m s	
June 28	I	Tr. ing. first contact	Altaz.	100	13 3 13	13 12	AD
„	I	„ last contact	„	„	13 8 57		
July 1	III	Ecl. reapp. first seen	„	„	13 6 46	13 8 35	C
Aug. 5	I	Ecl. disapp. first obs.	E. Eq.	140	13 6 16	13 8 20	AD
„	I	„ last seen	„	„	13 8 50		
„ (*a*)	I	„ first obs.	S.E. Eq.	220	13 6 4	13 8 20	M
„	I	„ dichotomised	„	„	13 7 14		
„	I	„ last seen	„	„	13 9 1		
Aug. 6 (*b*)	II	Occ. reapp.	E. Eq.	140	12 49 38	12 51	J
Aug. 21 (*c*)	I	„ bisection	„	„	13 57 32	13 58	BD
„	I	„ last contact	„	„	13 59 46		
Aug. 22	II	Tr. ing. last contact	Altaz.	100	9 4 53	9 4	R
„ (*d*)	II	Tr. egr. first contact	E. Eq.	140	11 52 26	11 55	„
„	II	„ bisection	„	„	11 54 6		
„	II	„ last contact	„	„	11 55 36		
Aug. 28 (*e*)	I	Ecl. disapp. last seen	„	„	13 21 27	13 21 32	„
Sept. 15	I	Ecl. reapp. first seen	„	„	8 24 29	8 24 25	AD
„	I	„ full brightness	„	„	8 26 50		
Sept. 25	III	„ first seen	„	„	13 13 0	13 15 30	J
„	III	„ half brightness	„	„	13 14 35		
„	III	„ full „	„	„	13 16 50		
Oct. 7	I	Tr. Ing. first contact	„	„	8 9 20	8 11	
„	I	„ bisection	„	„	8 12 19		
„	I	„ last contact	„	„	8 15 29		

Day of Obs.	Satellite.	Phenomenon.	Telescope.	Power.	Mean Solar Time of Observation.	Mean Solar Time from *N.A.*	Observer.
1879.					h m s	h m s	
Oct. 9	II	Ecl. reapp. first seen	E. Eq.	140	10 52 5	10 53 5	C
„ 15	I	„ „	„	„	10 35 17		
„	I	„ half brightness	„	„	10 36 36	10 35 8	J
„	I	„ full	„	„	10 37 21		
Oct. 20 (*f*)	III	Tr. Ing. first contact	S.E. Eq.	130	7 30 51		
„	III	„ bisection	„	„	7 34 55	7 32	M
„	III	„ last contact	„	„	7 39 30		
Oct. 25	II	Tr. ing. first contact	E. Eq.	140	6 2 7	6 3	C
„	II	„ last contact	„	„	6 6 21		
„	II	Tr. egr. bisection	„	„	8 49 10	8 55	„
„	II	„ last contact	„	„	8 51 39		
„	IV	Ecl. disapp. last seen	„	„	9 29 53	9 34 29	„
Nov. 7	III	Occ. reapp. bisection	„	„	8 15 9	8 20	AD
„	III	„ last contact	„	„	8 17 24		
„ (*g*)	III	Ecl. disapp. first obs.	„	„	9 49 24	10 6 52	„
„	III	„ last seen	„	„	9 55 8		
Nov. 11 (*h*)	IV	Ecl. reapp. first seen	S.E. Eq.	310	7 37 23 ±	7 38 50	WC
„	IV	„ full brightness	„	„	7 53 20 ±		
„	IV	„ first seen	E. Eq.	140	7 37 1	7 38 50	AD
„	IV	„ full brightness	„	„	7 43 0		
Nov. 14	III	Occ. disapp. last contact	„	„	8 39 6	8 38	T
„	I	„ „	„	„	9 11 31	9 11	„
Nov. 15	III	Tr. ing. first contact	„	„	6 19 23		
„	III	„ bisection	„	„	6 21 3	6 19	AD
„	III	„ last contact	„	„	6 22 38		
Nov. 19 (*i*)	IV	Tr. egr. first contact	S.E. Eq.	220	6 5 1	6 20	M
„ (*k*)	IV	„ last contact	„	„	6 16 0		
Dec. 2 (*l*)	I	Ecl. reapp. first seen	E. Eq.	140	5 36 12	5 35 51	AD
„	I	„ full brightness	„	„	5 39 22		
„	III	Tr. ing. first contact	„	„	6 40 12	6 44	„
„	III	„ last contact	„	„	6 51 40		
„	I	Ecl. reapp. first seen	S.E. Eq.	130	5 35 48		
„	I	„ half brightness	„	„	5 36 54	5 35 51	M
„	I	„ full brightness	„	„	5 38 44		
„	III	Tr. ing. first contact	„	„	6 45 33		
„	III	„ bisection	„	„	6 49 3	6 44	„
„	III	„ last contact	„	„	6 52 47		

Day of Obs.	Satellite.	Phenomenon.	Telescope.	Power.	Mean Solar Time of Observation.	Mean Solar Time from *N.A.*	Observer.
1879.					h m s	h m s	
Dec. 6 (*m*)	III	Ecl. reapp. first seen	„	285	5 27 20	5 30 42	„
„ (*n*)	III	„ „	T.C.	100	5 29 41	5 30 42	TP
Dec. 8 (*o*)	I	Tr. ing. first contact	E. Eq.	140	6 32 13	6 35	J
„	I	„ bisection	„	„	6 35 28		
„	I	„ last contact	„	„	6 38 48		

Notes.

(*a*) Definition very good; observed with the Airy eye-piece.

(*b*) Planet very tremulous; limb boiling.

(*c*) Tremulous.

(*d*) Definition good; the satellite was seen on *Jupiter* three or four minutes before the first contact.

(*e*) Disappeared close to the limb of *Jupiter*.

(*f*) Limb boiling.

(*g*) Observation not satisfactory; *Jupiter* was low down in the mist.

(*h*) Estimated time of reappearance; the satellite was first noticed one minute later, and was then quite faint.

(*i*) *Jupiter* very unsteady; limb boiling. The satellite was seen just for a moment, and was perhaps $\frac{1}{8}$ off the limb.

(*k*) Four minutes before, at $6^h\ 12^m$, the satellite was perhaps $\frac{3}{4}$ off. At $6^h\ 14^m$ it was suspected to be clear of the planet, but it seemed to join the limb again. Clouds were passing continually during the observation.

(*l*) Definition good at this and the following observation.

(*m*) The first faint sparkle of the satellite was admirably seen.

(*n*) Observed with the field half illuminated, while *Jupiter* was passing the wires in the Transit-Circle telescope.

(*o*) *Jupiter* very tremulous and ill-defined.

The clear aperture of the object-glass of the S.E. Equatoreal is $12\frac{3}{4}$ inches of the East Equatoreal 6·7 inches, of the Altazimuth $3\frac{3}{4}$ inches, and of th Transit-Circle 8 inches.

The initials WC, C, AD, M, T, W, BD, R, GP, J, and TP are those Mr. Christie, Mr. Criswick, Mr. Downing, Mr. Maunder, Mr. Thackers Mr. Wickham, Mr. Dennison, Mr. Robinson, Mr. Pearce, Mr. James, a Mr. Plucknett.

Royal Observatory, Greenwich,
1880, *January* 2.

On the Rotation-period of Jupiter.

By H. Pratt, Esq.

On the 24th July 1879, between $11^h\ 30^m$ and 12^h G.M.T., on turning my telescope on *Jupiter* for the first time this season, I saw, near the middle of the great white belt on the south of the equatoreal zone, a finely developed reddish spot, or short belt. At that time its apparent centre was about half-way from the central meridian to the preceding limb. From its definiteness of outline and vividness of colour, I concluded it would prove useful as a means of obtaining a new determination of the rotation-period; and accordingly made observations for this purpose on every available occasion, and from them have derived the results which are detailed below.

The only previous determinations of *Jupiter's* rotation-period with which I am acquainted, are as follows:—

Author.	Date.	Rotation-period. h	m	s	Authority.
Cassini	1665	9	55	49·5	Dick, *Celestial Scenery.*
„	„	9	56		Grant, *Hist. of Astron.*
„	1672	9	55	50	„ „
„	1692	9	50		„ „
Maraldi	1708	9	56	48	„ „
„	1713	9	56		„ „
Sylvabelle	1773	9	56		Arago, *Pop. Astron.*
Herschel, W.	1779	9	55	40	Grant, *Hist. of Astron.*
„	„	9	55	48	„ „
Schroeter	1786	9	55	33·6	„ „
„	„	9	56	56	Arago, *Pop. Astron.*
„	„	9	55	18	„ „
Schmidt	No date	9	55	28·7	Chambers, *Astron.*
Mädler	1835	9	55	29·9	Hind, *Solar System.*
Airy	„	9	55	21·3	*Mem. R.A.S.*, vol. ix.
Schmidt	1866	9	55	46·3	Webb, *Celes. Objects.*

Considering the three most recent determinations, when the observers, the instruments, and the methods were all of the best, it may seem folly even to attempt the work again. Indeed it may prove impossible ever to arrive at a nearer knowledge of the true rotation-period of the planet. Yet, as the harmony between those three results is not perfect, and in view of the suggested cause of discrepancy being a proper motion of the spots made use of, I beg to submit (with what value may be attached to them) the following observations and inferences therefrom.

The observations themselves consisted of the Greenwich Mean Times of the transits of the middle of the spot across *Jupiter's* central meridian. These were obtained by means of micrometer measures of the distances of the ends of the spot from the E. and W. limbs of the planet. When these distances were equal it was concluded that the middle of the spot was on the central meridian. My clock's rate had been kept corrected by the frequent use of my transit-instrument.

The times of transit of the spot, found in this manner, were:—

			h	m	s
1	1879	July 26	12	25	0
2	,,	Sept. 3	9	19	
3	,,	,, 10	10	1	30
4	,,	,, 27	8	59	
5	,,	Oct. 1	12	15	30
6	,,	,, 6	11	22	15
7	,,	,, 26	7	50	
8	,,	Nov. 2	8	35	20
9	,,	,, 5	6	4	20
10	,,	,, 12	6	50	
11	,,	,, 19	7	35	30
12	,,	Dec. 6	6	36	

Arranging these observations in two series; taking them consecutively in pairs; and designating the intervals in the first series, between 1 and 2 (a), 2 and 3 (b), 3 and 4 (c); and in the second series between 1 and 2 (α), 1 and 3 (β), 1 and 4 (γ), and so on, we have:—

Intervals.	No. of Rotations.	Time elapsed in Mean Solar Seconds.
(a)	94	3,358,440
(b)	17	607,350
(c)	41	1,465,050
(d)	10	357,390
(e)	12	428,805
(f)	48	1,715,265
(g)	17	607,520
(h)	7	250,140
(i)	17	607,540
(j)	17	607,530
(k)	41	1,465,230
(α)	94	3,358,440
(β)	111	3,965,790
(γ)	152	5,430,840

Intervals.	No. of Rotations.	Time elapsed in Mean Solar Seconds.
(δ)	162	5,788,230
(ϵ)	174	6,217,035
(ζ)	222	7,932,300
(η)	239	8,539,820
(θ)	246	8,789,960
(ι)	263	9,397,500
(κ)	280	10,005,030
(λ)	321	11,470,260

The method by which I have arrived at the final result has been to take, as an assumed period of rotation, 35,726 seconds, multiply it by the number of rotations, and, after having corrected it for the difference of the Jovi-centric longitudes of the Earth at the two stations, to subtract the total from the time elapsed. Then, if the assumed rotation-period is in accord with the observed times when the middle of the spot was found to be on the planet's central meridian (allowing for the equation of light), there will be no remainder. If it is not so, then the remainder divided by the number of rotations will give a result which, after it is corrected for the differences of the equation of light due to the varying distances between the Earth and *Jupiter* at the times of the two observations under discussion, will give the final correction to the assumed rotation-period which the case requires.

As this last correction is in every case *plus*, it remains merely to add it to the assumed period in order to obtain that value of the time of *Jupiter's* rotation which is indicated by that pair of observations.

Treating the whole of the two series, in pairs of observations, by this method, I obtain the following corrections:—

	secs.		secs.
(*a*)	7·06	(α)	7·06
(*b*)	4·86	(β)	6·71
(*c*)	9·73	(γ)	7·47
(*d*)	15·23	(δ)	7·99
(*e*)	8·13	(ϵ)	7·96
(*f*)	7·86	(ζ)	7·90
(*g*)	7·57	(η)	7·97
(*h*)	6·95	(θ)	7·92
(*i*)	7·70	(ι)	7·78
(*j*)	7·68	(κ)	7·72
(*k*)	7·60	(λ)	7·43
Mean	8·21	Mean	7·62

and

$$\frac{8·21 + 7·62}{2} = 7·91 \text{ secs.} = \text{the final correction.}$$

With regard to the relative value of the two series, undoubtedly that of the second is the greatest, for including as it does a much larger number of rotations of the planet between each pair of observations, its results must therefore be the more accurate of the two.

From the fairly close accord between the means of the two sets of double observations, we may assume with great probability *that the spot was without variable proper motion*; for what differences there are arise more probably from the unavoidable errors of observation. But I have thought it best to exhibit them thus plainly in order that a proper estimate may be formed of the whole matter; for, after all, the difference of the means does not exceed one second of time.

Therefore, if the mean of those two means, $= 7^{s}\cdot91$, is added to the assumed period, giving $9^{h}\ 55^{m}\ 33^{s}\cdot91$ of Mean Solar Time, that may perhaps be accepted as the nearest approximation which I am able to derive from the whole of my observations, which include 321 rotations of *Jupiter*.

By substituting for the assumed period this now corrected value of it, and by calculating what should have been, according to it, the times of transit of the spot across *Jupiter's* central meridian on the dates of observation, we arrive at the following discrepancies between prediction and observation:—

	In Time. secs.		In Jovi-centric longitude. °
(α)	+ 79	or	0·7
(β)	+ 132	,,	1·3
(γ)	+ 65	,,	0·6
(δ)	− 14	,,	0·01
(ϵ)	− 20	,,	0·2
(ζ)	− 9	,,	0·09
(η)	− 15	,,	0·01
(θ)	− 5	,,	0·05
(ι)	+ 33	,,	0·3
(κ)	+ 54	,,	0·5
(λ)	+ 154	,,	1·5

from which it is apparent that the greatest of these discrepancies is 154^{s} in time of observation, equal to $1^{\circ}\cdot5$ of longitude on *Jupiter*, which is well within the limits of possibility.

From the experience gained in making the original observations, I have concluded that it would have been impossible for me to have made an error exceeding two minutes, or two minutes and a half, in determining the times of central transit across *Jupiter's* disk. This amount doubled, as a possible excess or defect at the times of the first and last observations, amounts

but $\frac{1}{38,000}$th of the whole. From which it follows that *the probable error* in my determination is less than one second of time.

The rotation-period which is perhaps most frequently quoted in astronomical books is $9^h\ 55^m\ 26^s$. During the 321 rotations which I have observed during this series, that period would produce a discordance equal to $42^m\ 19^s$; and by that time, not only the middle, but the following end of the spot would have considerably passed the central meridian on the date of the last observation. It is therefore plainly evident that the most generally quoted value of the rotation-period is completely out of harmony with the present series of observations.

I gladly acknowledge the assistance derived from the perusal of the valuable paper, "On the Determination of the Rotation-period of *Jupiter* in 1835," to be found in vol. ix. of the *Memoirs*, by the Astronomer Royal, whose method to some extent I have followed.

18 *Preston Street, Brighton,*
1880, *January* 8.

Estimated Times of Transit of the Red Mark on Jupiter across the Central Meridian. By T. W. Backhouse, Esq.

1879.		G.M.T.			Long.			
Mo.	Day.	Pr. End. h m	Middle. h m	Foll. End. h m	Pr. End. °	Middle. °	Foll. End. °	Length. °
8	29		10 14			255		
9	22	9 29	9 33	10 23	245	261	278	33
	29		10 43			264		
10	6	10 59*	11 23		248	263		
	28			9 56			283	
11	7			8 17†			288	
	10			5 44			287	
	17	5 35‡			260			
	29		5 53§	6 21§		271	287	
12	6	6 14§	6 40§	7 10§	256	272	290	34
	13		7 35‡			278		
	23	5 18§	5 48§	6 15§	259	277	293	34
	25	7 1	7 27	7 56‡	262	278	295	33

* Main part. A slender point extends a little further pr.
† Thickish cloud.
‡ Definition bad.
§ Definition not good.

Estimations made with a $4\frac{1}{4}$-in. Refractor.

Sunderland.

Means of Daily Areas of Umbræ, Whole Spots, and Faculæ upon the Sun's Disk, as measured on Photographs taken at the Royal Observatory, Greenwich, for each Rotation of the Sun, from 1877, December 22, to 1879, December 28.

[In continuation of the results printed in the *Monthly Notices*, vol. xxxix., p. 515.]

(*Communicated by the Astronomer Royal.*)

The Mean Daily Areas have been formed by taking the Means of the Areas for each day of observation throughout each rotation of the Sun, and are expressed in millionths of the Sun's visible hemisphere. Usually two photographs are taken on each day of observation.

No. of Rotation.	Date of Commencement of each Rotation.		No. of Days of Obs.	Means of Daily Areas. Umbræ.	Whole Spots.	Faculæ.
65	1877	Dec. 22	7	0	5	128
66	1878	Jan. 16	8	8	66	81
67		Feb. 10	8	5	25	129
68		Mar. 8	6	11	50	0
69		Apr. 2	9	0	0	0
70		Apr. 28	10	0	0	11
71		May 23	8	16	75	347
72		June 17	17	6	35	105
73		July 13	16	0	1	64
74		Aug. 7	13	0	0	8
75		Sept. 1	16	19	90	180
76		Sept. 27	14	0	0	90
77		Oct. 22	10	6	23	35
78		Nov. 17	4	0	0	85
79		Dec. 12	2	0	0	0
80	1879	Jan. 6	2	0	0	0
81		Feb. 1	7	0	0	29
82		Feb. 26	10	0	0	0
83		Mar. 23	9	11	56	124
84		Apr. 18	11	4	17	51
85		May 13	10	0	0	63
86		June 8	12	12	47	60
87		July 3	7	2	15	370
88		July 28	12	11	29	80
89		Aug. 23	11	16	76	6
90		Sept. 17	12	14	45	2[illegible]
91		Oct. 12	9	45	206	3[illegible]
92		Nov. 7	9	25	108	[illegible]
3		Dec. 2	6	4	17	

Means of Daily Areas of Umbræ, Whole Spots, and Faculæ upon the Sun's Disk, as measured on Photographs taken at the Royal Observatory, Greenwich, for each Year, from 1873 to 1879.

The Mean Areas are expressed in millionths of the Sun's visible hemisphere.

Year.	Means of Daily Areas. Umbræ.	Whole Spots.	Faculæ.
1873	116	678	2882
1874	83	583	1095
1875	45	255	475
1876	25	132	226
1877	20	94	168
1878	5	25	84
1879	10	44	163

Many of the photographs taken during the early part of 1874 do not show the faculæ with sufficient distinctness to allow of their measurement; the mean area of faculæ given for that year is therefore too small.

The mean daily area of faculæ for the half-year beginning 1874, July 2, is 1257.

From these results it appears that the minimum both for Sun-spots and faculæ occurred about the end of 1878 and the beginning of 1879, and that there has since been a marked increase in the number of both.

Royal Observatory, Greenwich,
1880, *Jan.* 9.

Note referring to Observations and Estimations of the Brightness of Mars, which ought to be made in February and March 1880.

By A. Marth, Esq.

The apparent brightness of the planet *Mars* is now decreasing at such a rate, that in the course of February it will be reduced to that of some of the fixed stars of the first magnitude. The planet is unusually well placed for comparison with *α Tauri* and *α Orionis*, both stars of reddish hue and consequently better adapted for the purpose than others of different colour, and observers who may be favoured by clear evenings in the latter part of February and beginning of March ought therefore not to lose their opportunities for making such photometric measurements or naked-eye estimations as will serve to fix the time when *Mars* equals in brightness *α Orionis* and the time when it equals *α Tauri*. Seventy-nine years ago, on a certain evening in February 1801, Olbers estimated that *Mars* appeared just as

much brighter than *α Tauri* as it appeared fainter than *α Orionis*, and he made this estimate the foundation of his paper, "*Mars und Aldebaran*," in vol. 8 of Zach's *Monatliche Correspondenz*. The circumstances of the present apparition of the planet are not much different from those in 1801, and Olbers' old estimate accordingly suggests the time when *Mars* is likely to appear of the corresponding brightness. But since *α Orionis* is a variable or at least a suspected variable star, proper allowance must be made, and the comparisons must begin sufficiently early in February. The moonlight may be troublesome for some evenings, but this cannot be helped.

The ratio of the apparent brightness of *Mars* to that at mean opposition can only be given conditionally, since the real effect of the phase is not yet ascertained. If r is the distance of the planet from the Sun, Δ its distance from the Earth, and (in the notation of Lambert's photometric formula) v the phase-angle, or $180^\circ - v$ the areocentric angle between Sun and Earth, the light-ratio will be

$= \frac{\text{const.}}{r^2 . \Delta^2} \cdot \sin^2 \tfrac{1}{2}v$, in case the amount of light depends simply on the proportion of the illuminated portion to the whole disk;

or

$= \frac{\text{const.}}{r^2 . \Delta^2} \cdot \frac{\sin v - v \cos v}{\pi}$, in case Lambert's photometric formula is valid.

In order to refer the brightness to that at mean opposition, the const. c is put $= a^2(a-1)^2$, where a is the major semi-axis of the planet's orbit, or $\log c = 9{\cdot}803945$.

The following table gives, for Greenwich Noon, the values of $\log \frac{c}{r^2 . \Delta^2}$, of the phase-angle v, and of the logarithms of the light-ratios in the two cases:—

1880.	$\log \frac{c}{r^2 . \Delta^2}$.	v.	$\log \left(\frac{c}{r^2 . \Delta^2} . \sin^2 \tfrac{1}{2}v\right)$.	$\log \left(\frac{c}{r^2 . \Delta^2} . \frac{\sin v - v \cos v}{\pi}\right)$.
		°		
Feb. 18	9·22960	141·84	9·1805	9·1414
20	·21421	141·85	·1651	·1261
22	·19905	141·87	·1500	·1110
24	·18414	141·91	·1352	·0963
26	·16945	141·96	·1207	·0818
28	·15500	142·02	·1064	·0676
Mar. 1	·14077	142·09	·0923	·0537
3	·12676	142·18	·0735	·0400
5	9·11296	142·27	9·0650	9·0267

Since *Mars* is near quadrature, and the angle v varies litt' the uncertainty of the photometric formula will not sensi' affect the relative comparisons made during the interval.

The correction of the observations which takes into account the extinction of light in different zenith-distances may be diminished or avoided in case it is feasible to observe at the times when both objects are at the same zenith-distance. *Mars* and *α Orionis* allow this to be done till the beginning of March. If the angle τ is found from $\sin \tau = \nu \tan .$ latitude, and expressed in time, the two bodies will be at the same altitude at the local sidereal time $\theta_0 + \tau$. The values of $\log \nu$ and θ_0 are given for 8^h G.M.T. in the following table, together with the times of the occurrence at Greenwich:—

Mars and *α Orionis.*

8^h Gr.	$\log \nu$.	θ_0. h m	At Greenwich. h m	h m	°
Feb. 18	9·7387	4 20·8	7 15 Sid.T. =	9 24 M.T.	47·5 Z.D.
22	9·7810	4 20·9	7 34	9 26	49·2
26	9·8259	4 20·5	8 5	9 41	52·5
Mar. 1	9·8744	4 19·2	9 0	10 19	59·0
5	9·9264	4 16·7	...	...	...

Mars and *α Tauri* will not be observable in this way in our latitudes. But *α Tauri* and *α Orionis* will be at the same zenith-distance, if τ is found by $\sin \tau = [9·6506] \tan.$latit., at $4^h\ 47^m·6 + \tau$ local sidereal time, or for the latitude of London, at $7^h\ 4^m$ sidereal time, the zenith-distance being $46°·8$.

Observation of the Outer Satellite of Mars, made at the Royal Observatory, Greenwich.

(*Communicated by the Astronomer Royal.*)

The following micrometer measures of *Deimos* were obtained with the great Equatoreal ($12\frac{3}{4}$ inches aperture) on the evening of 1879, November 12. It was also seen for a few moments on November 7, but no measures could be obtained on that occasion. The inner satellite, *Phobos*, was never seen.

1879, Nov. 12.

Measures of Distance.

Greenwich Mean Solar Time. h m s	Greenwich Sidereal Time. h m s	Distance in Arc. ″
11 24 21	2 51 12	66·47
11 49 27	3 16 22	62·77
12 23 3	3 50 4	60·46
12 44 45	4 11 49	58·14

Measures of Position.

Greenwich Mean Solar Time. h m s	Greenwich Sidereal Time. h m s	Position-angle. ° ′
11 17 4	2 43 54	229 34
11 44 45	3 11 39	228 8
12 15 30	3 42 29	226 21
12 39 36	4 6 39	226 27*

The satellite, once found, was an easy object at greatest elongation. It became very difficult before the last observations, as the definition was then much impaired. It bore a fair amount of illumination in the field, but the measures were taken in a dark field, the wires only being bright. The determination of the position-angle was much more difficult and less certain than that of the distance. *Mars* was observed through a dark glass, which was placed in front of the eye-piece and covered half the field. Power employed, 295.

Observer, Mr. Maunder.

Royal Observatory, Greenwich,
1880, *Jan.* 9.

The Greenwich Standard Right Ascensions.

By A. M. W. Downing, Esq.

The Greenwich Standard Right Ascensions now in use are based on a determination of the right ascensions of clock-stars in which only those observations were included where the group of clock-stars extended over 12 hours at least. These results would presumably be independent of the assumed places of the clock-stars, and may be considered as the best standard places of fundamental stars which can be obtained from the Greenwich observations in the years 1868 to 1876.

These right ascensions are given in the Introduction to the Greenwich Nine-Year Catalogue.

I have thought that it would be interesting and instructive to compare these Greenwich places with the right ascensions of the fundamental stars as given by some of the most eminent authorities, in order to show the differences that exist, not only in the adopted equinoxes of the several Catalogues, but also in the places of individual stars. For this purpose I have selected for com parison the Standard Catalogues published by Newcomb, b Gyldén, and by Auwers. Newcomb's Catalogue is given in tl volume of *Washington Observations* for 1870, Gyldén's in tl *Monthly Notices* for 1875, May, and Auwers' in the *Publicati der Astronomischen Gesellschaft*, No. xiv.

* Rough bisection; satellite becoming very difficult to observe.

The comparison with Newcomb shows plainly enough, from the constancy of the differences all through, that the places in the two Catalogues are not affected by any appreciable error of a periodic character depending either on the right ascension or on the declination. The resulting mean correction to Newcomb is $-^s{\cdot}029$; this, therefore, is the difference between the adopted eqinoxes of the Catalogues. If this correction be applied, the individual results are found to agree very closely; the only cases of a correction exceeding $^s{\cdot}02$ being *α Canis Minoris* $+^s{\cdot}023$ (on account of its orbital motion, this star should perhaps be rejected altogether), α^2 *Libræ* $+^s{\cdot}027$, and *α Lyræ* $-^s{\cdot}024$. In *Monthly Notices*, 1875, Suppl. Number, Professor Newcomb points out that the mean correction to his right ascensions from the Greenwich observations of the Sun, 1836–1870, is $-^s{\cdot}028$, so that very little change seems to have taken place in the position of the Equinox as found from the more recent observations.

In the comparisons with Gyldén and Auwers, I have only used the stars that are given by Newcomb, as these are sufficiently numerous to determine the difference between the adopted equinoxes, and the Greenwich places may safely be assumed to be sensibly free from periodic error; so that the places of many stars are unnecessary, it being no part of my present object to determine the periodic errors (if any such exist) of the other Catalogues. Gyldén's right ascensions agree closely with Greenwich, the differences are nearly constant through the 24 hours, and the mean correction is $-^s{\cdot}016$; when this is applied the only individual stars that stand out are *β Orionis* $-^s{\cdot}024$, α^2 *Libræ* $+^s{\cdot}030$, and *β Aquilæ* $+^s{\cdot}026$. Auwers' *Catalog der Fundamental-Sterne* gives for mean correction $-^s{\cdot}026$, and we then have for *α Arietis* the correction $-^s{\cdot}021$, for *α Leonis* $+^s{\cdot}025$, and for *α Lyræ* $-^s{\cdot}028$.

It appears then that the most important difference between the Greenwich right ascensions and the places furnished by these distinguished astronomers, with which they are compared, is a difference in the adopted equinoxes, and this is really the puzzling circumstance in all determinations of right ascension. By the exercise of a due amount of care in the methods of observation and of reduction it is possible, practically, to get rid of the errors affecting the relative right ascensions of stars; but it has hitherto been found impossible wholly to eliminate the errors which affect the absolute right ascensions. As an illustration of the difficulty in this matter, I may refer to the table on page 3 of the Introduction to the Greenwich Nine-Year Catalogue, from which it appears that the corrections to the assumed equinox from observations of the Sun ranged from $+^s{\cdot}058$ in 1873 to $-^s{\cdot}049$ in 1874, and the mean error of the resulting correction from the nine-years' observations is as much as $\pm^s{\cdot}012$. The following results, extracted from Nyrén's *Das Æquinoctium für* 1865·0, in which that astronomer determines the equinox from the Pulkowa Observations of the Sun, 1861–1870, with his usual care and completeness, will also illustrate the difficulty of making a satisfactory determination.

Pulkowa (1865) – Greenwich (1836–70)	=	$+0^s.064$
– Pulkowa (1845)	=	+0·055
– Edinburgh	=	+0·036
– Cambridge	=	+0·030
– Paris	=	+0·011
– Washington	=	–0·002

This remarkable difference between Greenwich and Washington is also pointed out by Newcomb in the paper referred to above.

We must, I suppose, consider these discordances to arise from casual errors of observation. It is, however, much to be wished that more Observatories would co-operate in the work of continuous meridian observation of the Sun and principal stars, and especially valuable would be the results of observations made at stations where the Sun culminates at a considerable altitude when near the Equator.

Corrections to three selected Catalogues, derived from a Comparison with the Greenwich Standard Right Ascensions of Clock-Stars, based on 12-hour Groups.

Name of Star.	Newcomb (1872). I.	Newcomb (1872). II.	Gyldén (1875). I.	Gyldén (1875). II.	Auwers (1875). I.	Auwers (1875). II.
	s	s	s	s	s	s
α Andromedæ	–·034	–·005	–·007	+·009	–·034	–·008
γ Pegasi	·032	·003	·001	·015	·028	·002
α Arietis	·045	·016	·019	–·003	·047	·021
α Ceti	·047	·018	·034	·018	·035	·009
α Tauri	·026	+·003	·023	·007	·012	+·014
β Orionis	·020	·009	·040	·024	·016	·010
β Tauri	·026	·003	·022	·006	·018	·008
α Orionis	·027	·002	·036	·020	·018	·008
α Canis Min.	·006	·023			·006	·020
β Geminorum	·033	–·004	·034	·018	·033	–·007
α Hydræ	·018	+·011	·019	·003	·027	·001
α Leonis	·026	·003	·020	·004	·001	+·025
β Leonis	·012	·017	·009	+·007	·010	·016
α Virginis	·028	·001	·025	–·009		
α Boötis	·038	–·009	·012	+·004	·035	–·009
α^2 Libræ	·002	+·027	+·014	·030		
α Coronæ	·045	–·016	–·017	–·001	·036	
α Serpentis	·035	·006	·022	·006	·026	
α Scorpii	·027	+·002	·018	·002		
α^1 Herculis	·035	–·006	·020	·004	·034	

Name of Star.	Newcomb (1872). I. s	II. s	Gyldén (1875). I. s	II. s	Auwers (1875). I. s	II. s
α Ophiuchi	·036	·007	·026	·010	·042	·016
α Lyræ	·053	·024	·011	+·005	·054	·028
γ Aquilæ	·038	·009	·010	·006	·027	·001
α Aquilæ	·036	·007	·018	−·002	·027	·001
β Aquilæ	·025	+·004	+·010	+·026	·013	+·013
α^2 Capricorni	·018	·011	−·005	·011		
α Aquarrii	·026	·003	·006	·010	·028	−·002
α Piscis Aust.	·027	·002				
α Pegasi	−·023	+·006	−·001	+·015	−·015	+·011

The numbers under the columns headed I are the actual differences between the Catalogue places, those under the columns headed II are the differences when a constant correction of −^{s}·029 is applied to Newcomb, −^{s}·016 to Gyldén, and −^{s}·026 to Auwers.

1879, *Jan.* 8.

On the Effects of "Personality" on the Tabular Errors of the Moon. By E. Dunkin, F.R.S.

In Mr. Neison's interesting Paper on the effects of personality on the Greenwich tabular errors of the Moon (*Monthly Notices*, vol. xl. pp. 75–80) there are one or two points having reference to some expressions and deductions of mine contained in a Paper inserted in vol. xxix. of the *Monthly Notices*, on which I think I ought to make a few remarks, principally, however, in the way of explanation. The subject has for many years been one of considerable interest to me, since, in fact, the year 1848, when the remarkable personal equation in observing the limbs of the Moon between Mr. H. Breen and myself was first noticed.

The first point to which I wish to call attention is the paragraph in Mr. Neison's Paper in which he states that "it appeared that Mr. Dunkin had based his investigation on the assumption that 'the mean tabular errors are assumed to be constant throughout the mean lunation.' It is now known that this assumption is not a permissible one, for the tabular errors are not constant throughout the mean lunation, but that, owing to imperfections in the theory, they are systematically different before and after full Moon." Through a fault of my own in the construction of the sentence, Mr. Neison has been led unfortunately to misunderstand my meaning in the above quotation, for the information I intended to convey to the reader was, "provided the (relative) mean tabular errors are assumed to be constant

throughout the mean lunation." This remark, I contend, is sufficiently accurate for the purpose I had in view, knowing that the four observers had an equal and regular share in the observations. I was not concerned in determining the varying absolute amounts of tabular errors in the course of the year; all that I assumed was that, in the mean yearly groups of the different observers, the relative tabular error might be considered to be constant; and I had specially mentioned that, to ascertain whether the mean relative tabular errors represented the same point in the Moon's orbit, I had arranged all the errors for each observer according to the age of the Moon, when it was found that the mean age agreed sensibly for all, for observations both of the first and second limbs. It appears to me that the general accordance of the results exhibited in the tables given in my former Paper is a sufficient proof that they can only be slightly affected by any relative difference of tabular error; and probably, if I had to go over the work again, I should make no material modification which could sensibly influence the results.

I do not clearly understand the process by which Mr. Neison obtains the numbers inserted in the table on page 79. It appears to me that we are not at all certain that personal equations are so constant as to assume that no change has taken place in that of any person between 1863 and 1876; indeed there is some evidence that changes have actually occurred in personal equations when derived from the observations of the transits of stars. For this reason, I rather doubt the advisability of comparing, by an indirect method, the observations made by me in 1863–69 with those made by Mr. Thackeray in 1875–76, especially as the observations of the latter were far too few in these years to give a reliable determination. As an example of a progressive change of personal equation, I have only to refer to the case of Mr. Rogerson, in whom it varied in the eight years from 1846 to 1853 from $0^s.31$ to $0^s.67$, by a nearly equal increment each year. We ought therefore to be quite certain of our data before coming to a conclusion, especially in regard to such small quantities as we are obliged to deal with in investigations of this nature.

Now I must ask the question whether Mr. Neison has had reliable data before him with regard to the more recent observers, sufficient to prove that "the change of observers has therefore introduced a considerable increase in the mean tabular error of the Moon which has no real existence." (P. 79.) I think not. I have been led to this opinion from an examination of the tabular errors of longitude of the Moon determined in 1877 and 1878 when we have had a fair number of observations made with the Transit-Circle by Mr. Downing and Mr. Thackeray to deal with. I have grouped together the errors under the names of Mr. Lynn (who observed only in 1877 and in the early months of 1878), Mr. Criswick, Mr. Downing, and Mr. Thackeray; the means were then taken and referred to Mr.

Criswick's mean as the standard. The following is the result:—

	1877.	1878.	Mean 1877–78.
	"	"	"
Lynn —Criswick =	+0·54	...	+0·54
Downing — " =	−1·02	−0·27	−0·72
Thackeray— " =	−1·02	+0·95	−0·10

These numbers seem to show that we must look to some other cause than that assigned by Mr. Neison to explain any increase in the mean tabular error of the Moon. The figures point towards a decrease in the error rather than an increase; but they also prove that it is unsafe to base any particular theory on results liable to such fluctuations as those exhibited in this table.

Kidbrooke, Blackheath,
1880, *January* 8.

Note on the Remeasurement of a Lunar Photograph, in reply to Mr. Neison's Criticism. By Prof. C. Pritchard.

In the *Monthly Notices* of November last, Mr. Neison did me the honour of criticising a paper of mine laid before the Astronomical Society in June of the same year. In that paper I presented to the Society what seemed to me to be a question of considerable novelty and interest, viz. the measurement of the Moon's mean semi-diameter from a series of photographs taken by Mr. Jenkins with Mr. De La Rue's Reflector, the admirable defining power of which is generally acknowledged. I explained that the computation ef the Moon's diameter was not the ultimate object for which the photographs were originally taken, but that, if I found a considerable number of them, taken at random out of many hundreds, gave accordant results, this circumstance would give me confidence in applying the fine series of photographs in the possession of the University of Oxford to the determination of the lunar physical libration; the latter investigation, it may be observed, involving an enormous expenditure of labour, and not to be undertaken without adequate grounds of confidence. The results of the computation of this diameter exhibited an amount of accordance which not only satisfied, but surprised me, when I reflected on the discordances arising from other known methods of observing. These results of very laborious measurement and computation I laid before the Society, and I thought them creditable both to the photography and to the computation.

Mr. Neison, however, in his criticism thinks it right to say: "Until we know the method by which these measures have been reduced, it is impossible to say how far the above result indicates

the true photographic semi-diameter, or how far it may be vitiated by the effect of the inequalities on the limb. In the method employed by Wichmann, the effect of these inequalities is not eliminated; and if Professor Pritchard has employed the same method, they will not have been eliminated from the above result."

The method of measurement was stated by me in the second and third paragraphs of my communication of June last. Substantially it is Bessel's with some modifications suggested by the peculiarity of Mr. De La Rue's measuring engine. As to the mode of reduction of these measurements, this also is substantially Bessel's; and in fact is the only natural one that would suggest itself to any mathematical astronomer.

However, I hope before a few months are over, to lay the matter in all its details before astronomers, so soon as the second fasiculus of the work done in the last two years at the University Observatory of Oxford can be published. Still, as Mr. Neison's criticism seemed to me to have a tendency to throw some doubt over the substantial accuracy of the method having in view the inequalities of the lunar contour, I arranged for the remeasurement of the semi-diameter of some one photograph taken entirely at random, and that by both the Observatory Assistants, superadded to a measure of my own.

The photograph selected was taken on January 26, 1877, just three years ago. Mr. Plummer selected, at his own judgment, eight points on the limb, measuring their distances from the centre of *Hypatia* B. Mr. Jenkins selected seven other points on the same limb, and measured from the same spot. Measures taken by myself, Mr. Plummer, and Mr. Jenkins of the same distances were compared together and the means of our measures did not differ by more than 0″·15 in about 600″. Consequently no appreciable "personality" existed in the results of our mode of measuring these lunar distances. Neither can I see how such "personality" could exist in a microscope measure, when due care is taken to destroy parallax and to ensure all proper and obvious instrumental adjustments. The measures were then subjected to computations, the same as would be adopted by any competent astronomer.

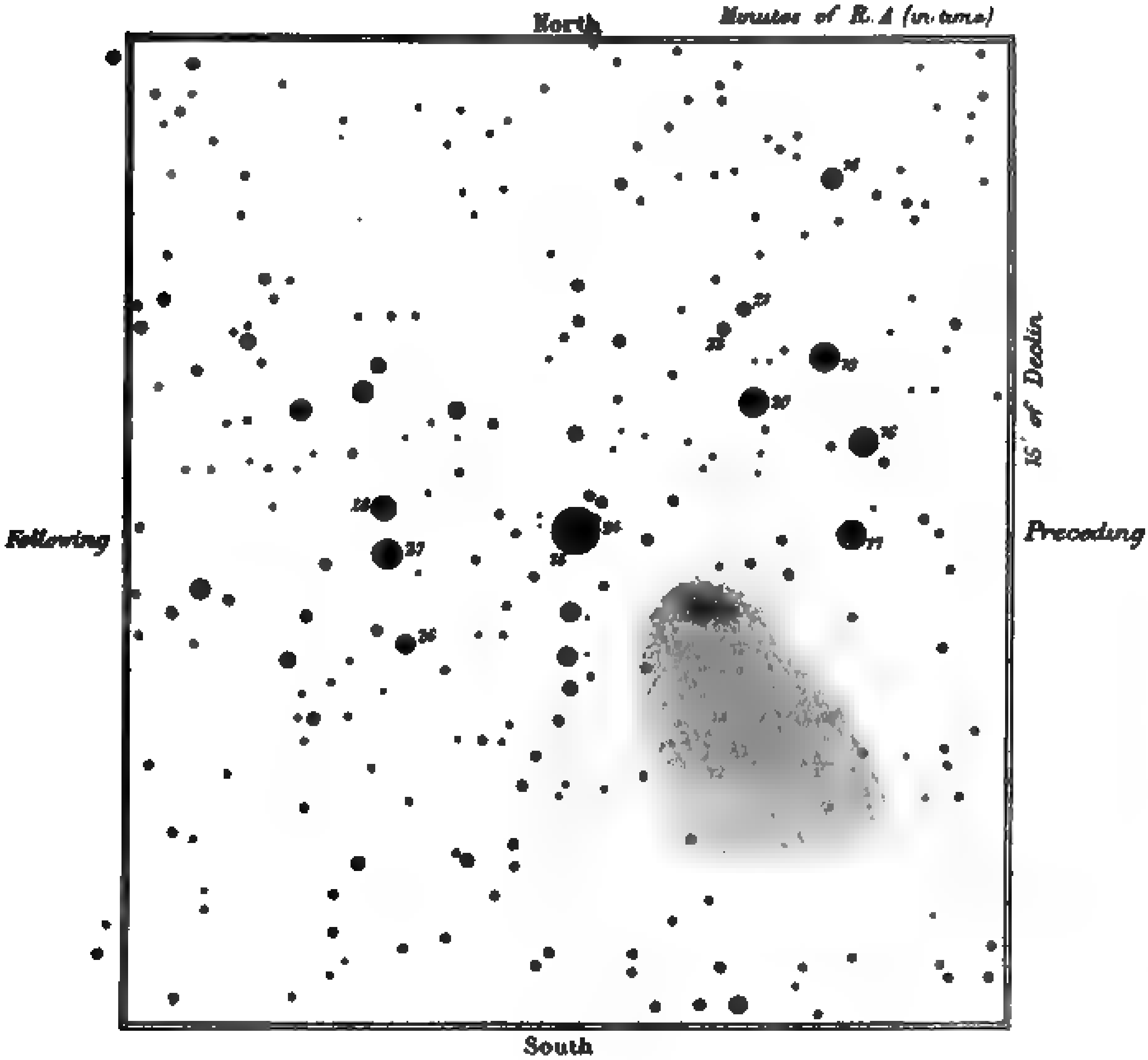
Now let it be admitted that there are irregularities in the lunar contour, as assuredly there are: nevertheless here are measures, taken from a well-defined point on the lunar photograph to 23 points on the limb, in parts of it of more than average evenness: and what is the result? The original photographic mean semi-diameter was, as is stated (page 440, line 14) 15′ 34″·26. The new measures of January 1880 give

For Mr. Plummer	15	34·58
" Mr. Jenkins	15	34·02
The mean is	15	34·30

THE PLEIADES

including the

NEBULA ROUND MEROPE.



(To be viewed at a distance of 6 feet.)

HALL
JAMAICA
1877 March 6

which shows, I think, conclusively how very little the first result is affected by the introduction of 15 fresh measures. Further still, I would appeal to astronomers who are experienced in delicate and accurate micrometrical measurement whether a greater degree of accordance could be expected even if the Moon's contour were absolutely uninterrupted. I trust Mr. Neison may not think it desirable here to repeat his former criticism of "fortuitous coincidence." I claim for them the natural results of some instrumental experience and some astronomical skill.

Oxford, Jan. 9, 1880.

The Lithograph of the *Pleiades*, including the Nebula round *Merope*, belongs to Mr. M. Hall's paper, "The Nebula in the *Pleiades*," printed in the December No., pp. 89–90.

Changes of Relative Brightness of Jupiter's Satellites.

By C. E. Burton, Esq.

Perhaps I may be permitted to call the attention of those Fellows of the Royal Astronomical Society who are interested in the changes referred to in the above title to an essay by Dr. Rudolph Engelmann, of Leipzig, *Ueber die Helligkeitsverhältnisse der Jupiter's Trabanten*, Leipzig, 1871, which treats very fully of the connection between the brightness of the satellites and their places in their orbits, establishing the reality of such a connection in the case of IV., even going so far as to identify the period of axial with that of orbital revolution with certainty for this satellite, and rendering it probable that the first satellite also behaves similarly.

Perhaps I shall be pardoned for calling attention to the concluding paragraph of a Note on IV. published by me in the *Monthly Notices*, vol. xxxiii. pp. 472–4, years before I heard of Engelmann's results or labours, and taking a somewhat different line of argument from his, which leads, nevertheless, to the same result.

Observations of Eclipses, Occultations, and Phenomena of Jupiter's Satellites, made with the 8-inch Equatoreal (Cooke) at the Adelaide Observatory, South Australia. By C. Todd, Esq., Director of the Observatory.

Longitude, $9^h\ 14^m\ 21^s.3$ E. | Latitude, $34^\circ\ 55'\ 33''.8$ S.

1878.

Ref. No.	Date 1878.	Observer	Phenomenon.		Phase of Phenomenon.	Adelaide Mean Time. h m s	Corresponding Greenwich Mean Time. h m s	Time by *Nautical Almanac.* h m s	Apparent Error of *Nautical Almanac.* m s	Power.
1	July 5	T	IV.	Sh. E.	About bisected	10 0 32·31	0 46 11·01	0 48	...	200
2	„	„	IV.	Tr. I.	External contact	10 5 7·30	0 50 46·00	...	...	„
3	„	„	IV.	Sh. E.	Disappeared	10 5 22·30	0 51 1·00	...	...	„
4	„	„	IV.	Tr. I.	Bisected	10 12 24·29	0 58 2·99	0 46	...	„
5	„	„		„	Internal contact	10 16 49·28	1 2 27·98	...	...	„
6	July 7	T	I.	Oc. R.	First seen	8 36 17·77	23 21 56·47			
7	„	„		„	About bisected	8 36 37·77	23 22 16·47	23 24	...	„
8	„	„		„	External contact	8 39 7·77	23 24 46·47			
9	July 21	T	I.	Ec. D.	Disappearance	9 43 31·36	0 29 10·06	0 28 45	−0 25·06	„
10	July 22	T	I.	Sh. E.	Internal contact	9 5 0·06	23 50 38·76			
11	„	„		„	About bisected	9 8 53·55	23 54 32·25	23 57	...	„
12	„	„		,	Limb complete	9 9 49·55	23 55 28·25			
13	July 22	T	I.	Tr. E.	Internal contact	9 10 36·55	23 56 15·25			
14	„	„		„	Bisection	9 12 37·05	23 58 15·75	0 2	...	„
15	„	„		„	External contact	9 15 38·05	0 1 16·75			

16	July 22	T	II.	Ec. D.	Last seen	10 31 18·95	1 16 57·65	1 17 3	+0 5·35	„
17	July 23	T	I.	Oc. R.	External contact	6 38 12·50	21 23 51·20	21 18	...	„
18	July 26	T	III.	Tr. I.	External contact	8 14 0·35	22 59 39·05			
19	„	„		„	Bisected	8 19 13·35	23 4 52·05	23 3	...	„
20	„	„		„	Internal contact	8 23 58·35	23 9 37·05			
21	July 26	T	III.	Sh. I.	First seen	8 28 38·25	23 14 16·95			
22	„	„		„	Bisected	8 29 38·25	23 15 16·95	23 11	...	„
23	„	„		„	Internal contact	8 32 28·25	23 18 6·95			

Remarks.

Ref. No.
1 Late; clouded.
2 Good.
3 Pretty good.
4 Late; doubtful; clouds passing, frequently hiding planet.
5 Very good.

Observations interfered with by passing clouds, but splendid glimpses occasionally, the planet and satellites being splendidly defined, the latter, excepting the fourth, having well-defined disks. The fourth satellite, before ingress, was noticed as faint, and, when on the planet's disk, it appeared as a black, well-defined spot on the northern white belt, almost as black as its shadow, which was then passing off the opposite limb. The transit occurred on the north bright belt.

6, 7, 8 Not good; planet very badly defined.
9 Very good, but satellite disappearing close to the limb might cause the time noted to be a little early.

Ref. No.
10, 11, 12, 13, 14 Clouds interfering; the shadow and the satellite traversed the north edge of the north equatoreal white belt, and were almost touching each other.
15 Very good observation.
16 Observation doubtful; clouds continually obscuring planet, which, however, emerged just before time noted, and I then caught sight of satellite as a mere speck close to, but not touching the edge or limb of planet; the time noted was when it disappeared.
17 Late; not good; planet dreadfully ill defined and unsteady.
18 Not very good.
20 Considered good.
21 Late; clouds interfering.
23 Difficult.

Ref. No.	Date 1878	Observer.	Phenomenon.		Phase of Phenomenon.	Adelaide Mean Time. h m s	Corresponding Greenwich Mean Time. h m s	Time by *Nautical Almanac.* h m s	Apparent Error of *Nautical Almanac.* m s	Power.
24	July 26	R	III.	Tr. E.	Bisection	11 51 57·85	2 37 36·55	2 41	...	200
25	„	„	III.	Sh. E.	Internal contact	11 55 17·85	2 40 56·55	...	...	„
26	„	„	III.	Tr. E.	External contact	11 56 7·84	2 41 46·54	...	...	„
27	„	„	III.	Sh. E.	Bisected	11 56 49·34	2 42 28·04	2 47	...	„
28	„	„		„	Limb complete	12 1 37·83	2 47 16·53	...	...	„
29	July 29	T	I.	Tr. I.	External contact	8 26 26·30	23 12 5·00			
30	„	„		„	Bisection	8 32 3·80	23 23 42·50	23 25	...	„
31	„	„		„	Internal contact	8 41 19·80	23 26 58·50			
32	July 29	T	I.	Sh. I.	First seen	8 45 54·30	23 31 33·00			
33	„	„		„	About bisected	8 47 38·30	23 33 17·00	23 32	...	„
34	„	„		„	Internal contact	8 49 46·30	23 35 25·00			
35	July 29	T	I.	Tr. E.	Internal contact	10 54 50·00	1 40 28·70			
36	„	„		„	About bisected	10 57 5·00	1 42 43·70	1 45	...	„
37	„	„		„	External contact	11 0 8·05	1 45 47·20			
38	July 29	T	I.	Sh. E.	Internal contact	11 1 4·00	1 46 42·70			
39	„	„		„	About bisected	11 2 13·00	1 47 51·70	1 52	...	„
40	„	„		„	Limb complete	11 4 31·00	1 50 9·70			
41	Aug. 7	T	I.	Tr. E.	Internal contact	7 4 45·68	21 50 24·38			
42	„	„		„	About bisection	7 6 58·68	21 52 37·38	21 55	...	„
43	„	„		„	External contact	7 9 0·18	21 54 38·88			

44	Aug. 9	R	II.	Ec. R.	First seen	7 48 25·60	22 34 4·30	22 34 52	+0 47·70	„
45	„	„		„	Full blaze	7 54 15·10	22 39 53·80	...	...	„
46	Aug. 13	T	I.	Oc. D.	External contact	9 23 21·67	0 9 0·37			
47	„	„		„	About bisected	9 25 12·17	0 10 50·87	0 11	...	„
48	„	„		„	Internal contact	9 27 58·67	0 13 37·37			
49	Aug. 13	T	III.	Ec. R.	First seen	9 54 32·10	0 40 10·80	0 43 28	+3 17·20	„
50	„	„		„	About bisection	9 58 49·10	0 44 27·80	...	...	„
51	„	„		„	Full blaze	10 1 59·10	0 47 37·80	...	...	„
52	Aug. 13	T	I.	Ec. R.	First seen	12 12 0·80	2 57 39·50	2 57 48	+0 8·50	„
53	„	„		„	Full blaze	12 16 33·80	3 2 12·50	...	...	„
54	Aug. 21	T	I.	Tr. In.	External contact	8 16 25·80	23 2 4·50			
55	„	„		„	About bisected	8 19 22·30	23 5 1·00	23 5	...	„
56	„	„		„	Internal contact	8 22 2·78	23 7 41·48			

Remarks.

Ref. No.	
28	When shadow disappeared the satellite was only about half its diameter outside the limb of the planet.
31	Satellite entered on northern edge of equatoreal bright belt.
32	Limb slightly indented.
35, 36, 37	Considered good.
39, 41	Appearing as a cup to satellite.
42, 43	Definition wretched; planet very unsteady; limb boiling; lost egress of shadow by clouds.
44	Very exact; seen instantly; definition good.
46, 47, 48	Very exact; planet and satellite very well defined; occultation complete at limb; satellite disappeared behind bright equatoreal belt, and a little south of equator.
49, 50, 51	Definition splendid; satellite well defined; and at time noted of bisection it appeared as a half-moon.
54	Not very good; planet unsteady.
56	Very good.

Ref. No.	Date 1878.	Observer.	Phenomenon.	Phase of Phenomenon.	Adelaide Mean Time. h m s	Corresponding Greenwich Mean Time. h m s	Time by Nautical Almanac. h m s	Apparent Error of Nautical Almanac. m s	Power.
57	Aug. 21	T	I. Sh. I.	First seen	8 58 55·20	23 44 33·90			
58	„	„	„	Bisected	8 59 29·70	23 45 8·40	23 44	...	200
59	„	„	„	Internal contact	9 1 33·70	23 47 12·40			
60	Sept. 25	T	III. Ec. R.	First seen	10 0 21·38	0 46 0·08	0 49 40	+3 39·92	„
61	„	„	„	Full blaze	10 12 38·38	0 58 17·08	...	...	„
62	Sept. 30	T	I. Ec. R.	First seen	7 9 57·13	21 55 35·83	21 55 50	+0 14·17	„
63	„	„	„	Full blaze	7 13 7·63	21 58 46.33	...	...	„
64	Oct. 1	T	II. Oc. D.	External contact	9 53 25·20	0 39 3·90			
65	„	„	„	About bisected	9 55 28·20	0 41 6·90	0 41	...	„
66	„	„	„	Internal contact	9 57 49·20	0 43 27·90			
67	Oct. 2	T	III. Oc. R.	First seen	8 54 39·30	23 40 18·00			
68	„	„	„	About bisection	8 57 28·30	23 43 7·00	23 46	...	„
69	„	„	„	External contact	9 1 34·30	23 47 13·00			
70	Oct. 2	T	III. Ec. D.	Disappearance	10 34 47·50	1 20 26·20	1 20 52	+0 25·80	„
71	Oct. 5	T	IV. Ec. D.	Minute speck	8 20 38·62	23 6 17·32	...	...	„
72	„	„	„	Still visible	8 22 15·12	23 7 53·82	...	...	„
73	„	„	„	Disappearance	8 22 37·62	23 8 16·32	23 10 19	+2 2·68	„
74	Oct. 21	T	I. Oc. D.	External contact	9 16 19·54	0 1 58·24			
75	„	„	„	About bisected	9 17 55·54	0 3 34·24	0 4	...	„
76	„	„	„	Disappeared	9 20 34·04	0 6 12·74			

77	Oct. 23	R	I.	Ec. R.	First seen	7 25 4·46	22 10 43·16	22 10 46	+0 2·84	„
78	„	„		„	Quite distinct	7 25 19·96	22 10 58·66	...	...	„
79	„	„		„	Full blaze	7 27 48·96	22 13 27·66	...	...	„
80	Oct. 28	T	I.	Oc. D.	External contact	11 13 8·22	1 58 46·92			
81	„	„		„	About bisected	11 14 38·22	2 0 16·92	2 0	...	„
82	„	„		„	Last seen	11 16 23·22	2 2 1·92			
83	Nov. 7	R	IV.	Oc. D.	External contact	8 34 29·72	23 20 8·42			
84	„	„		„	About bisected	8 37 33·22	23 23 11·92			
85	„	„		„	Seen through limb	8 39 22·71	23 25 1·41			
86	„	„		„	„ „	8 40 24·71	23 26 3·41	23 29	...	300
87	„	„		„	„ „	8 41 38·20	23 27 16·90			
88	„	„		„	Internal contact	8 41 54·69	23 27 33·39			
89	„	„		„	Last seen	8 41 59·69	23 27 38·39			

Remarks

Ref. No.

57 Late; limb indented.

58

59 Good observations.

62, 63 Good observations; well defined; first seen as a minute speck of light.

64

65, 66 All good; satellite disappeared behind equatoreal cloud belt.

68 Too soon; emerged from behind southern dark band, near its polar margin; satellite slightly separated from limb; observations considered good, but planet not very well defined except in glimpses.

Ref. No.

70 Very good and exact; fourth satellite very faint.

73 Considered exact, but lost sight of satellite several times before final disappearance; planet not well defined; Moon a little to east of planet.

74, 75, 76 Very badly defined, but observations pretty good; at times I thought satellites could be seen through edge of planet, but not certain.

77 Good, sharp, and exact; definition of planet fair.

78 As distinct as a small star, to the north, between two satellites to the east of No. 1.

80, 81, 82 Not very good; badly defined and very unsteady.

Ref. No.	Date 1878.	Observer.	Phenomenon.	Phase of Phenomenon.	Adelaide Mean Time. h m s	Corresponding Greenwich Mean Time. h m s	Time by *Nautical Almanac.* h m s	Apparent Error of *Nautical Almanac.* m s	Power.
90	Nov. 7	R	III. Ec. R.	First seen	10 8 33·53	0 54 12·23	0 57 39	+3 26·77	200
91	,,	,,	,,	Quite distinct	10 9 27·03	0 55 5·73	...	...	,,
92	,,	,,	,,	As bright as adjoining Sat. No. II.	10 12 39·53	0 58 18·23	...	...	,,
93	,,	,,	,,	Full blaze	10 15 0·52	1 0 39·22	...	...	,,
94	Nov. 13	T	I. Oc. D.	External contact	9 34 58·15	0 20 36·85			
95	,,	,,	,,	About bisected	9 36 51·15	0 22 29·85	0 24	...	,,
96	,,	,,	,,	Internal contact	9 38 55·15	0 24 33·85			
97	Nov. 15	R	I. Ec. R.	First seen	7 39 43·23	22 25 21·93	22 25 28	+0 6·07	,,
98	,,	,,	,,	Quite distinct	7 40 4·23	22 25 42·93	...	...	,,
99	,,	,,	,,	Full blaze	7 42 7·23	22 27 45·93	...	...	,,
100	Dec. 11	T	IV. Ec. D.	Disappearance	9 0 14·80	23 45 53·50	23 49 3	+3 9·50	125

Remarks.

Ref. No.

83 May be a little too soon.

88 Satellite passed behind white cloud belt at about lat. 25° or 30° S.

89 Fancied I saw it at 5″ after internal contact, but not very certain; definition good and steady. Looked for egress of shadow of first satellite, but could not see it.

90 Very minute speck of light; caught the first glimpse; very good observation.

91, 92, 93 Very good observations.

Ref. No.

94, 95, 96 Mere guesswork; planet and satellites most wretchedly defined.

97, 98, 99 Not considered first-rate; hazy, and definition bad.

100 Very good; a mere speck for last minute or more; planet dreadfully ill defined.

[*The Notes of the Physical Appearance of Jupiter etc., accompanying the foregoing observations are unavoidably deferred.*—Ed.]

MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

Vol. XL.	*February* 13, 1880.	No. 4.

Lord Lindsay, M.P., F.R.S., President, in the chair.

Walter H. Bartlett, Esq., 100 Abbey Road, Kilburn; and
Captain John Steele, Marine Board, Tower Hill,

were balloted for and duly elected Fellows of the Society.

Report of the Council to the Sixtieth Annual General Meeting of the Society.

Progress and present state of the Society:—

	Compounders	Annual Subscribers	Non-resident	Mathematical Society	Patroness	Total Fellows	Associates	Grand Total
December 31, 1878	218	362	4	6	1	591	40	631
Since elected ...	+ 7	+ 24	...	...	...	...	+ 4	...
Deceased	− 13	− 10	...	− 1	...	...	− 1	...
Removals	+ 3	− 3	...	...	...	...	...	...
Resigned	− 1	− 5	...	...	...	...	...	...
December 31, 1879	214	368	4	5	1	592	43	635

Mr. Barrow's Account as Treasurer of the Royal

RECEIPTS.

	£	s.	d.	£	s.	d.
Balance at Bankers' Dec. 31, 1878	605	0	3			
„ in hand of Secretary of Library Committee:						
On account of Turnor Fund	1	1	11			
On account of Library Expenses ...	16	12	8			
„ in hand on Petty Cash account	7	14	1			
				630	8	11
Dividend on £6,400 Consols...	94	0	0			
„ £5,700 New 3 per cent. Stock ...	83	14	5			
„ £6,808. 3*s*. 3*d*. Consols	99	19	11			
„ £5,700 New 3 per cent. Stock ...	83	14	5			
				361	8	9
Received on account of Subscriptions:—						
Arrears	135	9	0			
246 Contributions for 1879	516	12	0			
2 „ for 1880	4	4	0			
31 Admission Fees	65	2	0			
24 First Contributions	36	15	0			
				758	2	0
10 Composition Fees				210	0	0
Sales of Publications:—						
At Society's Rooms, 1879	43	19	7			
At Williams & Norgate's, 1878	36	17	9			
				80	17	4
Received from Mr. Edward Lee, being the amount of the purchase of the Advowsons of Hartwell and Stone, pursuant to the Resolution passed at the last Annual General Meeting				700	0	0
Due to Assistant Secretary on Petty Cash Account, Dec. 31, 1879				1	4	5
				£2,742	1	5

Astronomical Society, from Dec. 31, 1878, *to Dec.* 31, 1879.

EXPENDITURE.

	£	s.	d.	£.	s.	d.
Salaries:—						
Editor of *Monthly Notices*	60	0	0			
Assistant Secretary, one half-year, at £150 ...	75	0	0			
„ „ one half-year, at £225 (as per resolution of Council, Nov. 14, 1879) ...	112	10	0			
				247	10	0
Income Tax and House Duty				9	3	9
Fire Insurance				7	16	6
Printing: Spottiswoode & Co.				561	5	3
Lithography and Engraving:—						
Malby & Sons	64	4	3			
M. & N. Hanhart	12	14	0			
W. H. Wesley	31	6	0			
				108	4	3
Turnor Fund: Books purchased during year ...				32	15	3
Library expenses: Binding				94	5	5
Purchase of scarce volumes of the *Memoirs* ...				2	8	0
House expenses	31	4	6			
Wages	25	16	0			
Stamps and postage	54	9	7			
Carriage of books and parcels	4	12	8			
Stationery and office expenses	4	13	2			
Expenses of meetings	20	0	0			
Coals and gas	44	7	8			
Care of apparatus for extinguishing fire	3	16	8			
Fittings in library	16	19	6			
Sundry fittings and repairs	11	3	10			
Repairs to instrument	11	14	6			
Sundries	5	11	9			
Bankers' commission on cheques	0	1	0			
				234	10	10
Five Gold Medals				52	10	0
Mrs. Jackson-Gwilt's Annuity				8	19	0
Messrs. Merriman & Co.'s charges as to the Advowsons of Stone and Hartwell				50	7	3
Investments:—						
Purchase of £408. 3s. 3d. Consols, at 98, including commission	400	11	3			
Purchase of £691. 16s. 9d. Consols, at 97¼, including commission	673	13	9			
				1,074	5	0
Balance at Bankers' Dec. 31, 1879	217	7	0			
„ in hand of Secretary of Library Committee:						
On account of Turnor Fund	18	6	8			
On account of Library expenses ...	22	7	3			
				258	0	11
				£2,742	1	5

Examined and found correct,

JAMES CAMPBELL.
SIDNEY WATERS.
ROBT. J. LECKY.

Assets and present Property of the Society, January 1, 1880:—

	£	s.	d.	£	s.	d.
Balance at Bankers' Dec. 31, 1879	217	7	0			
, in hand of Secretary of Library Committee	40	13	11			
	258	0	11			
due to Assistant Secretary on Petty Cash Account...	1	4	5			
				256	16	6
Due on account of Subscriptions:—						
4 Contributions of 6 years' standing ...	50	8	0			
4 " 5 " ...	42	0	0			
7 " 4 " ...	58	16	0			
4 " 3 " ...	25	4	0			
31 " 2 " ...	130	4	0			
51 " 1 " ...	107	2	0			
Various amounts	19	19	0			
Two admission fees and first contributions	6	6	0			
	439	19	0			
Less two contributions paid in advance ...	4	4	0			
				435	15	0
Due for publications	4	7	0			
" from Williams & Norgate for sales during 1879	36	19	0			
				41	6	0

£7,500 Consols, including the Lee Fund (£300), the Turnor Fund (£450), and the Horrox Memorial Fund (£100).

£5,700 New 3 per cent. Stock, including Mrs. Jackson-Gwilt's gift (£300).

Astronomical and other MSS., books, prints, instruments, &c.

Five gold medals.

Unsold publications of the Society &c.

Report of the Auditors.

We, the duly appointed Auditors, beg to lay before this General Meeting of the Royal Astronomical Society the following Report:—

We have examined the Treasurer's account, and an account of the assets and property of the Society, and have found and certified the same to be correct.

The receipts and expenditure for the past year are as stated in the Treasurer's account. The cash in hand on December 31, 1879, including the balance at the Bankers', amounts to £258. 0*s.* 11*d.* The funded property of the Society is in a

satisfactory state, and the books, instruments, and other effects have been examined as far as possible and found in a satisfactory condition.

We have laid on the table a list of the names of those Fellows who are now in arrear for sums due at the last Annual General Meeting, with the amount due against each Fellow's name.

ROBT. J. LECKY.
JAMES CAMPBELL.
SIDNEY WATERS.

Stock in hand of volumes of the *Monthly Notices*:—

Vol.	At Society's Rooms.	At Williams & Norgate's.	Vol.	At Society's Rooms.	At Williams & Norgate's.
I.	79	1	XXII.	36	...
II.	79	1	XXIII.	22	...
III.	...	...	XXIV.	24	...
IV.	...	...	XXV.	7	...
V.	...	...	XXVI.	11	...
VI.	46	...	XXVII.	2	...
VII.	2	...	XXVIII.	78	1
VIII.	142	2	XXIX.	57	2
IX.	24	3	XXX.	72	4
X.	177	2	XXXI.	104	2
XI.	186	2	XXXII.	127	5
XII.	11	2	XXXIII.	118	2
XIII.	163	3	XXXIV.	92	3
XIV.	109	3	XXXV.	73	2
XV.	129	2	XXXVI.	41	2
XVI.	110	3	XXXVII.	55	1
XVII.	137	1	XXXVIII.	119	1
XVIII.	167	1	XXXIX.	125	12
XIX.	64	...	Index to *Monthly Notices*	601	...
XX.	37	...			
XXI.	20	...			

In addition to the above volumes of the *Monthly Notices*, the Society has a considerable stock of separate numbers of nearly all the volumes. With the exception, however, of Vols. XXXVI. to XXXIX., no complete volumes can be formed from the separate numbers in stock.

Stock in hand of volumes of the *Memoirs*:—

Vol.	At Society's Rooms.	At Williams & Norgate's.	Vol.	At Society's Rooms.	At Williams & Norgate's.
I. Part 1	9	...	XXIII.	159	1
I. Part 2	46	...	XXIV.	165	2
II. Part 1	61	...	XXV.	177	2
II. Part 2	25	...	XXVI.	181	2
III. Part 1	70	...	XXVII.	436	1
III. Part 2	92	...	XXVIII.	395	...
IV. Part 1	87	3	XXIX.	421	...
IV. Part 2	98	3	XXX.	172	...
V.	112	4	XXXI.	154	2
VI.	131	4	XXXII.	171	3
VII.	155	3	XXXIII.	178	2
VIII.	133	4	XXXIV.	177	8
IX.	142	3	XXXV.	127	3
X.	152	1	XXXVI. (with M.N.)	206	15
XI.	165	1	XXXVI. (without)	15	...
XII.	169	...	XXXVII. Part 1	373	8
XIII.	177	...	XXXVII. Part 2	316	6
XIV.	378	3	XXXVIII.	308	1
XV.	149	1	XXXIX. Part 1	285	2
XVI.	179	...	XXXIX. Part 2	296	3
XVII.	155	3	XL.	326	1
XVIII.	159	...	XLII.	299	3
XIX.	164	...	XLIII.	343	2
XX.	162	2	XLIV.	529	6
XXI. Part 1	314	...	Index to *Memoirs*	669	4
XXI. Part 2	99	...			
XXI. 1 & 2 (together)	68	1			
XXII.	163	2			

Instruments belonging to the Society.

The Council are glad to be able to state that the instrument catalogued in former reports as "Abraham Sharp's Quadrant" has been found. From notes in the handwriting of Dr. Priestley, now in the possession of the Society, it appears that the instru-

ment was at one time in his possession, and was described by him as "Abraham Sharp's Universal Quadrat."

Information has also been obtained with regard to the existence of "The Reade Transit Instrument" and *Sheepshanks'* Instrument No. 6 (formerly lent to the Rev. Jonathan Cape), both of which were mentioned in the Report of the Council of 1878 as missing. All the instruments referred to in the old Catalogues of the Society are thus accounted for, with the exception of Instrument No. 68, which is described as "A Thermometer."

No. 1. The *Harrison* clock.
" 2. The *Owen* portable circles, by Jones.
" 3. The *Beaufoy* circle.
" 4. The *Beaufoy* transit instrument.
" 5. The *Herschel* 7-foot telescope.
" 6. The *Greig* universal instrument, by Reichenbach and Ertel. The transit telescope, by Ultzschneider and Fraunhofer, of Munich.
" 7. The *Smeaton* equatoreal.
" 8. The *Cavendish* apparatus.
" 9. The 7-foot Gregorian telescope (late Mr. Shearman's).
" 10. The variation transit instrument (late Mr. Shearman's).
" 11. The universal quadrat, by Abraham Sharp.
" 12. The *Fuller* theodolite.
" 13. The standard scale, by Troughton and Simms.
" 14. The *Beaufoy* clock, No. 1.
" 15. The *Beaufoy* clock, No. 2.
" 16. The *Wollaston* telescope.
" 17. The *Lee* circle.
" 18. The *Sharpe* reflecting circle.
" 19. The *Brisbane* circle.
" 20. The *Baker* universal equatoreal.
" 21. The *Reade* transit.
" 22. The *Matthew* equatoreal, by Cooke.
" 23. The *Matthew* transit instrument.
" 24. The *South* transit instrument.
" 25. A quadrant, by Bird (formerly belonging to Captain Cook).
" 26. A globe showing the precession of the equinoxes.

The *Sheepshanks* collection:—

" 27. (1) 30-inch transit instrument, by Simms, with level and two iron stands.
" 28. (2) 6-inch transit theodolite, with circles divided on silver; reading microscopes, both for altitude and azimuth; cross and siding levels; magnetic needle; plumbline; portable clamping foot and tripod stand.

No. 29. (3) $4\frac{6}{10}$-inch achromatic telescope, about 5 feet 6 inches focal length; finder; rack motion; double-image micrometer; two other micrometers; object-glass micrometer; one terrestrial and ten astronomical eyepieces, applied by means of two adapters, with equatoreal stand, clock movement.

„ 30. (4) $3\frac{1}{4}$-inch achromatic telescope, with equatoreal stand; double-image micrometer; one terrestrial and three astronomical eyepieces.

„ 31. (5) $2\frac{3}{4}$-inch achromatic telescope, with stand; one terrestrial and three astronomical eyepieces.

„ 33. (7) 2-foot navy telescope.

„ 34. (8) A transit instrument of 45 inches focal length; with iron stand, and also Ys for fixing to stone piers; two axis levels.

„ 35. (9) Repeating theodolite, by Ertel, with folding tripod stand.

„ 36. (10) 8-inch pillar sextant, by Troughton, divided on platinum, with counterpoise stand and artificial horizon.

„ 37. (11) Portable zenith telescope and stand, $2\frac{3}{4}$-inch aperture and 26 inches focal length; 10-inch horizontal circle and 8-inch vertical circle, read to 10″ by two verniers to each circle.

„ 38. (12) 18-inch Borda repeating circle, by Troughton, $2\frac{1}{8}$-inch aperture and 24 inches focal length; the circles divided on silver, the horizontal circle being read by four verniers, and the vertical circle by three verniers, each to 10″.

„ 39. (13) 8-inch vertical repeating circle, with diagonal telescope, by Troughton and Simms; circle divided on silver, reading to 10″; a 5-inch circle at eye-end reading to single minutes; horizontal circle 9 inches diameter in brass, reading to single minutes.

„ 40. (14) A set of surveying instruments, consisting of a 12-inch theodolite for horizontal angles only, reading to 10″; two sets of adjusting plates; tripod stand with enclosed telescope; heavy stand for theodolite; Y piece of level; two large and three small ground-glass bubbles divided; level collimator, object-glass $1\frac{5}{8}$-inch diameter and 16 inches focal length; micrometer eyepiece, comb, and wires; mercury bottle and trough.

„ 41. (15) Level collimator with object-glass $1\frac{7}{8}$-inch diameter and 16 inches focal length; stand, rider-level, and fittings.

„ 42. (16) 10-inch reflecting circle, by Troughton, reading by three verniers to 20″; counterpoise stand; artificial horizon with mercury; two tripod stands.

„ 43. (17) Hassler's reflecting circle, by Troughton, with counterpoise stand.

No. 44. (18) 6-inch reflecting and repeating circle, by Troughton and Simms, contained in three boxes, two of which form stands. The circle is divided on silver, and is read to single minutes; two inside arcs divided to single degrees, 150 degrees on each side; artificial horizon and mercury.

„ 45. (19) 5-inch reflecting and repeating circle, by Lenoir, of Paris.

„ 46. (20) Reflecting circle, by Jecker, of Paris, 11 inches in diameter, with one vernier reading to 15″.

„ 47. (21) Box sextant; reflecting plane and level.

„ 48. (22) Prismatic compass, by Troughton and Simms.

„ 49. (23) Mountain barometer.

„ 50. (24) Prismatic compass, by Thomas Jones, mounted with a cylindrical lens.

„ 51. (25) Ordinary 4½-inch compass with needle.

„ 52. (26) Dipping needle, by Robinson.

„ 53. (27) Compass needle, mounted for variation.

„ 54. (28) Magnetic intensity needle, by Meyerstein, of Göttingen; a strongly fitted brass box with heavy magnet; filar suspension.

„ 55. (29) Box of magnetic apparatus.

„ 56. (30) Hassler's reflecting circle, by Troughton; a 10½-inch reflecting and repeating circle, with stand and counterpoise, divided on platinum with two movable and two fixed indices; four verniers reading to 10″.

„ 57. (31) Box sextant and glass plane artificial horizon, by Troughton and Simms.

„ 58. (32) Plane 2⅜-inch speculum, artificial horizon, and stand.

„ 59. (33) 2½-inch circular level horizon, by Dollond.

„ 60. (34) Artificial horizon, roof, and trough; the trough 8¼ by 4¼ inches. Tripod stand.

„ 61. (35) Set of drawing instruments, consisting of 6-inch circular protractor and common protractor, T-square: one beam compass.

„ 62. (36) A pentagraph.

„ 63. (37) A noddy.

„ 64. (38) A small Galilean telescope with object-glass of rock crystal.

„ 65. (39) Five levels.

„ 66. (40) 18-inch celestial globe.

„ 67. (41) Varley stand for telescope.

„ 69. (43) Telescope, with the object-glass of rock crystal.

„ 70. Portable equatoreal stand.

„ 71. Portable altazimuth tripod.

„ 72. Four polarimeters.

„ 74. Registering spectroscope, with one large prism.

„ 76. Two five-prism direct-vision spectroscopes.

„ 78. 9¼-inch silvered-glass reflector and stand, by Browning.

No. 79. Spectroscope.
„ 80. A small box, containing three square-headed Nicol's prisms; two Babinet's compensators; two double-image prisms; three Savarts; one positive eyepiece, with Nicol's prism; one dark wedge.
„ 81. A back-staff, or Davis' quadrant.
„ 82. A nocturnal or star dial.
„ 83. An early non-achromatic telescope, of about 3 feet focal length, in oak tube, by Samuel Scatliffe, London.
„ 84. A Hollis observing chair.
„ 85. A double image micrometer, by Troughton and Simms.
„ 86. A 4½-inch Gregorian reflecting telescope, by Short, with altazimuth stand and 6-inch altitude and azimuth circles and two eyepieces.
„ 87. A 3¼-inch Gregorian reflecting telescope with wooden tripod stand.
„ 88. A pendulum with 5-foot brass suspension rod, working on knife edges, by Thomas Jones.
„ 89. A Rhabdological Abacus. A contrivance invented by Mr. H. Goodwyn, consisting of a box filled with compartments, in which are square rods covered with numbers, which can be arranged so as to facilitate the labour of multiplying high numbers.
„ 90. An Arabic celestial globe of bronze, not quite 6 inches in diameter.
„ 91. An astronomical time watchcase, by Professor Chevallier.
„ 92. A 2-foot protractor, with two movable arms, and vernier.
„ 93. A beam compass, in box.
„ 94. A 2-foot navigation scale.
„ 95. Stand for testing measures of length.
„ 96. Artificial planet and star, for testing the measurement of a fixed distance at different position angles.
„ 97. A 12-cell Leclanché battery.

The following instruments are lent, during the pleasure of the Council, to the undermentioned persons:—

No. 4. The *Beaufoy* transit instrument, to the Observatory, Kingston, Canada.
„ 12. The *Fuller* theodolite, to the Director of the Sydney Observatory.
„ 22. The *Matthew* equatoreal, to Mr. Brett.
„ 23. The *Matthew* transit, to Captain Noble.
„ 74. Registering spectroscope, with prism, to Mr. Lecky.
„ 76. One 5-prism spectroscope, to Mr. Plummer.
„ 78. The 9⅓-inch reflector, to Mr. Neison.
„ 85. Double-image micrometer, to Mr. Common.

From the *Sheepshanks* collection:—

No. 30. (4) 3½-inch equatoreal and stand, to Mr. Sadler.
" 31. (5) 2¾-inch telescope and stand, to Mr. Birt.
" 34. (8) Transit instrument, to the Rev. Professor Pritchard.
" 35. (9) Repeating theodolite, to the Sydney Observatory.
" 39. (13) 8-inch repeating circle, to Mr. Plummer.
" 43. (17) Hassler's reflecting circle, to Mr. Gill.
" 69. (43) Telescope, with rock-crystal object-glass, to Dr. Huggins.

The Gold Medal.

No Medal has been awarded by the Council this year.

The Library.

During the year the binding of the Society's books has been proceeded with. Eight hundred and sixty-two volumes have been bound in a substantial manner, at a cost of £94. 5*s*. Altogether since the removal of the Society to Burlington House nearly three thousand volumes have been bound, at a cost of about £450. The Library now contains about eight thousand volumes, and of these nearly one thousand more require binding.

A catalogue, in which the titles of the books are written upon cards, has been commenced, and is nearly half completed. In the first instance the cards will be arranged in alphabetical order, under author's names and catchwords; and when this catalogue is printed the cards will be rearranged so as to form a subject index, which will be kept in the Library for reference.

Publications of the Society.

Vol. XLI. of the *Memoirs*, containing collated accounts of Observations made during total Solar Eclipses, is now ready for distribution amongst the Fellows.

Vol. XLIV. of the *Memoirs* has been published during the past year. It contains the following Papers:—

Mr. E. Neison. "On a General Method of Treating the Lunar Theory."

Mr. N. E. Green. "Observations of *Mars* at Madeira, August and September 1877."

Mr. Maxwell Hall. "Opposition of *Mars* 1877."

Mr. S. W. Burnham. "Double Star Observations made in 1877–78 at Chicago with the 18½-inch Refractor of the Dearborn Observatory."

OBITUARY.

The Council regret that they have to record the loss by death of the following Fellows and Associates during the past year:—

Fellows:—J. R. Christie.
W. K. Clifford.
J. T. Cooper.
George Creaser.
H. M. E. Crofton.
Richard Farley.
Isaac Fletcher.
H. J. Gibson.
J. G. C. C. Godsman.
Sandford Gorton.
The Very Rev. H. P. Hamilton.*
Sir Rowland Hill.
C. H. Johns.
W. E. Jones.
Charles Judd.
Rev. H. C. Key.
Sir Thomas Maclear.
Henry Mann.
Rev. J. N. Peill.
J. E. Richard.
Rev. A. Robertson.
Benjamin Templar.
John Waterhouse.
S. C. Whitbread.
Richard Wilding.
Associate:—Prof. J. Lamont.

JAMES ROBERT CHRISTIE, F.R.S., second son of the late Professor S. H. Christie, for some time Secretary of the Royal Society, was born at Woolwich on February 9, 1814, and early acquired from his father a taste for mathematics. In 1837 he was appointed a mathematical master at the Royal Military Academy, Woolwich, and was, ten years later, promoted to the post of first mathematical master in the same institution, an appointment which he held till 1865, when he retired on a pension. Mr. Christie was the author of an "Introduction to Practical Astronomy," in which is given a clear exposition of the principles used in the reduction of observations, and of a "Collection of Elementary Test Questions in Mathematics." He also wrote papers "On the Extension of Budan's Criterion for the

* An obituary notice of Mr. Hamilton will appear in the next *Annual Report.*

Imaginary Roots of an Equation" (*Phil. Mag.*, 1842), and "On the Use of the Barometric Thermometer for the Determination of Relative Heights" (*Phil. Trans.*, 1846). His chief work was in connection with the Royal Military Academy, where he ably seconded the efforts of his father to raise the character of the mathematical studies at that institution. He died at Norwood on February 28, 1879. He was elected a Fellow of the Society on January 13, 1854.

WILLIAM KINGDON CLIFFORD was born at Exeter on May 4, 1845. His father was a well-known and active citizen, and filled the office of Justice of the Peace; his mother he lost early in life. He was educated at Exeter till 1860, when he was sent to King's College, London. In 1863 he came into residence at Trinity College, Cambridge, having previously obtained a minor scholarship. He was elected a scholar of the College, and graduated as second wrangler in 1867, Mr. Charles Niven being the senior wrangler in that year. He was also second Smith's prizeman. In 1868 he was elected Fellow of Trinity and appointed Assistant Tutor. In 1871 he was elected to the Professorship of Applied Mathematics at University College, London, an office which he held till his death. On April 7, 1875, he married Lucy, daughter of Mr. John Lane, and granddaughter of Blandford Lane, of Barbados. In the spring of 1876 grave indications of a pulmonary disease were noted; these gradually increased, and in April 1878 he was compelled to leave England for the Mediterranean, when he visited Gibraltar, Venice, Malta, &c. There being signs of improvement, he returned to London in August 1878. A relapse, however, took place in September, and his strength began visibly to diminish. At the beginning of 1879 he sailed for Madeira, his friends hardly expecting him to survive the voyage. He arrived there, however, safely, and some weeks were thus added to his life, the change of climate enabling him to spend his last days in ease and comparative enjoyment. He died on March 3, 1879.

His first papers, published while he was an undergraduate, are "Analogues of Pascal's Theorem" (*Quarterly Journal of Mathematics*, vol. vi., 1863) and "On Jacobians and Polar Opposites" (*Messenger of Mathematics*, vol. ii., 1864); one of his last completed papers seems to have been that "On the Classification of Loci," which appears in the *Philosophical Transactions*, 1878. Most of his contributions to mathematics were communicated to the London Mathematical Society. He published in 1877 the first portion of a book entitled *The Elements of Dynamic*, but his health prevented him from completing it. Many of the new terms employed in this work are already coming into general use. He left in manuscript a considerable portion of a book *On the Common Sense of the Exact Sciences*, which is now being edited by Mr. R. C. Rowe, Fellow of Trinity College, Cambridge, and will shortly be printed.

Among the best-known of Clifford's general writings may be mentioned his lectures "On Some of the Conditions of Mental Development," delivered before the Royal Institution on March 6, 1868, and "On the Aims and Instruments of Scientific Thought," delivered before the British Association at Brighton on August 19, 1872. Several also of his addresses before the Sunday Lecture Society were published and attracted considerable attention. His *Lectures and Essays* have since his death been edited by Leslie Stephen and Frederick Pollock, and published in two vols. 8vo., with an introduction, partly biographical, by F. Pollock; a republication of his mathematical writings, edited by Mr. R. Tucker, is in the press.

Clifford was one of the few mathematicians who can with justice be said to have shown real mathematical genius: everything that he did was distinguished by an originality which rendered his work unique. He died at the early age of 34, and a great portion even of this short life was devoted to philosophy, metaphysics, and subjects of more general interest; so that it is difficult to realise the position he might have attained to in mathematical science had he confined his attention more exclusively to it, and had his life been longer. As it is, the scientific position he has gained is a permanent one, and it will always remain a matter of surprise how it was possible to write the brilliant papers he has left us among the distractions of so many other occupations.

He was elected a Fellow of this Society on December 12, 1873, and of the Royal Society on June 4, 1874. He was a member of the Sicilian Expedition to observe the Eclipse of December 22, 1870, and the account of his observations is published in vol. xli., pp. 310–311, of the *Memoirs* of the Society. A few remarks of his on the subject are also contained in vol. ii. of the *Proceedings of the Cambridge Philosophical Society* (February 27, 1871).

JOSEPH THOMAS COOPER was born in London May 25, 1819, and died November 17, 1879. At an early age he evinced considerable talent for music, and was placed under Mr. William Holmes, of the Royal Academy of Music; he finished his education with M. Moscheles, then resident in England, and who subsequently became Principal of the Conservatoire at Leipzig. In 1837 Mr. Cooper was appointed organist of St. Michael's, Queenhithe; in 1844, of St. Paul's, Ball's Pond, London; in 1866, of Christ Church, Newgate Street; and, in 1876, of Christ's Hospital. The two latter appointments he held until his death. He was a member of the old Mathematical Society, and became a Fellow of the Royal Astronomical Society in 1845, when the former society was merged into the latter. His time, however, was so occupied with professional duties, that he was

unable to devote so much attention to scientific pursuits as he desired. He was also a member of the Philharmonic and other musical societies. For several years he was musical editor of the periodical *Evening Hours.*

George Creaser, the fourth son of Francis and Elizabeth Creaser, was born at Scampstone Lodge, near Walton, on February 25, 1802. The first six or seven years of his life he spent at Scampstone; the family then removed to Milnsbridge, near Huddersfield, his father becoming the steward and surveyor of Sir J. Radcliffe. When a boy his health completely gave way. Up to the age of eighteen he manifested no signs of what he was likely to be; but one of his elder brothers, William Creaser, having begun to devote his leisure hours to making telescopes, George joined him in this work, and began the study of Optics and Astronomy, which he steadily pursued. He married, about 1830, Miss Turner, of Selby, who died about eight years afterwards. He then gave up his whole time to the manufacture of telescopes, microscopes, &c., and continued to do so till his death. Few men have done so much work of this kind, especially with such simple tools. He died at Meltham, near Huddersfield, on April 21, 1879. He was elected a Fellow of the Society on February 13, 1874.

Henry Morgan Earbery Crofton, eldest son of the Rev. Henry William Crofton, of Inchinappa House, Ashford, County Wicklow, Ireland, succeeded to the considerable family estates on the death of his father.

He was educated at Trinity College, Dublin, and at an early age showed a decided taste for practical astronomy; and shortly after the completion of his University career provided himself with an 8-inch Equatoreal by Cooke, of York, and other astronomical instruments.

He resided until his early and untimely death upon his Inchinappa estate, and took considerable interest in its cultivation and improvement.

For many years he owned a yacht, in which, during the summer, he annually attended the regattas in the South and West of England and on the Irish coasts.

He married the daughter of Major Townsend, of Wicklow, County Inspector. His wife, by whom he left a young and numerous family, survived him only by a very brief period.

He was elected a Fellow of the Society on January 10, 1863.

Solomon Moses Drach was born in Bury Street, St. Mary Axe, London, in 1835. He went to a school kept by Mr. Cutler, in Devonshire Street, and afterwards to one kept by Mr. Tait, where he remained two years. His own name was Solomon

Moses, and he took the name of Drach from his aunt's husband, Mr. Liepman Woolf Drach, who (after his aunt's death) left him his fortune. He married, on August 4, 1841, his cousin Rebecca, the daughter of another aunt who had also been adopted by Mr. L. W. Drach. His uncle died on April 18, 1840, and he continued his business of a commission agent till the death of his aunt, which occurred on June 9, 1847, when he devoted himself to study, being chiefly interested in mathematics, astronomy, and biblical and antiquarian subjects.

Mr. Drach's first published paper appeared in the *Philosophical Magazine* for 1839, and was entitled "On the use of Barometrical Formulæ for determining the Heights of Mountains." He published also, in the *Philosophical Magazine* for 1849, several papers on mathematical questions relating to the description of epicyclical curves. These also had reference to, and contained some account of, the epicycloidal curves which Mr. Perigal had traced geometrically in 1835, and by continuous circular motions in 1840. Mr. Drach published numerous short papers in the *Monthly Notices* of the Society. He was also a frequent contributor to the library of the Society, having presented at different times many volumes, chiefly old astronomical and mathematical works; and after the removal of the Society to Burlington House he passed much of his time in the library, working at his favourite subjects. Some years ago he presented to the Royal Society two large volumes containing resolutions of numbers into sums of squares and cubes; and he devoted much of his time to work of a similar kind. He died, at 23 Upper Barnsbury Street, Islington, on February 8, 1879. He was elected a Fellow of the Society on May 14, 1841.

Mr. Drach was a member of the old Mathematical Society, and became a life Fellow of this Society when the former was merged into it in 1845.

RICHARD FARLEY was for a long period connected with the *Nautical Almanac* Office, first as an ordinary computer and afterwards as First Assistant. He was one of a number of young computers collected together in 1831 by Lieutenant Stratford, Superintendent of the *Nautical Almanac*, for the calculation of the first volume of the enlarged series of that work, according to the recommendations drawn up by a Committee of the Royal Astronomical Society in November of the preceding year. Though little more than twenty years of age, Mr. Farley had at this time already attained a good reputation as an accurate computer, and he was at once intrusted by Lieutenant Stratford with the calculations of the places of the Sun and the lunar distances for the *Almanac* for 1834, as well as with the general examination of most of the computations. After the resignation in 1837 of Mr. Woolhouse, First Assistant, Mr. Farley was chosen to succeed him in that important and responsible office, which he filled more

than thirty years with the greatest devotion to the work, under the superintendence of Lieutenant Stratford and Mr. Hind, even beyond the ordinary business hours of the office, till his superannuation about ten years ago. He was succeeded by Mr. W. Godward, the present Chief Assistant.

Mr. Farley's skill in calculation was well known to a large number of astronomers, who have often availed themselves of his services to assist in carrying out the reduction of large masses of observations, especially in the formation of general catalogues of stars. His first employment on these miscellaneous calculations, few of which are known to the astronomical world, was his engagement by Mr. Baily to assist in completing what at that time was considered a very important publication, the *Astronomical Society's Catalogue of* 2,881 *Stars*. When Mr. Baily undertook to superintend the compilation of a larger general catalogue of stars, which afterwards assumed the form of the *British Association Catalogue*, he again secured the assistance of Mr. Farley, in whose hands the superintendence of the reductions was placed. In this very laborious work he was assisted by Mr. Russell and Mr. Alger, two of his colleagues in the *Nautical Almanac* Office, but on him rested the entire responsibility for the accuracy of the computations. The magnitude of the reductions was sufficient to absorb the whole of Mr. Farley's unofficial time; and the work was performed to the satisfaction of Mr. Baily, who has acknowledged that it is to the labour, care, and attention of Mr. Farley in particular that the public are indebted for this well-known and popular Catalogue.

The convenient tables of logarithms, in imitation of the tables of Lalande, issued in 1839 under the superintendence of the Society for the Diffusion of Useful Knowledge, but more commonly associated with the name of Professor De Morgan, was carried through the press by Mr. Farley, who, in his most careful manner, examined the proof-sheets with Lalande's work, and afterwards with Vega's edition of Vlacq. Before the tables were stereotyped the differences were all retaken and the work examined throughout by Mr. Farley. It is therefore to him that astronomers and others have been principally indebted for the use of this conveniently arranged pocket volume of tables of logarithms. The reprint of Barlow's useful tables of squares, cubes, square roots, cube roots, and reciprocals, issued in 1840 by the above Society, was also carried through the press by Mr. Farley, by whom all the proofs were carefully read, and the numbers examined by second and third differences. Other works of a similar kind have also had the advantage of his assistance.

Though Mr. Farley's astronomical labours have been chiefly devoted to the assistance of others, he has made original investigations on the orbits of *Pallas* and *Vesta*, taking into account the perturbations by the principal planets. The annual ephemerides for these two minor planets, given in the *Nautical Almanac*, have been calculated from his elements. Mr. Farley was elected a

Fellow of the Society on March 8, 1850, and served four years on the Council, 1860–1864.

Isaac Fletcher, F.R.S., of Tarnbank, Cumberland, was born on February 22, 1827. He was the second son of John Wilson Fletcher, of Tarnbank. His mother, Mary, was a daughter of John Allason, of Beech Hill. He married, on December 13, 1861, Esther, daughter of the late Mr. Joseph King, of Wassall Grove, Stourbridge. He unsuccessfully contested Cockermouth in April 1868, but was elected at the general election in the following November, and retained the seat till his death. His politics were liberal. He was a Justice of the Peace for Cumberland. He died by his own hand, in London, on April 3, 1879.

Mr. Fletcher was elected a Fellow of this Society on May 11, 1849. He at first used a telescope of 4·16 inches aperture, for which he built a small observatory, described in vol. x., p. 137, of the *Monthly Notices*; subsequently he obtained a telescope of 9½ inches aperture, the mounting of which is described in vol. xxv., p. 242. He chiefly devoted himself to micrometrical measurements of double stars, and several communications of his on this subject have appeared in the *Monthly Notices*. He was elected a Fellow of the Royal Society on June 7, 1855.

Sandford Gorton early evinced a taste for mechanical and scientific pursuits, and astronomy soon became the principal occupation of his leisure. Soon after his marriage he went to reside at Stamford Villa, Downs Road, Clapton, where he established an Observatory. His principal telescope, which was of 3⅞-in. aperture, by Ross, was first mounted on an equatoreal stand by Cooke, and afterwards upon a stand with clock motion by Dallmeyer. To his instruments and Observatory Mr. Gorton was constantly adding ingenious contrivances. He became an ardent and persevering general observer, keeping an accurate record of his work. He was an excellent draughtsman, and his delineations of sun-spots and planetary features are marked by careful execution and strict fidelity. In 1861 he presented a series of 111 Indian-ink drawings of *Jupiter*, made between 1839 and 1861, to the Society (*Monthly Notices*, vol. xxii., p. 60). It was during his residence in the Downs Road that Mr. Gorton determined to establish a purely astronomical periodical. In the address which accompanied the first number of the *Astronomical Register*, he states that it occurred to him that it would be very desirable "to collect together those stray fragments of information which, though not of sufficient importance possibly to occupy the pages of the *Monthly Notices*, may nevertheless, in the shape of passing observations, or occasional notes, be useful for future reference,"

and that he wished "to introduce a sort of astronomical *Notes and Queries*, a medium of communication for amateurs and others," believing that "many valuable contributions to the science are now scattered in different publications: collected in one periodical they would be of far more benefit to the astronomical inquirer than they are now." Mr. Gorton was also of opinion that an account of the discussions which took place at the meetings of this Society should be published, both for the sake of those who were unable to be present, and also in order that some permanent record of them should be preserved; and the admirable reports of our proceedings contained in the *Astronomical Register* have always formed one of its chief characteristics. The first number appeared in January, 1863, and the whole of the first volume was printed by Mr. Gorton himself, at his own private printing press. The frontispiece of this volume is a plate of *Jupiter*, from a sketch by Mr. Gorton.

On his removal to his residence, Parnham House, Pembury Road, Clapton, he gave up his Observatory, and was often heard to express his regret that he had been compelled to abandon it, after all the labour that he had devoted to its construction. His health at this time began visibly to decline, but he nevertheless continued to edit and manage the *Astronomical Register*, in which he took the greatest interest, until 1872, when he was compelled with much reluctance to relinquish it. Since this time it has been continued by the Rev. J. C. Jackson.

Mr. Gorton had always a great fondness for nautical matters, and during the last five or six years of his life he resided during the summer and early autumn months in his yacht, the "North Star"; there is no doubt that the change of scene and pure and bracing breezes which he thus met with somewhat prolonged his life. His debility continually increased, and was at length succeeded by utter helplessness. He died on February 14, 1879, in his 56th year. He was elected a Fellow of the Society on June 8, 1860.

Sir Rowland Hill, the third son of Thomas Wright Hill, was born at Kidderminster on December 3, 1795, in a house which had belonged to his forefathers for some generations. The war with France had caused the ruin of the business in which his father was engaged, and the family was reduced to great straits. In her desire to secure an education for her children, his mother persuaded her husband to give up trade, for which he was very little fitted, and establish a school near Birmingham. At the age of eleven Rowland began to assist his father in teaching, and a year later he had ceased to be a pupil and had become altogether a teacher. While still quite a youth he and his brother Matthew began to discover the deficiencies in their father's school, and to set about reforming them. Matthew chiefly concerned himself with improving the instruction, while Rowland dealt with the

discipline and organisation. "Organisation," he used to say in after life, "is my forte." He aimed at making the boys govern themselves. A constitution was promulgated, and a code of laws was made, which fills more than a hundred pages of a closely printed volume published in 1827 when they had been in operation for fully ten years. A complete democracy was established, each boy having even the right of being tried by a jury of his schoolfellows whenever a charge was brought against him by one of the masters. In the *Essays of a Birmingham Manufacturer*, Mr. W. L. Sargant gives an account of the school and the system, the chief objection which he makes to it being that the boys were made premature men.* In a volume entitled *Public Education*, written chiefly by his brother Matthew, Rowland's new system was made known to the world. The book at once excited considerable public attention, and an article upon it appeared in the *Edinburgh Review*. Rowland used to boast that at this time he had the largest school in Warwickshire. After living at Birmingham till he was more than thirty, he removed to the neighbourhood of London, where, with the aid of one of his brothers, he established a branch school at Bruce Castle, Tottenham. But by this time his health, which had always been delicate, began to give way, and at length broke down. He had certainly tried it ever since childhood by the severest and the most prolonged labour. His work as a schoolmaster also had become distasteful to him, and he longed for a change. His means were very small, but he did not hesitate to give up his business in the full conviction that, with the powers he knew he had, he was as certain of success in some other path as a man could be. He always preserved, however, the strongest interest in his school, which was carried on by his younger brother Arthur. Rowland, as soon as his health was re-established after a long period of rest, began to cast about for a new employment. He had long been known to many leading men among the advanced Liberal party, not only by his work as a schoolmaster, but also as an eager advocate of political and social reform. He had assisted in founding the Society for the Diffusion of Useful Knowledge. He had published a plan for the gradual extinction of pauperism and for the diminution of crime. Shortly after his retirement from the school an association was formed for the colonisation of South Australia on the plan of Mr. E. G. Wakefield. In this he took an active part, and when the Act was carried through Parliament and a Commission was appointed, he was named secretary. He held this post for four years, and during this time he gave up most of his leisure to the invention of a printing machine. While he was still labouring at this machine, he began to interest himself in postal matters. He hesitated for a long time between his printing press and his postal project, but at length, being obliged to make a choice, he preferred the latter.

* Essay III. *Middle Class Education*, vol. ii., p. 191.

It is not needful to dwell on the state of the Post Office before Rowland Hill reformed it. Its charges were high and arbitrary, and its services were limited and irregular. There were districts larger than the county of Middlesex in which the postman never set foot. For the 11,000 parishes of England and Wales there were only 3,000 post offices. A single letter from London to Edinburgh was charged *1s.* 1½*d.* If it contained the smallest enclosure—a receipt, for instance—it was charged the double, *2s.* 3*d.* Two separate pieces of tissue paper sent in one enclosure would have been charged twice as much as the heaviest letter that was written on a single sheet. The upper classes, through the right of franking which was enjoyed by every member of Parliament, had to some extent their letters carried free of charge; and the traders, by the help of illicit means of conveyance, were often able to evade the heavy tax. The poor man, however, was helpless; he could not afford to use the Post Office, and had no other means of sending a letter. Under this system the postal revenue had remained stationary for twenty years. In 1835 the general revenue of the country showed a large surplus, and Rowland Hill began to speculate how it might be best employed. No tax, on examination, proved so defective as the tax on letters, and he then applied himself to investigate the state of the Post Office by the aid of the Parliamentary Reports. When he first turned his attention to the practicability of reforming the Post Office, he had no idea of uniformity of rate. He found it most difficult to obtain accurate statistics; but at length he discovered that the cost of conveying a letter from London to Edinburgh was only the 36th part of a penny, and that the cost of conveyance was so insignificant that a uniform rate could not only be established, but was "absolutely fairer than any other." He also formed large and bold schemes for the reorganisation of the Post Office. In 1837 he published his plan in a pamphlet entitled "Post Office Reforms." Associations were formed to carry it through, and petitions to Parliament in its favour soon began to pour in. In 1838 a select committee of the House of Commons was appointed to consider the plan: uniformity of postage was carried only by the casting vote of the chairman. A twopenny rate of postage was recommended. The Ministry still seemed indisposed to adopt the plan; but after a deputation of 150 members of Parliament, all supporters of the Government, had waited upon the Premier, Lord Melbourne, they yielded, and penny postage came into effect on January 10, 1840.

In order to carry out the reforms, Rowland Hill was appointed for two years to an office in the Treasury, at a salary of £1,500 a year. He had very little power given to him, and all the officers of the Post Office were against him, so that his position was very trying and painful. On Sir Robert Peel coming into office his services were dispensed with by the Government; but a national testimonial was raised, and at a public dinner he was presented with a cheque for £13,000. He then became a director,

and afterwards chairman, of the London and Brighton Railway Company, and it is stated that on his recommendation as chairman the first excursion train and the first express train were run.

In 1846, when the Liberals returned to power, he was offered an office in the Post Office itself. He was appointed, however, not Secretary to the Post Office, but Secretary to the Postmaster-General, so that the old double government was continued; and it was not till 1854, fourteen years after penny postage had been established, that, by his appointment as sole secretary, he was free to carry out his plans. He was in this greatly aided by his youngest brother, Frederick, who was transferred from the Home Office to the Post Office.

In 1860 he was made a K.C.B., and in the same year the late Lord Stanley of Alderley was appointed Postmaster-General. Differences arose between him and Sir Rowland, chiefly upon the question of promotion by merit, which the latter had succeeded in introducing into the Post Office, with the sanction of his previous chiefs, in place of the system of patronage. Sir Rowland appealed in vain to the Treasury, and accordingly, his health also failing him, he sent in his resignation in 1864. The House of Commons granted to him, on the recommendation of Her Majesty, the sum of £20,000, without a division, and in addition he received his full salary of £2,000 a year for life. In 1865 he was appointed a member of a Royal Commission on Railways. He received the degree of D.C.L. from Oxford, and only a few weeks before his death the freedom of the City of London. He was elected a Fellow of the Royal Society on June 11, 1857.

He was one of eight children, six sons and two daughters, all of whom reached adult age. He married, in 1827, Caroline, eldest daughter of Mr. Joseph Pearson, of Graisley, near Wolverhampton, who survives him, and by whom he had four children, one son and three daughters. He passed away unconsciously, at his residence at Hampstead, at half-past four o'clock in the morning of August 27, 1879, and was buried in Westminster Abbey on September 4. Sir Rowland was one of the oldest Fellows of this Society, having been elected more than 57 years ago, viz. on November 8, 1822. At the time of his death there were but two older Fellows of the Society, and one of these, the Rev. H. P. Hamilton, Dean of Salisbury, has since died.

William Edward Jones, being then resident in Leamington, began his astronomical pursuits about 1860, with a fine 4-inch Refractor equatoreally mounted on a moveable tripod. This he afterwards replaced by a 5-inch instrument by Dallmeyer, which he erected first at Brighton and subsequently at his residence in Gloucestershire. Two years ago he added a 9-inch Browning-With Reflector, but his failing health allowed him to make but little use of this. He was chiefly interested in double stars. He died at Hyères after a short illness, at the age of 57. He was elected a Fellow of the Society on May 12, 1865.

CHARLES JUDD was born at Edmonton on March 23, 1826, and was educated at the Edmonton Grammar School, then called Latymer's School. At the age of 16 he went to the Westminster Training College. Upon resigning an appointment which he had obtained in the Rev. Mr. Bertie's school at Ilford, he became the head master of the "Yellow School," at Cirencester. Here he laboured assiduously for some years, undertaking, in addition to his ordinary work, the delivery of public lectures on Astronomy, Physical Geography, and other branches of science. In 1854 he resigned this appointment, and became an assistant master in Denmark Hill Grammar School. In 1858, after a brief interval, in which he was connected with Stockton Grammar School, he was elected second master of the Foundation School, Whitechapel, and about ten years subsequently became head master. This post he retained until his death, which took place after a short illness at Freiburg, Baden, to which place he had gone to recruit his strength during the autumn vacation. Mr. Judd was a member of the Numismatic Society, an Associate of King's College, a Fellow of the Geological Society, and had been for some years a member of the Council of the College of Preceptors. He was also for several years an active member on the London Committee for the Oxford Local Examinations. He leaves a widow and four children. He was elected a Fellow of the Society on June 14, 1867.

HENRY COOPER KEY, M.A., was the eldest son of the celebrated surgeon, C. Aston Key, of Guy's Hospital. He was born in 1819, and educated at private schools, and at Christ Church, Oxford, where he took his degree in 1842. He early showed a taste for astronomy, and while quite a boy constructed a telescope. After being four years in orders, he was presented by Guy's Hospital to the living of Stretton Sugwas, in Herefordshire: the parish being very small, he had leisure for scientific pursuits.

In 1859 he began his attempts to grind glass mirrors for Newtonian reflectors, then a most difficult task, the process having been almost lost; but after some years' persevering efforts, in co-operation with Mr. George With, of Hereford, he succeeded in inventing a method by which great accuracy of figure could be obtained.

After making several mirrors of increasing diameters, he attained the object of his ambition, a speculum of 18 inches diameter and 11 feet focus. On achieving this success, he discontinued the mechanical work of mirror-making, and used his splendid instrument in observing. Its great light-collecting power and admirable definition rendered it especially suitable for studying faint objects. He discovered the remarkable depression in the Moon's surface which has been named after him, and contributed observations of comets, nebulæ, &c. to the *Monthly Notices* at intervals. He was much interested in other branches of science, such as ancient chronology, geological periods, the tidal wave in the Medi-

terranean, &c., and was engaged in the last weeks of his life in observing the periodical earth-vibrations noticed by Plantamour and D'Abbadie, which he expected would have thrown light on obscure geological and meteorological questions. From careful observations made with accurate instruments, he believed that, the elements from which a law might be deduced had been obtained.

After a short illness, he died on December 25, 1879, aged 60. He was elected a Fellow of the Society on November 9, 1860. His paper "On a mode of figuring glass specula for the Newtonian telescope" appeared in vol. xx., pp. 199–202, of the *Monthly Notices*, and that "On certain depressions on the Moon's western limb" in vol. xxiv., pp. 20–23.

Sir THOMAS MACLEAR, formerly Her Majesty's Astronomer at the Cape of Good Hope, was the eldest son of James Maclear of Newton Stewart, Tyrone, Ireland. He was born on March 17, 1794, and died at his residence, Grey Villa, Mowbray, Cape of Good Hope, on July 14, 1879. His early childhood gave promise of the talents that distinguished the future man, and his proficiency in Latin when only seven years old caused his father to wish him to enter the Church. An attempted coercion in this matter ended finally in a breach between father and son; the funds required for his education as a medical man were placed in the hands of friends, and he was dismissed to England, at the age of 15, to the care of his maternal uncles, Sir George and Dr. T. Magrath, both eminent medical men.

He studied at Guy's and St. Thomas's Hospitals, and passed his examinations in all branches of his profession with high honours. Brilliant prospects were opened before him in London; but he preferred a quieter life, where he could follow with more advantage his mathematical and astronomical studies, which already were becoming to him a necessity of life. He was accordingly elected house-surgeon of the Bedford Infirmary, where, in the congenial society of Admiral Smyth and others, he combined with the practice of his profession a gradually increasing study of astronomy.

In 1823 he moved to Biggleswade, where he practised his profession; and in 1825 he married Mary, third daughter of Mr. Theed Pearse, for many years clerk of the peace of the county of Bedford. His astronomical pursuits daily absorbed him more and more, and he built a little observatory in his garden, in which he spent every moment he could spare.

In 1833 he was appointed Her Majesty's Astronomer at the Cape of Good Hope. He reached his new sphere of labour on January 5, 1834, and took up his residence at the then desolate-looking Observatory.

Ten days afterwards Sir John Herschel arrived at the Cape, and the next four years were spent in happy mutual intercourse

between the two astronomers, each assisting with heart and soul the labours of the other. From the date of Mr. Maclear's arrival the records of the Observatory bear ample testimony to his devotion and zeal, and show that he applied himself to the execution of his duties as laid down in his official instructions, with the untiring energy which distinguished him.

The Transit Instrument and Mural Circle were kept in constant use, and a large amount of most valuable material was accumulated. Under the clear skies of the Cape it was inevitable that with a man of such a temperament observations would far exceed the computing powers of his staff. The personal establishment of the Observatory at the time was much too limited to enable the astronomer to reduce and publish the great mass of observations which he accumulated. The actual force on duty at the Observatory was as follows: from January 1834 to December 1834, the astronomer and one assistant; December 1834 to October 1835, the astronomer; October 1835 to October 1839, the astronomer and one assistant; October 1839 to October 1845, the astronomer and two assistants.

In 1838 the first part of his great work, *The Verification of Lacaille's Arc of the Meridian*, was commenced. The measurement of this arc and its extension were commenced in 1840, and the field work was finished in 1847.

It is impossible to convey within the limits of this notice any adequate idea of the indomitable energy and perseverance with which this operation was carried out, of the difficulties surmounted, and of the extent and value of the work accomplished with limited means. That all this was fully recognised at the time is sufficiently testified by the fact that for this work he received the Royal Medal of the Royal Society of London and the Lalande Medal of the Institute of France.

Besides the great value of this survey in a geodetic sense, it forms the groundwork of the survey of the colony, and has given to that operation in certain parts of the country a character and completeness which it would otherwise have lacked.

In 1847 a 46-inch Achromatic Telescope by Dollond was mounted equatoreally, and in 1849 an Equatoreal by Merz, of 7 inches aperture and 8½ feet focal length, was added to the instrumental equipment of the Observatory.

These instruments were vigorously employed in the observation of double stars, comets, and nebulæ, and of occultations of stars by the Moon. The results of many of these observations have been published from time to time in the *Memoirs* of this Society, but some of the most valuable series still remain to testify still more strongly to the unwearied activity and skill of Mr. Maclear as a practical astronomer. The original records show that the observations were sustained nearly all night long, and there are frequent notes to the effect that the observations have been brought to a close by the rising Sun.

Simultaneously with these observations the meridian instruments were worked with redoubled energy, and during the years 1849–53 the whole of the stars of the B.A.C. having south declination were observed generally three times in both coordinates. The energy with which this series of observations was carried on is shown by the fact that in the year 1852 between 9,000 and 10,000 observations of right ascension were made with the Transit-Instrument, on some nights over 100 stars having been observed. The reduction of these observations, with a view to their early publication as the "Cape Catalogue for 1850," now occupies the attention of Her Majesty's Astronomer.

In 1855 the new Transit-Circle (a facsimile of the Greenwich Transit-Circle) arrived. It was duly mounted with the assistance only of local masons and labourers, and observations were commenced with it at the end of the same year.

In 1859 Mr. Maclear paid a visit of a few months to England, and keenly enjoyed the seeing of old friends and making the personal acquaintance of many men only known to him by repute or by correspondence.

He returned to the Cape in 1860, and in June of the same year he received the honour of knighthood, a well-merited recognition of his labours in science. After the year 1860 Sir Thomas Maclear's attention was chiefly directed to the reduction of his previous observations. He reduced the valuable series of observations made in the years 1835–40, which have since been revised by Mr. E. J. Stone, Sir Thomas Maclear's successor, now Radcliffe Observer, and formed by him into the Cape Catalogue for 1840, containing 2,892 stars. Sir Thomas also partially reduced the observations made with the new Transit-Circle in the years 1856–60, a work completed by his successor, and published under the title of the "Cape Catalogue for 1860."

All comets visible in the southern hemisphere were diligently observed by him with the Equatoreal, and the places of the comparison stars observed with the Transit-Circle. The results of these observations were always promptly reduced and published in the *Memoirs* or *Monthly Notices* of this Society.

Amongst these may be quoted the following series: 1835–36, Halley's Comet; 1842, Encke's Comet; 1844, Mauvais' second Comet; 1844–45, Wilmot's Comet; Comet of 1843; 1846, Biela's Comet; 1850, Petersen's Comet; 1853–54, Klinkerfues' Comet; 1855, Encke's Comet; 1857, D'Arrest's Comet; 1858–59, Donati's Comet; Comet III, 1860; Comet I, 1861; 1861–62, Encke's Comet; Comet II, 1862; Comet I, 1865; 1865, Encke's Comet.

He determined the parallax of *α Centauri*, confirming Henderson's result of an annual parallax amounting to nearly 1″ of arc. He accumulated a fine series of observations of the most interesting binaries in the southern heavens, which is still unpublished.

He made a long-continued series of observations of the Moon

and of Moon-culminating stars for the purpose of determining the longitude of the Observatory and the parallax of the Moon.

Besides these varied astronomical labours he gave much attention to meteorological, magnetic, and tidal observations. He was successful in exciting an interest throughout the colony in meteorological observation, and was always ready to lend a helping hand to any student of science.

He threw himself with heart and soul into all measures by which he could promote the well-being of the colony. He was a member of the Examining Board; for many years he assisted in the establishment of lighthouses, and was the originator of, and took part in a commission on standards of weights and measures. He felt the keenest interest in sanitary matters, and in cases of emergency has lent his medical assistance.

He was the intimate friend of Livingstone. Their acquaintance commenced in 1850, when Livingstone came to him for assistance as to the best means of ascertaining his position when on his travels. Livingstone's quickness and aptitude for the work won Maclear's heart: the men were kindred spirits, and their firm friendship continued to the end.

The reduction of Livingstone's observations was performed at the Observatory, and formed a serious item in the work undertaken; but the labour was a labour of love.

The year 1861 was shadowed by a sad bereavement, which caused him the keenest sorrow. He occupied himself still more closely with his official duties, and in the various colonial matters in which he took a part, till, in 1870, he retired from the Observatory, and lived at his residence in Mowbray, about a mile from the scene of his former labours. Latterly his sight failed him, and in 1876 he became totally blind. In his declining health he was tenderly nursed by a devoted family; he kept up his interest in science and politics with unabated vigour, his daughters reading to him for hours together. He was particularly interested in all matters connected with the exploration of Africa, and the last occasion on which he left his house was to attend a meeting held in Cape Town when Stanley visited the colony.

No name was better known or better loved in the colony than that of Sir Thomas Maclear. On the occasion of his last public appearance, which we have just mentioned, he was received with even greater applause than that which greeted Stanley himself.

Sir Thomas Maclear gently breathed his last on July 14, and his remains were interred in the Observatory grounds beside those of his wife, not far from the spot where Fallows—the first astronomer at the Cape—is buried.

Sir Thomas Maclear was elected in 1828 a Fellow of this Society; in 1831, a Fellow of the Royal Society; in 1835, a Member of the Academy of Sciences, Palermo; in 1858, a Corresponding Member of the Imperial Geological Institution

and Geographical Society of Vienna; and in 1863, a Corresponding Member of the Institute of France. In 1860 he received the honour of knighthood, in 1867 he received the Lalande Prize of the Institute of France, and in 1869 a Royal Medal of the Royal Society.

The House of Assembly at Cape Town agreed to the following resolution on July 17, 1879:—"That this House desires to express its deep sense of the signal services rendered by the late Sir Thomas Maclear, Knt., F.R.S., F.R.A.S., to the general cause of astronomical and geographical science while in charge of the Royal Observatory, Cape Town, and also to the material interests of the colony in the practical application of his researches; and, furthermore, its high appreciation of his devotion for a long period of years to the cause of South African exploration and civilisation; and that this resolution be recorded in the journals of the House."

Henry Mann, late of Spern Bank, near Cleckheaton, died at 15 Phillimore Gardens, Kensington, on August 20, 1879, aged 73. He was attached to astronomy, and possessed a valuable instrument; he was also a distinguished amateur musician. He was elected a Fellow of the Society on May 12, 1871.

John Netwon Peill, B.D., was born at Liverpool on December 14, 1808. He was educated at the Royal Institution in that town, and afterwards at Queen's College, Cambridge. He graduated as Seventh Wrangler in the Mathematical Tripos, 1831, and was soon after elected a fellow of his college, of which he became also tutor and bursar. He was rector of St. Botolph's, Cambridge, from 1843 until 1853, when he was presented by his college to the rectory of Newton Toney, in the diocese of Salisbury. He became also rural dean of Amesbury, and diocesan inspector of schools. A most active and energetic clergyman, he was well known for his administrative abilities, especially in connection with schools for the poorer classes. He possessed some good astronomical instruments, and made many observations in his little country Observatory, but the mathematical side of astronomy had for him the most interest. He died on June 12, 1879. He was elected a Fellow of the Society on January 12, 1869.

John Edmund Richard was the son of John Richard, of St. Martin's Lane, and was born in London in 1818. He received the greater part of his education privately, and early in life joined his father in the old-established firm of Richard & Wilson, wholesale stationers, St. Martin's Lane, London. Mr. Richard's love of scientific pursuits was manifested in his younger days, and he appears to have dipped into most sciences; but circum-

stances having thrown him much into the society of astronomers, he for many years took up that branch of science as his favourite recreation. He built himself an observatory in the grounds surrounding his house at West Hill, Wandsworth, and furnished it with one of Cooke's Equatoreals and a Transit Instrument by Simms. Here he carried on a series of observations which probably were known to but few even of his intimate friends, his retiring nature preventing him from giving that publicity to his works which some of them deserved.

His delight was to surround himself with men interested in science, and his great hospitality and warm-heartedness were of assistance to many. He enjoyed the satisfaction of seeing a family grow up around him who took great interest in their father's scientific pursuits. He died on October 24, 1879, after an illness, not of long duration, but of intense suffering. He was elected a Fellow of the Society on June 10, 1859.

JOHN WATERHOUSE, F.R.S., was born at Halifax, Yorkshire, on August 3, 1806.

His father, John Waterhouse, of Well Head, was the representative of a family which, for 400 years, had been intimately connected with the prosperity of the town and neighbourhood.

Very early in life he evinced a decided taste for scientific studies, and the training which he received at school only served to increase this preference, and enabled him to obtain a sufficient knowledge of mathematics, which he turned to good account in after years in the various branches of physical research to which he gave attention.

A certain weakness of constitution, which prevented him in his youth from great physical exertion, only seemed to stimulate his mental activity; and when, in search of change of climate with a view to invigorated health, he undertook a voyage round the world, the training which he had received and the bent of his mind enabled him to record his observations in a journal which is a storehouse of scientific facts and notices, and which, had not his modesty shrunk from having it printed, would have proved the record of a "scientific expedition" when such journeys were far less numerous and attended by far greater inconveniences than at present. During this voyage his love of nature and the wide range of his scientific tastes acquired an increased stimulus; and when he returned home his experience in observation and his knowledge of natural phenomena in different parts of the world enabled him to enter with renewed pleasure into the less active study of the physical sciences.

He established an astronomical and meteorological observatory, and in connection with the latter published a few years ago a complete work on the "Meteorology of Halifax," which may be regarded as a model for all such local observations.

Practical botany also engaged his attention, and his gardens

were distinguished throughout the neighbourhood for the rich variety of their contents, of which he was justly proud.

His favourite studies were astronomy, geology, electricity, and light, and in connection with the latter he was identified with the early progress of Photography, and with the discovery by the Rev. J. B. Reade, F.R.S., of the method of taking portraits, first upon leather and afterwards upon paper, instead of upon silver plates or glass, and also with the chemical means of giving permanence to such images.

He was also specially interested in the progress of microscopy, and was himself both a skilful observer and an adept at those manipulations which are necessary in the preparation of objects for examination. He was also extremely fond of music, and was a skilful performer on the violoncello. Indeed, he seemed able to turn his hands to any pursuit, and such was the aptitude which he possessed for grasping the general principles upon which any practical operation depended, that he speedily was enabled to do with proficiency work which required under ordinary circumstances years of patient labour and practice. Few men could handle their tools better than he could; for, in addition to his scientific acquirements, he was a good mechanician, and many of his turnings in ivory were almost unrivalled for their beauty and skilful execution.

Those who knew him best can best appreciate the many-sided features of his genius, and will long remember the evenings spent at Well Head, where, although reticent in public, he would converse with ease upon the various departments of mathematical or physical investigation with an originality of illustration which showed that he was practically, as well as theoretically, acquainted both with the facts and principles of science.

As might be expected, he was also identified with those movements which had for their object the spread of scientific knowledge; and, in connection with the local Literary and Philosophical Society (of which he was one of the founders and for many years the president), he lectured on more than one occasion on various scientific subjects. He also enriched the museum with many choice objects of natural history, collected during his travels.

He was also connected with the Mechanics' Institute during its early years, and was active as a magistrate, being for many years Chairman of the County Bench at Halifax, and a deputy lieutenant for the West Riding.

In later years a stroke of paralysis, which compelled his retirement into private life, only made him appreciate his gardens the more, until a severer form of his malady prevented all mental pursuits, and finally terminated his life on February 12, 1879, in the 72nd year of his age.

He was a Fellow of the Royal Society, of the Royal Microscopical Society, of the Geological Society, and several others.

Although a certain timidity of disposition prevented him from making original discoveries, few men were better acquainted with the whole range of scientific inquiry; and his kind and generous disposition, as well as the means at his command, enabled him to liberally assist many who were pursuing the difficult path of original investigation.

He was elected a Fellow of the Society on May 12, 1837; and at the following meeting on June 9 contributed an account of his observations of the eclipse of May 15, 1836 (*Monthly Notices*, vol. iv., p. 89). This was the only communication he made to the Society.

Samuel Charles Whitbread, F.R.S., of Southill Park and Cardington, in the county of Bedford, Justice of the Peace, was the second son of Samuel Whitbread, M.P., the well-known statesman and adherent of Fox. His mother, Lady Elizabeth, was the sister of Charles, Earl Grey, who was Prime Minister. He was born on February 16, 1796. From 1820 to 1830 he was M.P. for Middlesex, and in 1831 he served as High Sheriff of Bedfordshire. He succeeded to the large landed estates of his family in 1867, on the death of his brother, William Henry Whitbread, M.P., of Southill. On June 28, 1824, he married the Hon. Juliana, second daughter of Henry Otway, twenty-first Lord Dacre, by whom he had one daughter, Juliana, late Countess of Leicester, and one son, Samuel Whitbread, now M.P. for Bedford. His wife died in 1858. On February 18, 1868, he married Lady Mary Stephenson, daughter of William Charles, fourth Earl of Albemarle, who survives him. Mr. Whitbread took considerable interest in astronomy, and built an observatory at his seat at Cardington: a few observations made there were published in the *Monthly Notices*. He was elected a Fellow of the Society on January 12, 1849, and succeeded the late Mr. George Bishop as Treasurer in 1857. This office he held till 1878, when he was reluctantly compelled by the state of his health to relinquish it.

Mr. Whitbread also was one of the three founders of the Meteorological Society in 1850, the late Dr. John Lee and Mr. James Glaisher being the other two. He was always interested in the prosperity of the Society, and a long series of careful meteorological observations were made at Cardington under his direction. He was elected a Fellow of the Royal Society on June 1, 1854.

The brewery, in connection with which the name of Whitbread is so well known, was founded by Mr. Whitbread's grandfather, who is remembered as a benefactor to the town of Bedford.

Mr. Whitbread died at his London residence, 49 St. George's Square, on May 27, 1879. He had been for some time seriously ill, but his death was somewhat sudden.

Professor JOHANN VON LAMONT was born at Braemar, Aberdeenshire, on December 13, 1805. He was the second of three sons, of whom the eldest and youngest are still resident at Goderich, in British North America. After having received his early education at a private school in his native county, young Lamont, at the age of twelve, left his parents' home to become a pupil in the gymnasium attached to the Scotch Benedictine Monastery at Ratisbon, Bavaria, which was established by the monks especially for the education and support of the sons of Scotch Catholics. In this school he was very successful as a student, and generally maintained a foremost place among his fellow-pupils; a position he owed, in some measure, to the interest taken in him by the Prior, Father Deasson, an excellent mathematician and practical mechanician, who was one of the first to notice the talents possessed by young Lamont. After a most regular attendance at the gymnasium for several years, he, in 1824–1826, also passed through the more advanced philosophical and theological courses at the Lyceum with much distinction.

During his career as a student, Lamont had always exhibited a strong inclination for astronomical studies, and it was a great delight to him, in the summer of 1827, to have an opportunity of visiting the Royal Observatory at Bogenhausen, near Munich, then under the direction of Professor Soldner, the Conservator. Soldner was much impressed with the sound knowledge on most astronomical subjects possessed by his young visitor, and this ultimately led to a closer intimacy between them, and to the appointment of Lamont, in March 1828, to the office of Assistant Astronomer at the Munich Observatory. During these first years of Lamont's career as an astronomer, he gave many indications of his future success, and in various scientific journals of the time several astronomical papers may be found on subjects which occupied his attention while an assistant at the Observatory. One of the first, published in 1829, was his note on the observed diminution of the periodic times of Encke's comet. In 1833 he gave an account of the results of his observations of the solar eclipse of July 16 in that year, which was of some interest. On the death of Professor Soldner, in 1833, the office of Conservator of the Munich Observatory remained vacant for some time; and it was not till July 13, 1835, that it was filled up by the appointment of Dr. Lamont, who had virtually the direction of the Observatory after the death of Soldner. In 1852 Dr. Lamont was appointed Professor of Astronomy in the University of Munich.

Though Professor Lamont has been more generally known in the scientific world by his numerous contributions to the theory and observation of terrestrial magnetism, in which he was always considered one of the principal authorities, he never gave up his interest in astronomical observations, which continued to be the standard work of his Observatory during the larger portion of his directorate; the results have been published in

his series of volumes of the *Annalen der Königl. Sternwarte bei München.* In addition to his zone-work, Lamont for many years took a special interest in various miscellaneous subjects of astronomical research, among which may be noticed his long series of observations of Halley's comet in 1835; and, in the following year, of the second and third satellites of *Saturn*—*Enceladus* and *Tethys*—from which he calculated the elements of their orbits. In 1836 he made some measures of *Pallas*, with the object of determining its diameter; and in the same year he undertook an examination of a few of the principal nebulæ and star-clusters, from which he constructed charts of the popular clusters in *Scutum* and *Perseus.* He afterwards published papers on the results of his observations of Moon-culminating stars; on the atmosphere of the Moon; on the rings of *Saturn*; and his important memoir on the mass of *Uranus*, inserted in the *Memoirs of the Royal Astronomical Society*, vol. xi. Professor Lamont's determination of the mass of *Uranus* is based on observations made with the Munich Refractor of 15 feet focal length and $10\frac{1}{2}$ inches aperture, on a few very favourable nights in the autumn of 1837, when he was able to obtain some satisfactory observations of the two outer satellites, *Titania* and *Oberon.* His result, $\frac{1}{24905}$, was not without interest, for it gave a value for the mass of *Uranus* nearly one-fourth part less than that obtained by Bouvard, by the perturbations of *Jupiter* and *Saturn*, which up to that time was generally adopted. Though the value of the mass deduced by Professor Lamont was so much smaller than that previously determined by Bouvard, it was looked upon with some favour as evidence that the mass hitherto in use ought to be diminished, but probably not by so large an amount as that indicated by the difference between the two values. This belief is confirmed by modern investigations. Professor Newcomb's most recent determination, by observations of the two outer satellites in 1874–1875, is $\frac{1}{22738}$.

Professor Lamont was fortunate on several occasions in observing the phenomena visible during eclipses of the Sun, especially in the total eclipses of July 8, 1842, and July 18, 1860, of which he has given interesting detailed accounts. In the *Fortschritte der Physik*, vol. xvi., he has published a valuable *résumé* of all the observations of the eclipse of 1860, made along the shadow-path from the northern to the eastern coasts of Spain, in which the principal phenomena recorded by the different observers are clearly notified, and the general results discussed. Lamont entertained a notion that the red prominences were produced by clouds or other vapours floating in the Earth's atmosphere, and his remarks on the subject give an illustration of the uncertain opinions of astronomers before 1860 as to the physical origin and constitution of these solar appendages. Writing to the Astronomer Royal in the latter end of 1859, Lamont remarked: "I suppose them to be produced by thin clouds, or masses floating in our atmosphere and condensed by

the depression of temperature in the shadow of the Moon." The eclipse of 1860 was favourably observed by him at Castellon de la Plana, and in his *résumé* of the observations Lamont refers to this opinion as one no longer tenable: "With regard to the hypothesis advanced by myself that the colours of the protuberances are produced by inflexion of light at the Moon's limb, and their forms by small masses of vapour floating in our atmosphere, this has been set aside by the circumstance that the same protuberances were seen at different places. Nevertheless, I cannot yet entirely give up the opinion that the vapours of an atmosphere—that is, the condensations caused by reduction of temperature next to the inmost shadow—do exercise a very considerable influence upon the phenomena of total eclipses, and especially upon the forms of the protuberances."

In 1840 a very important international system of magnetical observations was organised, and Professor Lamont, who had previously established a magnetical Observatory, entered into the scheme with great earnestness and zeal, and his experience and counsel were found to be of considerable service in arranging the plan of observation. The question was taken up with energy by the Council of the Royal Society, who appointed a strong committee of English physicists, by whom a series of elaborate instructions were drawn up for the guidance of the observers, and to insure order and regularity in the observations. The Government had previously sanctioned the equipment of an Antarctic expedition for scientific objects, under Captain James Clark Ross, and, in connection with this expedition, the establishment of Observatories in different parts of the globe for the special observation of magnetical and meteorological phenomena. The observations were simultaneously made at all the stations, the time chosen being the commencement of each two-hourly interval during the twenty-four hours, Göttingen mean solar time. No one took a greater interest in the success of this great scientific expedition than Professor Lamont, whose co-operation in the undertaking proved to be of great value. His own observations during the first three years of the experiment are contained in the *Abhandlungen* of the Munich Academy of Sciences, vol. iii., 1837–43.

In the *Catalogue of Scientific Papers* the titles of 107 separate investigations, distributed in various scientific publications, are attached to the name of Professor Lamont. These, however, take no account of the successive volumes of the *Annalen* and their contents. The greater number of his miscellaneous papers are devoted to magnetical or meteorological researches, but the record of astronomical work in the *Annalen* is a sufficient proof that practical astronomy was not neglected at the Munich Observatory. Lamont's first magnetical paper whose title is recorded in the Scientific Catalogue is *Bestimmung der Horizontal Intensität des Erdmagnetismus nach absolutem Maase*. This contribution to an almost new science was followed by a long se

of miscellaneous researches on magnetical and meteorological observations and instruments, all of which bear the marks of his great experience. He has also published an exhaustive treatise on the science, which has gone through several editions, the last of which appeared in 1867 under the title of *Handbuch der Magnetismus.*

Though the name of Professor Lamont may be best known to posterity in association with magnetic science, it must also be permanently connected with the history of astronomy of our time by the excellent series of zone-observations of telescopic stars carried out under his direction at the Munich Observatory. This valuable collection of reduced mean places of small stars is contained in six volumes, published as supplements to the *Annalen der K. Sternwarte*, in which the original observations are given in successive volumes. The total number of stars observed are 34,674, contained in ten catalogues, each extending over a zone of six degrees of declination, and a supplementary catalogue containing stars accidentally omitted in their proper zone. The portion of the heavens included in this survey of Lamont extends from 27° north declination to 33° south declination; but his principal attention was given to the stars in the five zones between +15° and −15°, nearly five-sixths of the whole being included between these parallels. The resulting mean right ascensions and declinations are reduced to 1850. We have found, from a comparison of the mean places of several corresponding stars in the Munich and Greenwich Catalogues, that they agree generally within reasonable limits, and thus for most purposes we believe that the Munich places may be adopted with confidence. Professor Lamont has himself compared his catalogues with those of Lalande, Bessel, Rümker, and Schjellerup, and the differences between the corresponding right ascensions and declinations are inserted in the zone catalogues. Excepting in cases where there is an evident error in the observations, these differences appear to be as small as might be expected, considering that a large number of the places of the stars depend on one observation. The stars observed are all invisible to the naked eye, being of the eighth and ninth magnitudes, with a few of the tenth. The insertion of the magnitudes of so many telescopic stars may occasionally be found useful in investigations on the periodicity of variable stars, and the whole work is a valuable supplement to the more extensive zone-observations of Argelander and Bessel. The six volumes were published in different years, the first in 1866 and the last in 1874.

In the *Monthly Notices*, vol. x., p. 42, Mr. Hind points out that the planet *Neptune* was unconsciously observed at Munich as a star on two occasions—on October 25, 1845, in zone 338, and on September 7, 1846, in zone 379—before its discovery by Dr. Galle. The apparent R.A. and N.P.D. of *Neptune* for these days were deduced by Mr. Hind from a comparison of the observed positions of the planet with those of several of the brighter stars

in each zone whose places had been determined at Greenwich or the Cape.

Professor Lamont was a member of most of the principal Scientific Academies and Societies of Europe. In 1852 the Royal Society elected him a Foreign Member, and he was connected with our Society, as one of its Associates, from May 12, 1837. His scientific services were frequnently honoured by Foreign Governments, from which he received various orders of knighthood. He died at Munich on August 6, 1879, at the age of seventy-four.

E. D.

Proceedings of Observatories.

The following Reports of the proceedings of Observatories during the past year have been received by the Council from the Directors of the several Observatories.

Royal Observatory, Greenwich.

The Transit-Circle and Altazimuth have been used regularly, as in past years, for observing the Sun, Moon, planets, and selected stars. Much attention has also been given to occultations of stars by the Moon and phenomena of *Jupiter's* satellites. To facilitate these observations the "Naylor" 6-inch Equatoreal by Cooke, returned from the late Transit of *Venus* Expedition, has been recently mounted in the grounds, for use when the Great Equatoreal is not available. A new determination of the flexure of the Transit-Circle telescope was made in April 1879, which gave the value $+0''.16$, whilst that hitherto adopted was $-0''.13$. Under these circumstances no flexure correction has been applied to the observations since the beginning of 1879. In connection with the question of refraction, to which allusion was made in the last report, thermometers have been mounted in various parts of the Transit-Circle room, and in the front court, at some distance from the buildings, in order to obtain data as to the temperature of the air in the neighbourhood of the instrument. Mention has been made in a former report of the observation of stars by reflexion in R.A. A discussion of the results for 50 stars, thus observed in 1878, shows that there is no sensible error in the adopted level corrections.

A complete discussion of the observations of the Moon's diameter in azimuth and zenith distance, with the altazimuth from 1864 to 1878, has been made by Mr. Lynn. This shows the existence of large personalities, and it is therefore proposed in future to use for each observer the value of the diameter deduced from his own observations, the probable error of the result being, as a rule, but a small fraction of the differences between different observers.

With the Great Equatoreal a few micrometer measures of *Saturn's* satellites, of the outer satellite of *Mars*, and of the red spot on *Jupiter* have been made, as also several sketches of *Mars* and *Jupiter*.

Some extensive alterations were made in the Great Equatoreal in the spring of last year, to make it more conve-

nient for use with the long half-prism spectroscope. With this object the telescope tube was shifted longitudinally 30 inches, bringing the eye-end nearer to the centre and allowing more room between it and the floor, and some other changes were made. The spectroscopic observations suffered some interruption in consequence, and were not actively resumed till the summer. The chromosphere was examined on 27 days (between 1879, June 30, and 1880, January 31), on four of which no prominences were seen. Three of these days, however, were very unfavourable for observation. The spectroscopic determination of star motions in the line of sight was resumed at the end of last May, but again intermitted during the opposition of *Mars*. Measures of the displacement of the F or *b* lines have been made in the case of 28 stars, 8 of which had not previously been examined. Up to the present time 63 stars have been thus observed at Greenwich; and as it is found practicable with the half-prism spectroscope to measure the displacement in the case of stars of the fourth magnitude, it is expected that in course of time the motions of some 300 stars may be determined by Dr. Huggins' method. For the present a working list of 150 stars has been formed.

The Sun-spot of 1879, August 27–September 8, Brorsen's comet, and the two components of μ *Draconis* have been spectroscopically examined.

During the year 1879 photographs of the Sun were taken on 128 days; on 83 there were no spots, and on 63 no faculæ. The last four months of the year show a remarkable increase in the number of spots and faculæ as compared with the beginning of 1879, and it is evident that the Sun-spot minimum is now passed. This is shown clearly in the Tables of "Means of daily areas of Spots and Faculæ," 1873 to 1879, communicated to the Society. The photographs have been measured in duplicate up to the end of 1879, and the measures completely reduced. The copy for press, 1879, has been prepared, and is now in the printer's hands.

All the reductions are in a forward state. The altazimuth, spectroscopic, and photographic observations are completely reduced to the end of 1879, and the corresponding MSS. have been sent to the printer. The volume for 1877 has been distributed, as well as separate copies of the Introduction and Results.

The Observatory has to regret the loss of the services of Mr. Lynn, who has at length been compelled by ill-health to resign his post. His successor has not yet been appointed.

The meteors of August (the Perseïds) were well observed; the circumstances at the period of those of November (the Leonids) were exceptionally good, but few meteors were seen. The anticipated Biela's comet shower was carefully watched for on November 27 and 28, but no meteors were seen; the sky was

generally clear on the nights mentioned, but otherwise the circumstances were very unfavourable, owing to the presence of a bright full moon.

Cambridge Observatory.

The astronomical work of this Observatory is still almost exclusively devoted to the zone observations. Notwithstanding the exceptionally unfavourable weather during the past year, 3,220 observations of zone stars, 605 of standard stars, and 132 of *Polaris* have been made with the Meridian Circle.

The total number of stars in the zone is 10,299; of these 6,677 have now been observed three or more times, 1,114 stars have been observed twice, 1,316 once, and 1,192 stars are unobserved. In order to complete the zone observations so that each star shall have been observed at least three times, 7,322 observations are still required.

The Nadir Point is observed almost invariably at the close of each night's work, and also after the observation of *Polaris* when taken by day; there are thus 214 observations for Nadir Point, 191 for collimation, and 147 for level.

A careful watch for *Vulcan* was kept up on March 19, as long as the Sun was above the horizon, but without any result.

Very satisfactory observations of the occultation of *Antares* on July 28 were made by two observers independently of each other. The observed times were almost identical.

A redetermination of the value of one revolution of the N.P.D. Micrometer of the Transit-Circle gave very nearly the same result as that which has been used hitherto.

10 differences with the W. circle give	1^r =	37·066
10 „ „ E. „ „	1 =	37·058
Mean		37·062
The value used hitherto is		37·058

The pivots were re-examined from June 9 to 14 at circular intervals of 30°. The instrument was turned 10 times round and the horizontal coordinates of the dots observed, and again 10 times round and the vertical coordinates observed. On the 9th and 10th of June the observations were made by daylight, an observer being at each end of the axis; but the light was not good, and the observations, especially of the east pivot, were unsatisfactory. After several trials, a mode of illumination by means of the axis lamp and lens was devised which showed the dots and circle beautifully. After this, on June 13 and 14, 480 measures of coordinates were taken by a single observer so as to make the results as far as possible homogeneous and reliable, and

24 measures of the diameter of each small circle on the end of the pivot were also made, to obtain the value of 1 revolution of each micrometer. The measured coordinates were reduced to a mean centre for each pivot, and the deviation of the instrument eastward from the meridian, consequent on the inequalities of the pivot, were calculated for the 12 symmetrical positions of the instrument at which they were observed.

Putting e for the deviation, in units of 15″ of arc, and z for the zenith distance, these 12 deviations are accurately represented by the circular function,

$$\begin{aligned} e = {} & \cdot 0063 \sin (213^\circ\, 57' + z) + \cdot 0050 \sin (32^\circ\, 44' + 2z) + \cdot 0050 \sin (49^\circ\, 54' + 3z) \\ & + \cdot 0055 \sin (58\ 46 + 4z) \\ & + \cdot 0013 \sin (131\ 59 + 5z) \\ & + \cdot 0024 \sin (90 \quad + 6z) \end{aligned}$$

or, in terms of the North Polar Distance (Δ),

$$\begin{aligned} = {} & \cdot 0063 \sin (176^\circ\, 10' + \Delta) + \cdot 0050 \sin (317^\circ\, 10' + 2\Delta) + \cdot 0050 \sin (296^\circ\, 33' + 3\Delta) \\ & + \cdot 0055 \sin (267\ 37 + 4\Delta) \\ & + \cdot 0013 \sin (303\ \ 3 + 5\Delta) \\ & + \cdot 0024 \sin (223\ 17 + 6\Delta) \end{aligned}$$

On June 28 a series of bisections were made with the centre wire on the north and south collimators, first moving the R.A. wire toward the micrometer-head, and afterwards moving from the micrometer-head to make the bisection. The mean of 20 observations each way gives, for wire moved

	r
Toward head	0·1157
From ,,	0·1164

The S. collimator was very unsteady; yet the results seem to prove conclusively that it is indifferent which way the Micrometer is turned in making the bisection.

Between August 6 and 11 a series of observations were made in order to find the corrections to be applied to certain arcs as determined by the west or fixed circle in consequence of errors of division. The readings were compared with those of the east or movable circle in six different positions of the latter with respect to the telescope tube, the circle being turned through 15 degrees in passing from one position to the next. The final results are:

Correction of Arcs determined by W. Circle on account of Errors of Division.

	″
From N. Pole to Zenith	−0·16
,, Zenith to 55° N.P.D.	+0·29
,, ,, to 0° ,,	+0·36

The coefficient of flexure for the tube of the Transit-Circle was carefully determined by direct observations of the collimators, made on sixteen separate days, from August 23 to September 9. The result is that, if we assume the correction for flexure to be as the sine of the zenith distance, the observed zenith distances, southward, are too great by

$$(0''.936 \pm 0''.024) \sin z.$$

It was assumed that the bend of the telescope is the same whichever side is uppermost.

To test the efficiency of the protection afforded by the covering of the telescope from the heat of the observer's body in taking the Nadir Point, observations were made on September 12, 13, 15. The results indicate that the Nadir Point taken with the observer on the S. side of the tube exceeds that taken with the observer on the N. side by

$$0''.3067 \pm 0''.0190.$$

With regard to the difference resulting from the direction in which the telescope is turned on the Nadir, the observations on September 13 and 15 render it probable that the Nadir Point, when the instrument is turned from the S. horizon, exceeds that obtained when it is turned from the N. horizon by

$$0''.076 \pm 0''.033.$$

To ascertain whether the instrument is injuriously affected by the ropes used for setting in the zone observations, it was put in harness on October 1, and three determinations of the Nadir Point were made with the ropes quite as tight as they ever are when used for setting the instrument in ordinary zone work, and three with the ropes quite slack. The mean of the results for Nadir Point gave, for

	°	′	″
Ropes tight	337	47	25.40
,, slack			25.42

which seem to indicate that discrepancies in zenith distance are not to be attributed to this cause.

A discussion of the N.P.D. of *Polaris*, obtained from the observations by the Transit-Circle in 1878 seems to show that the Berlin declination, as given in the Catalogue of 539 Stars, is too great by 0″.73, or, as corrected by the Zone Committee, it is too great by 0″.36, and that the *Nautical Almanac* declination is too great by 0″.32.

It seems to show, moreover, that the assumed co-latitude of the Cambridge Observatory (37° 47′ 8″.4) ought to be increased by 0″.46, making it

$$37^\circ\ 47'\ 8''.86.$$

In confirmation of this, it may be worth mentioning that a

careful reduction and examination of the standard clock stars, assuming the *corrected* Berlin places to be true, gives for the co-latitude

$$37^\circ\ 47'\ 9''.02.$$

Vol. xxi. of the *Cambridge Observations* is now completed, and a large number of copies have been distributed.

Radcliffe Observatory, Oxford.

Mr. Stone arrived in England to take charge of the Observatory at the end of June 1879.

The staff then consisted of only one assistant, Mr. Bellamy, whose time was chiefly occupied in making and reducing the regular meteorological observations and observing stars for clock-error. The staff has since been completed by the appointment of Mr. A. Bowden, as First Assistant, and Mr. Robinson as an assistant.

The position-circle of the Heliometer has been redivided by Mr. Simms, and the instrument cleaned and put in good working order. The cloudy weather which has prevailed at Oxford throughout the year has prevented much work being done with the instrument, but some measures of the diameters of *Mars* have been made, and the distances and positions of some double stars have been measured.

The Transit-Circle has been cleaned, readjusted, and the tube strengthened, and a large number of reflexion observations made to determine the amount of a systematic discordance which existed between the nadir point determinations. A large discordance of about $1''.5$, which has existed between the nadir point determined by the reflecting eye-piece and from reflexion observations of stars, since the erection of the instrument at Oxford, was a source of continual anxiety to Mr. Main.

These discordances appear now to have been traced to their sources and reduced to small limits, if not entirely removed.

It is intended to employ the instrument, amongst other work, upon regular observations of the Moon. These observations have been rather neglected of late years, except at Greenwich, and, now that questions of apparent changes in the mean motion are raised, it is certainly desirable that the discussions should not depend upon the work of a single instrument.

The Photographic Meteorological Instruments, which had been working continuously since 1854, required extensive repairs, and a modification of the exposure of the thermometers was certainly necessary. It has therefore been thought better to replace the old instruments by a new set, on the Kew pattern, which have been kindly lent for the purpose by the Meteorological Commission.

The new instruments have been brought into regular work. It is hoped that these instruments may be kept in use for two years; but if a comparison of the diurnal and annual changes thus obtained should generally confirm the old results, it is doubtful whether photographic registration should be permanently continued with the present staff of the Observatory.

The Catalogue of stars for 1866, from observations made 1862–1870, which was left in an advanced state at the time of Mr. Main's death, is approaching completion. The precessions and secular variations for all the stars contained in the Catalogue which had not been recently published elsewhere have been computed since last June, and there only remains the general revision of the work for the detection of errors and the preparation of an Introduction.

The Eye Meteorological Results for 1876, 1877, 1878, and 1879 are nearly ready for press, and will be included in a single volume.

The Cape Catalogue, 1880.

With the exception of the Introduction, the work has been completed and revised for press.

The sheets containing the positions of 4,320 stars have already been passed through the press. The total number of stars contained in the Catalogue is 12,438.

The examination of the work and the reading of the proof sheets has occupied, and must continue for some time longer to occupy, a very considerable portion of Mr. Stone's time.

The Oxford University Observatory.

The astronomical work undertaken since the last Annual Report, at this Observatory, is as follows:—

I. Observations of the following comets: Tempel, 1867, II., Swift, Brorsen, Hartwig, and Palisa.

II. A third set of the relative positions of 40 stars in the *Pleiades* with the duplex micrometer. These are now reduced to the Mean Equinox, 1879, and it is hoped that the results will be communicated to the Society, together with a description of the new micrometer, which has been successfully used in these observations. The rigorous reductions have been found troublesome, on account of the unusual largeness of the arcs measured, amounting in some instances to 20′.

III. Three observations of the outer satellite of *Mars* have been obtained with the large Refractor. It seems likely that such observations may be profitably discontinued with telescopes not of extraordinary aperture.

IV. Observations of the length of the shadows of lunar

mountains, for the purpose of comparison with those given in lunar photographs taken at corresponding times.

V. The relative positions of about 250 stars in the group Messier 39, in *Cygnus*, are in course of observation with the De La Rue Reflector and a modified adaptation of the Bar Micrometer. It was soon found that these observations required an undesirable stress of mind in observing the times of the transits of these very contiguous stars, to obviate which a form of chronograph has been devised specially adapted to this particular purpose. It is very inexpensive and sufficiently effective. This little instrument will be exhibited at the next meeting of the Society.

VI. The series of lunar photographs has been maintained, but the number has been reduced.

VII. Measures of the diameters of these lunar photographs have been taken and reduced, with the view of testing what amount of reliance can be placed on photographic measures generally. The results appear to justify the necessary reliance. Some of these results have been submitted to the Society. The calculations for the determination of the selenocentric coordinates are proceeding, preliminary to the formation of the equations of condition, the solution of which will tend to decide the fact of a lunar physical libration or otherwise.

VIII. For the more immediate purposes of the educational work which forms part of the business of the Observatory, a set of models of Ptolemy's instruments has been constructed. A convenient form of precessional celestial globe has also been devised for the mechanical solution of remotely ancient stellar phenomena of configuration at any place on the Earth.

The Observatory, Dunsink, Co. Dublin.

During the past year a set of observations for the annual parallax of 61 (B) *Cygni*, by the method of differences of declination, has been completed. Observations were made on 38 nights since September 18, 1878.

A series of observations for the annual parallax of Groombridge 1618 has also been completed. This series commenced on March 4, 1878, and the distance and position angle of the neighbouring star $+50°$, 1724 has been measured on 59 nights. A second series of measures of this object was commenced last autumn and is now in progress.

Observations of the star P. III. 242 recommended as a suitable parallax object by O. Struve (*Monthly Notices*, vol. xx., p. 8) were commenced in January, and were almost completed at the close of the year.

Parallax observations of the red star Schjellerup 249 (a) ($= +34°$, 4500) were commenced in October and are at present in progress.

A large number of observations in search of stars with large parallax have been made. The method adopted is described in Part III. of the *Dunsink Observations*, which has been recently issued.

It is expected that Part IV., containing the meridian observations of red stars, will be ready for press in the autumn.

Royal Observatory, Edinburgh.

The ordinary daily business of this Observatory has been going on as usual during the past year. Observations for time, and its daily distribution by ball, gun, and controlled clocks, have been attended to as regularly as ever; and the meteorological observations at 55 of the stations of the Scottish Meteorological Society have been computed and discussed for the Registrar-General in Scotland, and printed by that officer in his monthly and quarterly returns. The credit for all this work is chiefly due to Mr. Alexander Wallace, M.A., First Assistant, and Mr. Thomas Heath, B.A., Second Assistant.

The deep-rock thermometers, so calamitously destroyed in 1876, have, after long and careful remaking and preparation, been successfully replaced during the last year by Mr. Wedderburn, of Messrs. Adie & Son; they are again therefore under regular observation, and are to be reported on to Government next June, according to the contract terms.

Glasgow Observatory.

The operations at the Glasgow Observatory during the past year have been mainly devoted to the completion of a Star Catalogue which is intended to embrace all the star observations made with the Transit-Circle at the Observatory during the period 1860–80. The number of stars is 6,350, ranging chiefly between the sixth and ninth magnitudes. The epoch of reduction is 1870. The place of each star is in general determined by at least three observations. Steps have been taken to have the Catalogue printed.

The transmission of Greenwich time to the city and port of Glasgow by the electrical control of clocks, which was originally introduced at the Observatory in 1863, has been in active operation during the past year.

Kew Observatory.

The astronomical work of this Observatory has been confined during the past year to the completion of the measurements of

the ten-year sun-pictures, as noticed in the last report, and to the preliminary reductions of $\frac{r}{R}$ as far as August 1871 of the above measurements, which has been carried on at Mr. De La Rue's expense and under his direction.

It is expected that the above preliminary reductions will be completed in the course of two or three months.

The reduction of the measurements to heliocentric elements has been continued by Mr. Marth for Mr. De La Rue.

The eye observations of the sun, after the method of Hofrath Schwabe, as described in the report for 1872, have been made on 151 days, in order to maintain for the present the continuity of the Kew records of sun-spots. The sun's surface was observed to be free from spots on 113 of those days.

The Campbell Sunshine Recorder continues in action, and the instrument in its improved form, giving a separate record for every day of the duration of sunshine, has been regularly worked throughout the year and its curves tabulated.

Two papers based upon these records have been read by the Superintendent before the Meteorological Society and published in their quarterly journal.

The magnetical and meteorological work, to which the attention of the Observatory is chiefly directed, has been prosecuted continuously, and the Verification Department has been fully occupied the whole year.

Liverpool Observatory, Bidston, Birkenhead.

The object for which this Observatory was originally established has been steadily kept in view.

In 1873 a formula was published by the Mersey Docks and Harbour Board, at the recommendation of the Marine Committee, for calculating the corrections to be applied to the rates of chronometers under varying influences of temperature. Upwards of three thousand chronometers have been tested at this Observatory in such a way as to supply the necessary data for making these calculations. It has, however, been found necessary, in order to meet the requirements of the mariner, to furnish him with the calculated rates for the various temperatures to which his ship is likely to be exposed and to supply him with the means of keeping a systematic record of the performance of his chronometers at sea. At the request of the Directors of the "Pacific Steam Navigation Company," this has been done for their officers during the past two years with the most satisfactory results. The ships belonging to this Company carry three chronometers each, which may be called *a*, *b*, *c*. The rates of these instruments are supplied from this Observatory for every 5° of Fahrenheit from 45° to 95° inclusive. By means of these rates the Greenwich time for each chronometer is obtained daily by merely

adding the rate for the observed temperature to the error of the instrument for the preceding day; *a* is then compared first with *b* and then with *c*. In this way the differences of Greenwich mean time between *a* and *b* and between *a* and *c* are found daily, both by calculation and by comparison with each other. The whole of this process is only the work of a few minutes, and a record of the Greenwich time and of the performance of the instruments relatively to each other is preserved for future reference. The Mersey Docks and Harbour Board have recently ordered to be printed and distributed amongst mariners an example showing the daily performance of three chronometers in one of the Pacific Steam Navigation Company's ships during a voyage of 107 days. This ship crossed the Equator twice, and twice passed through the Straits of Magellan, and the error of longitude by the mean of the three chronometers appears to have scarcely exceeded two or three miles at any time during the voyage. In order to show that this is not an exceptional case, a table follows, which gives the error of longitude as shown by the chronometers for each of twenty voyages of the Pacific Steam Navigation Company's ships, taken in succession; these results having been obtained in the same way as shown in the example given in detail.

The mean error of longitude from these twenty voyages is $5\frac{1}{2}$ geographical miles on the Equator for an average voyage of 102 days; for ten of the twenty voyages the largest error is under $4\frac{1}{2}$ miles; for eighteen of the twenty voyages the largest error is under 9 miles, and for two voyages the errors are $17\frac{1}{2}$ and 18 miles respectively. The last two errors were produced by an unusual change of rate in one of each three chronometers; by rejecting these two instruments, and taking in each case the mean of the other two, the error of longitude is $1\frac{1}{2}$ instead of $17\frac{1}{2}$ miles for one ship, and 8 instead of 18 for the other. By using one uniform rate in variable temperatures a considerable majority of the timekeepers may be influenced in the same direction; but when the rates are corrected for change of temperature, the variations arising from other causes have a tendency to neutralise each other. Of the twenty voyages with three chronometers in each ship, there are ten in which the timekeepers put the ship to the east on the average four miles, and ten which put the ship to the west on the average seven miles. These are the averages of the accumulated errors for voyages of from three to four months' duration. The errors of the chronometers on Greenwich time are generally checked two or three times during each voyage, and the results entered in the error and rate book. The records contained in these books, together with the rates obtained at the Liverpool Observatory in definite temperatures, supply a continuous record of the performance of each timekeeper, and the data thus obtained are used in the calculations for predicting the most probable rates. By means of three fairly good chronometers treated in this way,

the Greenwich time at sea is known from day to day with a degree of accuracy which, without an examination of these records, might appear incredible.

From the performance of the chronometers employed in the twenty voyages before named, it appears that the average error due to change of temperature between 45° and 95° Fahrenheit is $2^{s}\cdot 8$ a day; but when corrected for change of temperature the average error for three chronometers on the sea voyage is only $0^{s}\cdot 22$ a day. It might therefore be reasonably expected that, by correcting the rates for change of temperature, a great improvement would be effected in the method of finding the Greenwich time at sea by means of chronometers.

Stonyhurst Observatory.

The large automatic spectroscope by Browning, which was in course of construction last year, has been completed, and is now in daily use at the Observatory. The instrument consists of six prisms of 60°, the light passing twice through each prism. As any number of the prisms can be employed, and the number changed without loss of time, an object may be very readily examined with different dispersive powers. The power generally used in the daily observation of the chromosphere is that of eight prisms of 60°, but a single prism twice used suffices to show the prominences. The adapter of the spectroscope is furnished with a pair of screws by which the slit of the instrument can be placed either radially or tangentially on any part of the edge of the Sun, whilst the centre of the Sun remains at the middle of the field of view of the telescope; and then, by aid of rack and pinion, the slit can be swept round the solar limb without altering the position of the Equatoreal. The spectroscope is provided with a micrometer for measuring the distance of the lines of the spectrum; but it became at once obvious that the only practical way of observing the varying height of the chromosphere is by the use of a photographic scale of tenths of millimetres, the readings being taken to hundredths. The slit carried by the collimator is 2·5 millimetres in length; but the extraordinary height of some of the solar prominences has rendered it necessary to provide a second slit in which the length is less limited, but in which the definition may not be quite so perfect throughout. A small photographic camera is adapted to the spectroscope. The instrument arrived at the Observatory at the beginning of March, but, as several changes were found necessary in the adapter, it was finally completed only in the month of October.

In November and December the chromosphere was mapped on 18 different days, showing that a very fair amount of this work can be effected even during the winter months.

The observations of *Jupiter's* satellites, of occultations of stars by the Moon, and of meteors, have been carried on as usual,

but the double star measures have been less numerous than in preceding years.

No change has been found necessary in the meteorological and magnetic branches of the establishment. A full discussion, undertaken here, of the meteorological observations made at Kerguelen Island on the occasion of the Transit of *Venus* Expedition in 1874 has just been published by the Meteorological Office.

Temple Observatory, Rugby.

The work of the Observatory during the year 1879 has been the continuation of the measurement of positions and distances of double stars. The opportunities have been few after deducting the time given up to school work, but 342 complete sets of measures have been made by Mr. Seabroke, Mr. Percy Smith, and Mr. Hodges, each set of measures consisting generally of four measures of position and two double measures of distance. Each star, with few exceptions, has been observed on three different nights.

The spectroscope on the 12-inch Reflector has been used for the measurement of the motion of stars in the line of sight, and the results, together with those of former years, have been brought before the Society.

The usual attention has been given to the use of the Observatory for educational purposes.

Mr. Barclay's Observatory, Leyton, Essex.

The large Refractor has been employed during the past year in a continuation of the measures of double stars commenced here some years since, and the satellites of *Saturn* have been observed when possible, Mr. Marth's ephemeris being used for identification.

The number of cloudy nights has been unusually large, and the meridian observations of the Sun fewer than in almost any preceding year.

Mr. Common's Observatory, Ealing.

In August last a 36-inch Equatoreal Reflector was completed and is at work. The definition is quite equal to the 18-inch erected in 1876; the mirrors of both instruments were made by Mr. Calver, the mountings by Mr. Common himself.

On September 21 *Deimos* was found, and the principal work done since has been observing *Deimos*, as well as the inner satellite of *Mars*. *Mimas* and *Hyperion* have also been well seen.

When convenient the power of the mirror has been tested photographically with the following results:—

With *Jupiter* an exposure of 1^s to 2^s gives a strong image with some little detail; the red spot coming out well, as also the darkening towards the north pole. The satellites give a dense image in 20^s, and it was found that, by placing a bar over *Jupiter* and exposing the satellites for 15^s to 20^s and withdrawing the bar about $1\frac{1}{2}^s$ before the finish of the exposure, the planet and satellites could be got on one plate in their proper places.

Saturn and *Mars* were photographed in from 5^s to 8^s, but without any appreciable detail, except, perhaps, in the case of *Saturn*.

With the Moon the exposure cannot be made by hand with sufficient quickness, and some mechanical means will have to be used.

An attempt was made by exposing for 20^m to get a picture of the nebula of *Orion*, but without success. When an electrical control is completed to the driving clock a longer exposure will be tried.

Two plates of the *Pleiades* have been taken, showing all, or nearly all, the stars given in the maps of this cluster, with $1\frac{1}{2}^m$ exposure, the three faint stars near *Alcyone* showing plainly.

The great amount of light tells very much on the fainter nebulæ, and several have been seen that are not in Herschel's or Dreyer's Catalogues, and there is little doubt a systematic search would add greatly to the number already known.

Colonel Cooper's Observatory, Markree.

Double stars within 25° of the pole have been observed at this Observatory, and, besides, a number of difficult and important binary stars have been occasionally examined. At the latter end of the year a number of sketches of planets were secured, and drawings of a few nebulæ have been commenced.

Mr. Edward Crossley's Observatory, Bermerside, Halifax.

There has been very little change in the work of this Observatory during the past year. As heretofore, the measurement of double stars has occupied most of the small number of nights when the sky was clear. More attention than usual has been paid to the delineation of the markings on *Jupiter's* disk, and to the phenomena of *Jupiter's* and *Saturn's* satellites. The list of satellite observations will shortly be submitted to the Society for publication. The Transit Instrument and the Equatoreal are in good working order.

Mr. Huggins's Observatory, Upper Tulse Hill.

Up to the present time photographs have been obtained of the spectra of the stars *Sirius*, *Vega*, *Rigel*, *a Cygni*, *a Virginis*, *η Ursæ Majoris*, *a Aquilæ*, *Arcturus*, *β Pegasi*, *Betelgeux*, *Capella*, *a Herculis*, and *a Pegasi*.

The spectrum apparatus consists of one prism of Iceland spar and lenses of quartz. A slit is employed $\frac{1}{350}$ of an inch in length. The length of spectrum taken with this apparatus is about half an inch from G to O in the ultra-violet. The definition is so good that in photographs of the solar spectrum at least 7 lines can be counted between H and K. The apparatus is adapted to a Cassegrain Reflector with a metallic speculum of 18 inches diameter. The small mirror was removed and the slit of the spectrum apparatus placed at the principal focus of the speculum. A very successful method was adopted by which the image of a star could be brought exactly upon the slit and retained there during the whole time of exposure, sometimes for more than one hour, by a system of continuous supervision and instant control by hand when necessary. The slit is provided with two shutters. By means of these a second known spectrum may be taken upon the plate for comparison.

Various photographic methods were tried, but the great sensitiveness which may be given to gelatine plates, together with the special advantages of dry plates for long exposures, led finally to the exclusive adoption of this method.

The photographs were examined and the lines measured by means of a micrometer attached to a microscope of low power. These measures were reduced to wave-lengths by the help of solar and terrestrial spectra, use being made of M. Cornu's map of the ultra-violet part of the spectrum, and of M. Mascart's determinations of the wave-lengths of cadmium.

The spectra of *Sirius*, *Vega*, *a Cygni*, *a Virginis*, *η Ursæ Majoris* *a Aquilæ*, and *Arcturus* have been laid down in a map on the scale of M. Cornu's map of the ultra-violet part of the solar spectrum.

The stellar spectra extend from about G to O. Six of these spectra belong to stars of the white class. In 1864 Mr. Huggins pointed out the features in common in the visible spectra of these stars.

The photographs present a remarkable typical spectrum, consisting of twelve strong lines. The least refrangible of these is coincident with the hydrogen line (γ) near G; the second with "*h*," also a line of hydrogen; the third with H. K, if present, is thin and inconspicuous.

The lines H and K are coincident with lines in the calcium spectrum, and are usually attributed to the vapour of this substance. There is another pair of strong lines in the calcium spectrum, which on M. Cornu's map have the wave-lengths

3736·5 and 3705·5. There are no strong lines in these stars coincident with these lines.

The twelve strong lines, together with the two other lines of hydrogen, C and F, in the visible spectrum, form a group in which the remarkable arrangement suggests that all these lines belong to one substance. Four at least, and probably a fifth also, belong to hydrogen.

In the photographs of *Vega* and *Sirius* the continuous spectrum extends far into the ultra-violet, beyond S; but in these stars no other lines are present beyond the last line of the typical group which occurs at λ 3699.

For the sake of convenience of reference, these lines have been distinguished by the letters of the Greek alphabet in the order of refrangibility, beginning with the first line beyond K of the solar spectrum.

The wave-lengths of these lines are

1	(Hydrogen near G)	4340	7	δ	3767·5
2	*h*	4101	8	ε	3745
3	H	3968	9	ζ	3730
4	α	3887·5	10	η	3717·5
5	β	3834	11	θ	3707·5
6	γ	3795	12	ι	3699

In all the stars of the white group the line K is absent or thin compared with its condition in the solar spectrum.

In the spectrum of *Arcturus*, which belongs to the solar type, this line exceeds in breadth its condition in the solar spectrum.

The white stars may be arranged in a series, in which the line K passes through different stages of thickness, at the same time that typical lines become narrower and more defined and other finer lines present themselves in increasing numbers.

Arcturus appears to present a spectrum on the other side of that of the Sun in the order of changes from the white star group.

The spectra of the planets *Venus*, *Mars*, and *Jupiter* were taken on the plan suggested by the author in 1864, in which a daylight spectrum is observed and photographed together with that of the planet. These show no sensible planetary modification in this region of the spectrum.

Numerous spectra of small areas of the lunar surface ha been taken under different conditions of illumination. T are negative as to the existence of a lunar atmosphere.

Mr. Knott's Observatory, Cuckfield, Sussex.

The year has not been a very favourable one for Obse work. A few measures of double stars have been tak the 7½-inch equatoreal and filar micrometer. Of these, tw

measures of γ^2 *Andromedæ* give P = 98°·9 for the epoch 1879·81, a result which is in fair agreement with other recent measures and is confirmatory of the view that the angle of position is slowly but decidedly decreasing.

Observations have also been made of a small number of variable stars. Among these may be mentioned Mr. Hind's U *Geminorum*, a maximum of which was observed on October 24 ±. From the appearance which it presents in the telescope, and from the irregularity of its period (which has a range, according to Dr. Schönfeld, of from 70 to 150 days), this star would seem to belong rather to the class of so-called "new stars" than to the ordinary variables.

The elliptical red spot on *Jupiter* was observed on several occasions. On December 6, six measures were taken of the length of the spot, when near the centre of the disk, the mean result being 13″. At 7h. 15m. G.M.T. on the same evening the *following* extremity of the spot was observed to be on the central meridian of the disk of the planet.

Lord Lindsay's Observatory, Dun Echt, Aberdeen.

This report is for the last two years. In 1878 the transit of *Mercury* was observed, in accordance with a plan detailed in a circular which was distributed shortly before the transit. In addition to the usual contact and diameter determinations, the projection of the planet on the chromosphere was spectroscopically observed by Lord Lindsay. Accounts of these observations, as well as others of the region near the supposed new lunar crater *Hyginus* N., and of meteors on November 27, have been published in the *Monthly Notices*. From the beginning of the year daily readings of the usual meteorological instruments have been made to supplement those of a King's barograph and a standard anemograph.

In 1879 the spectrum of Brorsen's comet was measured five times, and Palisa's comet was observed with the spectroscope on two occasions, and for place on thirteen nights. From the first three observations provisional elements and an ephemeris (published in the *Astronomische Nachrichten*) were deduced. During the latter part of its visibility, when it was so faint as probably to be a difficult object in small telescopes, Swift's comet was compared with neighbouring stars on four nights with the 15·06-in. Refractor. In the autumn a plan was devised for the more rapid and complete distribution of urgent astronomical news amongst British observers by means of a circular posted to all applicants. Two notices have been already distributed, viz. of the Rev. T. W. Webb's minute planetary nebula (the exact nature of which was decisively shown at Dun Echt), and of a hitherto unobserved star discovered in *Canis Minor* by Mr.

Baxendell. Co-operation has been promised from many sides, and a similar scheme has been started in France.

After a revision of the large Refractor, and more especially the removal of every trace of grease from the surfaces of the objective, the outer satellite of *Mars* was readily seen, and measures of its relative place taken on four occasions.

The neighbourhood of Schmidt's *Nova Cygni* was repeatedly scrutinised, and the list of stars within a radius of $7\frac{1}{2}'$ augmented to 97. The reductions of the measures, except to epoch, are all finished. Special attention has been given to the magnitudes.

The Transit-Circle has been used for time determinations, and latterly for fixing the places of a small cycle of occultation and comparison stars. The faintness of these led to a great improvement in the illumination of the wires in a dark field. The four prisms in the cube which reflect light down to the eye-end are so far from the centre-line of the axis as to be partly out of the range of the direct rays from the illuminating lamp. By inserting a group of four prisms of small angle in the axis, so that the rays from each of them cross the centre-line on their way to the prisms in the cube, this difficulty is completely overcome. The rays, too, are kept practically parallel, and so fall only on the internal prisms. The illuminating power thus gained is very great, but is to a considerable extent negatived owing to the light falling at an angle of 45° to the sets of wires, although nearly in the same plane. The full intensity of the illumination is manifested by the brightness of dust particles which happen to be normal to the rays. The systematic division errors of the circle generally used have been determined for every five degrees. The application of these corrections will complete the reduction of the observations of *Mars* and comparison stars made in 1877. The periodic errors of the declination, collimation, and microscope screws have been also determined, as well as the values of the first two.

The Foucault Siderostat has been fitted with slow-motion rods, accessible from any part of a room 68 ft. in length, and the opening through which the rays enter arranged to receive any of the telescope or photographic lenses. This room can be darkened by a single shutter. The electrical control of the chronograph and refractor driving clock (see *Monthly Notices*, vol. xxxiv., p. 35) has been much simplified without impairing its accuracy. The essential parts are reduced to a single commutator wheel, one ordinary Siemens' relay, and a single brake. To avoid passing a strong current through the sidereal clock, an extra relay is introduced.

On December 18, 1879, it was noticed that the last rays of the Sun descending behind a hill five miles off gave rise to a phenomenon closely akin to the "shadow bands" seen in some solar eclipses. A detailed account of the phenomenon was published in *Nature*.

Investigations and reductions connected with the transit of

Venus still absorb the greater part of the Observatory's working power. Twelve of the best photographs have been remeasured, each on two days. These measures, as well as former ones made by Mr. Gill, have been reduced to the neighbouring lines on the fiducial plate, and show no marked personality in the fixation of the limbs of the Sun or planet, nor does the probable error of a complete measure appear to be so large as $\pm 0''\cdot 3$. The base bars and auxiliary iron rod with which the distance of the photographic objective from the camera was ascertained have been compared with a copy of the *Mètre du Conservatoire* at temperatures of 55° and 90°. This work involved testing 20 thermometers and finding the lengths of two auxiliary bars for connecting *à trait* and *à bout* measures, as well as an examination of the periodic and other errors of the Fromont dividing engine.

The double-image micrometer has been subjected to a close examination. The periodic errors of both screws were determined for three parts, and observations made for defining the variation of the scale-value with increasing distances have been made by means of artificial stars. The coefficient of expansion of the bar in which these are formed, as well as their intervals and distances from the telescope, have been found. Many separate portions of the whole work have been finally prepared for the press.

The Earl of Rosse's Observatory, Birr Castle.

During the year Parts I. and II. have been published of the Catalogue of Nebulæ and Clusters observed with the 6-ft. and 3-ft. Reflectors from the year 1848 up to about the year 1878. These include 0^h to 14^h R.A. Part III., which completes the work, is in the printer's hands, and will probably be issued in the course of the next few months.

The planet *Mars* was observed several times during the late opposition; one satellite was seen, but no measures of its position were obtained.

A few photographs of the Moon on ready prepared gelatine plates were taken, but with only average results.

Colonel Tomline's Observatory, Orwell Park, near Ipswich.

The work of this Observatory during the past year has been almost entirely confined to the observation of comets, which is held to be the most useful sphere of work to which its appliances can be devoted. An excellent series of observations of Brorsen's Comet has been secured, and the reductions are being now completed; they will be sent to the editor of the *Astronomische Nachrichten* for publication without delay. The observations of Tempel's Comet were meagre and unsatisfactory, owing to its low altitude

and the presence of haze near the horizon at that season of the year. Later on the Comets of Swift, Hartwig, and Palisa have been fairly well observed, but it is to be regretted that the series of the two latter commence much later than might have been the case if intelligence of their discovery had reached the Observatory at an earlier date. These observations will be published as soon as the reductions are effected.

The only other work that has engaged the observer is the determination of the geographical coordinates of the Observatory, which, after some difficulty, has now been completed. It is intended, however, to supplement these results by observation of a certain number of occultations during the current year.

Royal Observatory, Cape of Good Hope.

At the end of 1878 Mr. E. J. Stone accepted the post of Radcliffe Observer at Oxford, and in consequence resigned his position as Her Majesty's Astronomer at the Cape of Good Hope. Before quitting the scenes of his labours for the past nine years he completed the work which led him to the southern hemisphere. This work was the construction of a Catalogue of all stars down to the seventh magnitude between N.P.D. 120° and N.P.D. 180°, and all Lacaille's stars beyond these limits of N.P.D. The whole Catalogue contains 12,400 stars, almost all of which have been observed three times. The Lords Commissioners of the Admiralty have arranged for the work to be printed in England under the superintendence of Mr. Stone, as also the annual volumes for the years 1877–78 and the first half of 1879.

Besides the construction of this Catalogue, Mr. Stone completed the reduction and published the annual volumes of observations from 1856–60, and formed the first Cape Catalogue (epoch 1860) from the material contained in these volumes. He also revised and published the Catalogue for 1840 from observations in 1834–40; the printing of this volume was completed before Mr. Stone's departure for England.

Mr. Stone left for England at the end of May 1879, and was succeeded in his post by Mr. David Gill, who reached the Cape on the day preceding Mr. Stone's departure. Among the old work of the Observatory still remaining for publication must now be mentioned the Catalogue for 1850, containing all B.A.C. stars having south declination. This Catalogue was found by Mr. Gill in a partially reduced state, and it has been pushed forward to completion as vigorously as possible. Nearly the whole of the computations are now finished, but a very heavy labour in examination still remains, because much of the early part of the work, such as reduction to the middle wire and the application of the instrumental corrections, previously assumed to be correct, appears to require careful re-examination. This work

has been much delayed by the illness of Mr. Pett, third Assistant, who was before exclusively employed in the work. His illness has necessitated a voyage to England. The Catalogue, however, will be completed as soon as possible consistently with the reduction of current work.

Besides this Catalogue, there is a valuable series of observations of occultations of stars by the Moon, made by Sir Thomas Maclear, extending from 1834 to 1870. In general, the errors of the observing chronometers have not even been computed, though there are records of careful comparisons with the standard clock before and after each observation. Some progress has been made in the reduction of these observations, but it is not intended to publish them till the determination of a telegraphic longitude of the Cape has given them their full value.

The completion of the submarine cable connecting Aden with D'Urban, and a system of land lines from D'Urban to Port Elizabeth, and from Port Elizabeth to the Royal Observatory, Cape of Good Hope, offer the means of determining the absolute longitude at an early date, because Aden has been already connected with Greenwich by two closely accordant and independent operations, viz:

1. Lord Lindsay's Mauritius Expedition in conjunction with the German Transit of *Venus* Expedition, when Aden-Suez and Suez-Alexandria were connected by Dr. Löw and Mr. Gill, and Alexandria-Malta and Malta-Berlin by Dr. Auwers, Dr. Löw, and Mr. Gill.
2. Sir George Airy's connection of Suez and Alexandria with Greenwich by the British Transit of *Venus* party in Egypt, and the after-determination of Suez-Aden under General Walker by officers of the G.T. Survey of India.

The necessary preliminary inquiries have been instituted as to instruments and telegraphic possibilities; when these have been completed, plans and estimates for the operation will be submitted to the authorities.

The important work completed by Mr. Stone has led Mr. Gill to consider very fully the proper objects to which the use of the Transit-Circle should be devoted in future. To continue the same class of observations as those of Mr. Stone would be useless; indeed, for the particular class of observation which he undertook, Mr. Stone has completed the work of the southern heavens for the next fifty years.

Two separate lines of meridian work seemed peculiarly open for solution :—

1. The construction of a limited Fundamental Catalogue of the Southern Heavens, including observations of the Sun and observations for latitude and refraction.
2. The observations of successive zones of stars down to the 9th magnitude from N.P.D. 120° on the plan of the *Astronomische Gesellschaft.*

After full consideration, Mr. Gill adopted the former pro-

gramme, because it not only appeared the more pressing and necessary in the present state of Astronomy, but also because Dr. Gould is known to be occupied with the latter work at Cordoba.

As part of the fundamental work, the observation of 250 stars (to be used by Dr. Schönfeld as fundamental stars in his Durchmusterung from N.P.D. 90° to 120°) has been undertaken.

For this work it was found that considerable changes were necessary in some details of the Transit-Circle.

The circle microscopes were defective in some points of construction, and the screws seemed to be worn, as different zenith points were found by the use of different parts of the screws. No means existed for determining the errors of the screws. The microscopes were accordingly sent to Mr. Simms to have new screws made and to be altered in some other respects, and an apparatus was contrived by Mr. Gill for determining the screw errors. The latter apparatus is now in course of construction by Mr. Simms. The improved microscopes, with their new screws, and adapters fitted for the screw-error apparatus, have been recently received, and are now mounted and adjusted. The arrangement of the webs has been altered from the now obsolete form of cross wires to that of parallel wires just embracing the division, and this, with a thorough revision of the focussing and illuminating arrangements, has very greatly increased the precision of the circle readings.

A re-examination of the division errors of the circle has been undertaken and completed as far as each 20th degree. Three different observers have taken part in the work, and the error of each division has been arrived at by three independent processes by each observer. The results by all methods and by all observers are in admirable agreement, but in some divisions there is considerable discordance from the previous determinations of division error, in which cross wires were employed. The determinations of the errors of the 5° spaces is now being carried out.

The new eye-end, which was received in the early part of 1879, but which had not been employed by Mr. Stone, was not found to be at all satisfactory; and particularly its drawtube did not fit, having been made to measure, and Mr. Simms having had no opportunity of trying it *in situ*. The old eye-end was accordingly returned to Mr. Simms to be provided with new micrometer screws and otherwise repaired.

Its arrival, now rather unduly delayed, is anxiously expected. An arrangement has been contrived by Mr. Gill for determining the astronomical flexure of the Transit-Circle at different altitudes, and plans and estimates have been prepared for the erection of an accessible meridian mark in the focus of a lens of 300 feet focal length. It is hoped that these proposals will meet with the approval of the authorities.

The deficiencies of the Transit-Circle eye-end have prevented much being done in the way of fundamental work, but considerable progress has been made with the Right Ascensions of Professor Schönfeld's stars.

A very extended series of observations on personal equation has been carried out. The results are of great interest, and will be shortly published.

α and β *Centauri* have been observed at every culmination when weather permitted. On 71 days observations of both stars have been already secured, and it is intended to continue the observation of both stars regularly in R.A. and N.P.D., in order, not only to test independently whether the results of Henderson and Maclear for the parallax of α *Centauri* will be confirmed, but also whether (in conjunction with the long series of similar observations of these stars to be found in the records of the Observatory, both published and unpublished) it may not be possible to determine the motion of the centre of gravity of the system α_1 and α_2 *Centauri*, and thus, in combination with the existing and future measures of position angle and distance of the stars, determine their relative masses.

With regard to extra-meridian observations. The extent of meridian work undertaken and carried out by Mr. Stone rendered it impossible for him to attempt extra-meridian work; and accordingly, though a new mounting had been provided for the Equatoreal, the instrument had never been brought into working order. Much time and labour were required to get the instrument into good adjustment, and considerable alterations had to be made before the whole was completed. As no skilled workmen can be obtained at the Cape, the work consumed a good deal of Mr. Gill's time. The illumination of the field is still very bad; but arrangements have been planned, and are now being executed by Mr. Simms, which, it is believed, will be satisfactory in this respect. The object-glass, though by Merz, is not of the highest quality, and the aperture (6·9 inches) is much too small for many purposes of modern research. The telescope may be usefully enough employed for observations of occultations of stars by the Moon, of the phenomena of *Jupiter's* satellites, and, when the illumination is arranged, for measures of not too difficult double stars; but for the more refined investigations of parallax and for stellar spectroscopic work the instrument is not adequate.

At the request of Professor Newcomb, the regular observation of all occultations visible at the Cape of stars in the *Nautical Almanac* occultation list has been undertaken. The Lords Commissioners of the Admiralty have provided for the prediction of these occultations, from the 1st January 1880, for the Royal Observatory at the Cape in the same manner as for the Royal Observatory, Greenwich. The regular observation of the eclipses of *Jupiter's* satellites has been commenced, and will form part of the work of the Equatoreal.

A determination of the parallax of α *Centauri* by differential measures of distances, or of declination from neighbouring stars, was looked upon by Mr. Gill as one of the most immediately interesting and important problems presented to an astronomer in

the southern hemisphere. There are, unfortunately, no stars suitably situated near *a Centauri* which are at the same time bright enough for measurement with so small an aperture as 6·9 inches, though for an Equatoreal of large aperture there are stars suitable for both methods. Before leaving England Mr. Gill received an unexpected and generous offer in the shape of a loan of Mr. Newall's 25-inch Equatoreal and its Observatory for use at the Cape during a period of seven years, on the simple conditions that at the end of the period in question the instrument should be returned to Mr. Newall, if required, free of expense. With similar generosity, Mr. James Nasmyth offered to contribute £1,000, and Dr. Siemens £250, towards the cost of transport and erection. The matter only came before Mr. Gill on the eve of his departure from England, and Messrs. Spottiswoode, Huggins, Nasmyth, De La Rue, and Siemens, together with Lord Lindsay, kindly agreed to form a committee to make the necessary inquiries and decide what should be done under the circumstances. It was found that the total cost of dismounting and altering the instrument, together with packing and transport of the observatory and instrument, erection at the Cape, and providing for re-transport and re-erection at the end of seven years would amount to about £3,000. The committee, seeing that a new Equatoreal equal to the requirements of modern times was in any case necessary, thought it a pity that so much money should be expended on a borrowed telescope, and accordingly it was not thought desirable to accept Mr. Newall's generous offer. They also thought that the new Equatoreal should be of not less than 20 inches aperture, and Mr. Nasmyth intimated his desire to subscribe £1,000 towards the cost of such an instrument whenever its construction should be decided upon. Preliminary correspondence has been opened on the subject, but no steps have as yet been officially taken in the matter. It is a matter of regret that some time must elapse before the Observatory can be provided with a suitable Equatoreal.

Mr. Gill has purchased from Lord Lindsay the Heliometer used in the *Juno* and *Mars* observations, and a new equatoreal mounting of great rigidity is now being made for it to Mr. Gill's order by Mr. Howard Grubb, of Dublin. The Lords Commissioners of the Admiralty have granted free transport of the instrument; an observatory is prepared for its reception, and observations will probably be commenced before the middle of the present year. The instrument will be devoted in the first place to the observation of ζ *Indi* and one or two other stars having large proper motion, for determination of their parallax

Melbourne Observatory.

The meridian work with the Transit-Circle at this Observat during the past year has been principally confined to complet the series of observations of all stars observed for the next T year Catalogue.

The reductions of the observations obtained in the zones 150° to 160° S.P.D. have been considerably advanced, and out of the 48,672 observations recorded 36,000 have been reduced in both R.A. and P.D.

The Great Telescope has been kept unremittingly at work on the revision of Sir John Herschel's southern nebulæ. During the year 60 of the smaller nebulæ and clusters contained in the Catalogue have been observed, compared, and drawn.

A large part of the drawings of the nebulæ made with this telescope, from its erection in 1868 up till the middle of 1878, are now being printed, some of which will be issued in parts, with notes, in a few weeks.

Among the occasional work of the year may be noted the conjunction of *Mars* and *Saturn* on July 1 last, which was satisfactorily observed with both the large Reflector and 8-inch Refractor; the occultation of 64 *Aquarii* by *Jupiter* on September 14; and a search for *Mars*' satellite "Deimos" in October and November.

With regard to the latter observations, it will be remembered that *Mars*' satellites could not be found with the Great Telescope during the opposition of 1877. At the present opposition, however, the outer satellite was observed on several occasions in November, namely, the 4th, 6th, 13th, 15th, and 20th; but, except in one instance, it was always too faint to obtain any satisfactory measures. In the instance referred to, November 13, the position was approximately measured as 54° 15′ at 10h. 40m. Melbourne mean time.

The publications issued from the Observatory during the past year are volume 5 of *Results of Astronomical Observations*, which includes the work of the years 1871-2-3-4-5, and the usual *Monthly Records of Meteorology and Terrestrial Magnetism.*

Notes on some points connected with the Progress of Astronomy during the past year.

Discovery of Minor Planets.

A considerable addition to the list of minor planets has been made since the date of the last Report, owing principally to the observing activity of M. Palisa, of Pola, and Dr. C. H. F. Peters, of the Litchfield Observatory, Clinton, New York, to whom the discovery of nineteen during the year is due. The remaining two were discovered at Marseilles by M. Coggia and M. Borrelly.

No.	Name of Planet.	Date of Discovery.	Name of Discoverer.	Mean Distance. (Earth = 1).
		1879		
(192)	Nausicaa	February 17	Palisa	2·401
(193)	Ambrosia	February 28	Coggia	2·626
(194)	Procne	March 21	Peters	2·653
(195)	Eurycleia	April 22	Palisa	2·874
(196)	Philomela	May 17	Peters	3·082
(197)	Arete	May 21	Palisa	2·753
(198)	Ampella	June 13	Borrelly	2·479
(199)	Byblis	July 9	Peters	3·206
(200)	Dynamene	July 27	Peters	2·741
(201)	Penelope	August 7	Palisa	2·676
(202)	Chryseis	September 11	Peters	3·076
(203)	Pompeia	September 25	Peters	2·742
(204)	Callisto	October 8	Palisa	2·673
(205)		October 13	Palisa	2·689
(206)	Hersilia	October 13	Peters	
(207)		October 17	Palisa	2·283
(208)		October 21	Palisa	2·872
(209)	Dido	October 22	Peters	3·168
(210)		November 12	Palisa	2·754
(211)		December 10	Palisa	3·048
		1880		
(212)		February 6	Palisa	

The relative positions of the new minor planets in the group of asteroids may be inferred by a reference to the last column in the table, remembering that the mean distance of *Flora*, the nearest minor planet to the Sun, is 2·201, and that of *Hilda*, the farthest from the Sun, is 3·950.

The following planets, discovered in preceding years, have only been recently named:—

No. (174)	Phœdra.	No. (182)	Elsbeth.
„ (175)	Andromache.	„ (183)	Istria.
„ (179)	Clytemnestra.	„ (187)	Lamberta.

The Comets of 1879.

Five comets have been under observation during the past year, including Brorsen's comet of short period and Tempel's periodical comet of 1867. The remaining three are believed to be new comets.

1. Brorsen's periodical comet was first detected by M. Tempel, at the Arcetri Observatory, Florence, on January 14, 1879, when quite close to the horizon. The comet rapidly increased in brightness after it was first seen, being six times as bright on March 15, and brighter still in April. It passed its perihelion on March 30. A valuable ephemeris of Brorsen's comet was computed by Professor Schulze in anticipation of its appearance.

2. Tempel's first periodical comet (Comet II, 1867) was detected on April 24, at the Arcetri Observatory, Florence, where a long series of observations was made by M. Tempel during the two following months. The comet, when first noticed, was extremely faint and small. Viewed with Mr. Common's 18-inch Reflector on May 25, its nucleus appeared to be considerably out of the centre. On June 10 the comet was round and diffused, about 30″ in diameter. Observations of Tempel's comet have been made at several Observatories both in Europe and America. At Rio Janeiro it was observed by M. Cruls, of the Imperial Observatory, to whom it also appeared as a faint round nebulosity, with a slight condensation in the centre. According to the elements calculated by Dr. Raoul Gautier, this comet passed its perihelion on May 7.

3. A telescopic comet was discovered by Mr. Lewis Swift, at Rochester, N.Y., on June 15. The nucleus was of the ninth magnitude, and the comet about 3′ in diameter. Elements of this comet were computed by Dr. Küstner, under Professor Winnecke's superintendence, which appeared to represent the observations very closely. The perihelion passage, according to these elements, occurred on April 27. This comet was observed through telescopes with small aperture until the end of July.

4. A comet was discovered by M. Palisa, at Pola, on August 21. It appeared with a round bright disk, and was easily visible in small telescopes. According to elements calculated by Mr. S. C. Chandler, its perihelion passage took place on October 4.

5. The fifth comet was discovered by Dr. Hartwig, at Strasburg, on August 24. It was a very faint object, and, owing to the unfavourable weather, was not much observed in England.

Mr. G. H. Darwin's Researches on the History of the Solar System.

In the Annual Report for last year a short notice was given of Mr. Darwin's investigations on the influence of geological changes on the rotation of the Earth. Mr. Darwin was thus led to examine the consequences of an absence of perfect rigidity in the Earth's mass, and the results of his researches have been communicated to the Royal Society in four Memoirs, entitled "On the bodily tides of viscous and semi-elastic spheroids, and on the ocean tides upon a yielding nucleus," "On the precession of a viscous spheroid, and on the remote history of the Earth," "Problems connected with the tides of a viscous spheroid" (*Phil. Trans.*, 1879), and "The determination of the secular effects of tidal friction by a graphical method" (*Proc. Roy. Soc.*, 1879).

The first of these Memoirs relates chiefly to the forced oscillations of a viscous sphere, or of a sphere whose elasticity breaks down under continued stress. Supposing the forced oscillations to be produced by the attraction of a satellite revolving about the sphere or planet, the author investigates the law which governs the retardation and height of the tide, and also the acceleration and reduction in height of the tide in an ocean superimposed upon the viscous nucleus. Numerical calculations show that the bodily tides are at present very small in amount, and the author infers that the actual effective rigidity of the Earth's mass must be very large as compared with that of known viscous substances. The method employed is based upon a somewhat similar investigation by Sir William Thomson in regard to an elastic sphere.

A consequence of the retardation of the tides in the viscous planet is found to be that the attraction of the satellite upon the planet is not equilibrating, and that the rotation of the planet is therefore modified. The author is thus led to the investigation contained in the second paper, which has for its object the examination of the effect of frictional tides both upon the planet in which they are raised and upon the satellite which raises them. It appears from the investigation, which is carried out by means of analytical developments of considerable complexity, that the obliquity of the planet's equator will increase or decrease according to conditions set forth in the paper; and that, if the planet and satellite revolve in the same direction, and the satellite revolves slower than the planet rotates, the angular velocity of the planet's rotation will decrease and the mean distance of the satellite increase. By means of the equations which give the rate of retardation of the planet's rotation and of the

satellite's mean motion, a relation is found between the heights and retardations of the several tides existing in the planet and the acceleration of the satellite's motion which would appear to take place to an observer whose measure of time was derived from the planet's rotation. The author then applies these considerations to the hypothesis that the hitherto unexplained part of the secular acceleration of the Moon's mean motion is due to the action of bodily tides in the Earth, from which it appears that the tides in the Earth must either be of very small amount or must be subject to very small retardation. This would seem to confirm the opinion that the Earth's mass at the present time possesses great effective rigidity.

In this paper the investigation was intended to apply especially to the case of the Earth, Moon, and Sun; but the introduction of the solar attraction, which was found to be of considerable importance, rendered the differential equations so complex that the author has substituted for the complete analytical solution a retrospective arithmetical integration by the method of quadratures. The integration is carried back in detail until the day is reduced to six hours and the sidereal month to twelve hours and the obliquity of the Ecliptic to 14°. In this investigation the inclination of the lunar orbit to the Ecliptic is neglected, and it is stated that the simultaneous changes in the plane of the lunar orbit would be such as to modify the result in the case of the obliquity. (In a subsequent paper, however, communicated to the Royal Society in December 1879, Mr. Darwin has taken account of the inclination and eccentricity of the lunar orbit.) The detailed integration is not carried further back, but, by means of the equation of conservation of moment of momentum, the author is enabled to determine approximately the initial configuration from which, according to the theory, the Moon and Earth had their origin. Neglecting the obliquity of the Equator, the equation of conservation involves two variables, viz. the rotation of the planet and the mean motion of the satellite; and if one of these be put equal to the other, the equation becomes a biquadratic which has two real roots.

The physical interpretation of these two roots is that a system of a planet and satellite may be started revolving, as the parts of a rigid body, in two different ways, and that one of these configurations involves a maximum of energy and the other a minimum.* As the system is by hypothesis subject to friction, it must necessarily degrade from the position of maximum energy to that of minimum. In the detailed integration already referred to the energy of the system continually increases as we go further back, and therefore the earliest state to which it would admit of being traced is that of maximum energy. This state of maximum energy in the case of the Earth and Moon is one in which the

* This subject is considered more especially in the paper above referred to in the *Proceedings of the Royal Society*.

Moon is very nearly in contact with the Earth. The author also shows that an inferior limit to the time requisite for the degradation of the system from the configuration of maximum energy to its present condition is 54,000,000 years. The proximity of the Moon to the Earth and the rapid revolution of the system in this initial state led the author to suggest that the Moon had its origin in the rupture of a primeval planet, partially fluid and partially solid, and that the Moon has attained its present distance from the Earth by means of the reaction of the tides in the lapse of ages. Reference is made to the applicability of similar views to the cases of the other planets, but a complete discussion of the matter is postponed for a future occasion. The paper also contains discussions of the modification of the precession of a spheroid in which tides are being raised, the influence of solar tidal reaction on the length of the year, the influence of tides of the second order, the influence of tides produced by an annular satellite, and the general case of the orbital revolution of two bodies, each of which raises tides in the other.

The third paper contains investigations upon certain problems connected with the tides of a viscous spheroid. If the rotation of a viscous spheroid be retarded by the attraction of a satellite upon the lagging tide raised by that satellite, it is found that the distribution of the tidal frictional couple in various latitudes is not such that the rotation of the planet can be retarded exactly as though it were a rigid body. This gives rise to a screwing motion of the whole mass of the planet, and produces a slow motion from west to east of the polar regions relatively to the Equator. This distortion however must, at least in the later stages of the Earth's history, have been exceedingly small in amount, and cannot be regarded as affording any explanation of the observed crumpling of strata. Assuming the evolution of the Earth and Moon already explained, this screwing action may once have had some importance, and it is suggested as at least possible that the northerly and southerly trend of our great continents may be due to the effects of this cause in the remote past.

It has been already mentioned that in consequence of the friction of the tides the energy of the system degrades: this is only true of the kinetic and potential energy of the system. The principle of the conservation of energy necessitates that the lost energy should reappear in the form of heat. The second portion this paper contains an investigation of the distribution of heat generated by friction within a tidally distorted viscous spheroid. It appears that by far the larger part of the heat is generated in the central regions.

If the Earth and Moon have passed through the supposed history, then it is shown that the whole heat generated in the Earth from first to last would give a supply of heat at the present rate of loss by cooling for 3,600 million years. The author first believed that this prodigious amount of heat might be an im-

portant cause of the increase of temperature in borings and mines, and thus that Sir William Thomson's investigation of the secular cooling of the Earth might require to be largely modified; but an investigation specially undertaken to decide this point shows that the present increase of underground temperature due to this cause could hardly exceed 1° Fahr. for every 2,600 feet, whereas we know that the actual amount is 1° Fahr. in 50 feet.

The third question relates to the part played by inertia in the forced oscillations of viscous fluid and elastic spheres. In the theory of tides employed by him in his investigations up to this point the inertia is neglected, and the author here shows that the introduction of inertia will not sensibly affect the results.

Mr. Darwin has also communicated to the Royal Society a fifth Memoir, of which the abstract alone has as yet appeared. In this Memoir the inclination and eccentricity of the orbit of the tide-raising satellite are the subjects principally considered, and the author is led to some remarkable results relating to the history of the solar system.

Oppolzer's Treatise on the Calculation of the Orbits of Comets and Planets.

In the course of the year Professor Oppolzer, of Vienna, has issued the second volume of his *Lehrbuch zur Bahnbestimmung der Kometen und Planeten*, and although it is not usual in this Report to notice the publication of astronomical treatises, the contents of the present work seem likely to be so useful to the mathematical astronomer, and to meet a want that so many mathematicians must have felt, that it seems desirable to direct attention to it.

The first volume was issued in 1870, and contained 353 large octavo pages; the second, which has just appeared, contains 635 pages, of which 120 are devoted to tables, some of which have been calculated especially for the work.

The first chapter, on numerical differentiation and integration, relates to the general application of mathematics to numerical work. The second chapter, which occupies more than 200 pages, is devoted to the explanation of the method of calculation of special perturbations. As an example, the perturbations of *Erato* (62), due to *Jupiter* and *Saturn*, are worked out in full numerical detail by Encke's method, and also by the Hansen-Tietjen method. The third chapter contains an elaborate account of the practical application of the method of least squares to the treatment of astronomical observations; and the fourth to the deduction of the elements from a great number of observations. The account of the method of least squares, though of course somewhat resembling that given by Encke in the *Berliner Jahrbuch*, and reproduced by Chauvenet in his

Astronomy, is much more full and detailed as regards the actual numerical formation and solution of the equations. The theoretical basis on which the method rests is, however, as was to be expected in a practical treatise, not specially considered; but there is no other work in which so complete an explanation of the processes is to be found. Lambert's equation is also treated at length, and there is added a table of a function which occurs in connection with it, calculated by Herr F. K. Ginzel. Some portion of the scope of the work is covered by Mr. T. C. Watson's *Theoretical Astronomy*; but as regards fulness of mathematical and numerical detail, Dr. Oppolzer's work is unique.

Such a book is not only useful to the practical astronomer, but is also in the highest degree valuable to the mathematician who desires to know the exact manner in which the dynamical formulæ he is acquainted with are actually applied in practice to the calculation of orbits &c. There is a considerable interval between the analytical formulæ of disturbed elliptic motion and the actual methods in use for practically determining the perturbations of the heavenly bodies; and the present work may be regarded as supplying the connection between the two.

Auwers's "Fundamental-Catalog für die Zonen-Beobachtungen am Nördlichen Himmel."

This Catalogue has been recently published by the *Astronomische Gesellschaft* in furtherance of the work of zone-observations of stars down to the 9th magnitude, which has been in progress for some ten years. The task of compiling the Catalogue has been entrusted to Dr. Auwers, and has been executed by that astronomer in his usual careful and complete manner. The Catalogue contains the places, for the epoch 1875·0, of the 336 Pulkowa fundamental stars, and of certain additional stars, bringing the total number up to 539. The places of these stars may probably be considered to be as accurately determined as is possible in the present state of astronomical science, especially when it is remembered that Dr. Auwers uses the proper motions of the Bradley stars in the Catalogue (which constitute a very large proportion of the entire number) determined from a comparison of his, as yet unpublished, Bradley Catalogue for 1755 with the best modern observations.

Dr. Auwers' places are derived from a combination of the observations made at Pulkowa, Greenwich, and Cambridge, U.S., for the right ascensions; and for the declinations he has used, in addition to these, observations made at Leipzig and at Leiden. The Pulkowa observations made use of are: 1st, the right ascensions and declinations for 1845, the former deduced from observations made during the years 1842–1853, the latter from observations made 1842–49; and, 2ndly, a MS. Catalogue by

Herren Wagner and Nyrén, for the epoch 1871, deduced from the observations 1869–1874. The Greenwich observations used are: 1st, a MS. Catalogue of the Bradley stars observed at Greenwich, 1836–1872, and reduced to 1861; and, 2ndly, the Nine-year Catalogue for 1872, the star-places of which were forwarded to Dr. Auwers by the Astronomer Royal some time before its publication. The Catalogue places mainly depend, as might be expected, on these Pulkowa and Greenwich observations, though exception might, perhaps, be taken to the great weight which has been assigned to the Pulkowa results.

The publication of this Catalogue will doubtless add materially to the already great reputation of its author, and will do much for the efficient progress of the important work which is being carried out under the auspices of the *Astronomische Gesellschaft.*

Uranometry.

Our knowledge of the brightness and distribution of the visible stars, designated by the title Uranometry, has made some highly important advances in the last few years. The valuable series of observations by Mr. Peirce, carried out under the auspices of the Harvard College Observatory, which was noticed in the Council's Report last year, was a distinct advance in the photometry of naked-eye stars. During the past year Professor Pickering has published, in Part I. of vol. xi. of the *Annals of Harvard College Observatory*, a very extensive series of photometric observations of the visible stars, made with photometers of peculiar and original construction on the 15-inch Equatoreal Telescope, during the years 1877–79. With these observations are included a comparison of the relative light of the components of 239 bright double stars. The value and importance of this work will be more fully appreciated when the reduction of the observations, which will be published in Part II. of this volume, is in the hands of Astronomers, when a detailed examination of the whole can be properly and satisfactorily made.

The contributions, however, of the Harvard College Observatory to Uranometry do not stop here, for during the past year Professor Pickering has undertaken the determination of the light of all the stars visible to the naked eye in the latitude of his Observatory. For this purpose a Catalogue of about 4,000 stars has been formed, arranged in the order of R.A. for the epoch 1880·0. It includes all the stars in the *Uranometria Nova* of Argelander, all the stars in the *Atlas Coelestis Novus* of Heis, and in the *Durchmusterung* to the 6th magnitude inclusive, and those in Behrmann's *Atlas des Südlichen gestirnten Himmels* north of 30° S., besides a few stars from other authorities. A novel photometric method is adopted which possesses many important advantages. In his Annual Report

Professor Pickering says: "As most of these stars are inconspicuous objects, much time would be lost in identifying them in the field of a photometer mounted on an ordinary stand. This is avoided by observing them in the meridian as with a transit instrument. The photometer consists of a horizontal telescope pointing to the west and having two objectives. By means of two prisms mounted in front of the telescope, the Pole Star is reflected into one object-glass and the star to be measured into the other. The cones of light are made to coincide by a double-image prism, the extra image being cut off by an eye-stop. The star to be measured is thus seen in the same field with the Pole Star, with the same aperture and magnifying power. Many of the errors to be apprehended in the use of the Zöllner photometer, and other instruments in which the comparison is made with an artificial star, are thus eliminated. To determine the relative transparency of the air at different altitudes, a list of 100 circumpolar stars has been prepared to be observed at both upper and lower culminations." The results of this original method of observation will be looked forward to with peculiar interest.

In 1878 M. Houzeau published, in vol. 1 of the new series of the *Annales de l'Observatoire Royale de Bruxelles*, an interesting "Uranométrie Générale," contained in a Catalogue of the magnitudes and positions of nearly 6,000 stars observed by him with the naked eye in both north and south hemispheres. During a residence of some years at Jamaica, M. Houzeau was able to construct a chart of stars visible to the naked eye, which he plotted down without distinction of magnitude. A rapid re-examination of them during a period of 13 months enabled him to estimate and record their respective magnitudes. The results are contained in four Catalogues, each arranged in order of R.A.

°		°		
+90	Decl. to	+45		885 stars
+45	"	0		2031 "
0	"	−45		2053 "
−45	"	−90		750 "
			Total	5719

The Catalogues give approximate places for 1880, magnitudes, date of observation, and name. The most ancient Catalogues of stars with which we are acquainted expressed brightness in thirds of magnitudes. For instance, Ptolemy designates stars as either 3rd mag., large 3rd mag., and small 3rd mag., corresponding to Argelander's notation of 3 mag. 3–2 mag. and 3–4 mag. This example was continued by the Orientals, and has been imitated in the present day in the Uranometries of Argelander, Heis, and Behrmann. It is unfortunate that M. Houzeau has expressed his results only to half magnitudes, and the value of the work is thereby much impaired. A large number of the

stars are given without name or designation, and consequently their identification is difficult. The No. in the B.A.C. might here have been advantageously inserted. The Catalogues are accompanied by five charts, two of which are devoted to the north and south polar zones to P.D. 45°, and the other three to a cylindrical projection of the equatoreal zone. M. Houzeau has appended some interesting chapters on the Milky Way, and on the distribution of the visible fixed stars, considered: first, with reference to the plane of the Sun's equator; secondly, to the point in the heavens towards which the solar system is moving; thirdly, at right angles to this direction; and fourthly, with respect to the Milky Way. Tho conclusion arrived at is expressed by the author as follows: 'Les étoiles les plus brillantes et les étoiles les plus faibles sont celles qui se montrent le plus soumises à l'influence de la Voie Lactée; tandis que celles d'un éclat moyen, vers la limite de la visibilité à l'œil nu, en sont les plus affranchies.'

But the work recently published by Dr. Gould, of Cordoba, is perhaps the most important contribution to Uranometry that has appeared since the *Uranometria Nova* of Argelander. The title is "*Uranometria Argentina: Brightness and Position of every Fixed Star down to the Seventh Magnitude within One Hundred Degrees of the South Pole.* By Benjamin Apthorp Gould. With an Atlas. Buenos Ayres. 1879." 4to. pp. 385. It forms vol. i. of "Resultados del Observatorio Nacional Argentino en Córdoba."

In 1870 Dr. Gould proceeded to Cordoba to establish an Argentine National Observatory, and experiencing numerous delays in carrying out the work, he determined to devote the time of himself and his assistants to the formation of a uranometry of the southern hemisphere analogous to that of Argelander in the northern. As he decided to adopt the scale of the *Uranometria Nova*, he devoted considerable time to the determination of standard stars. For this purpose he selected only such stars as are situated within about 5° of that parallel of declination which has the same altitude at Cordoba and at Bonn. Hence his typical magnitudes are contained in a zone between 5° and 15° of north declination. To quote his words: "For each order of magnitude below the second, those stars were carefully selected whose brightness was found best to represent the average of those assigned to the same order by Argelander; and taking this degree of brightness for a point of departure, estimates of magnitude to the nearest tenth of a unit were made for all the stars of his Uranometry within the type-belt, and these estimates were then repeatedly revised in such manner as best to represent the scale of Argelander as a complete system." Four observers were engaged in the work, and only those stars were adopted as standards of magnitude regarding which the four observers all agreed. It was found that it was too difficult to compare the the typical stars with those near the South Pole, and therefore a series of new standards much nearer the Pole were determined

as before. Upon these typical magnitudes all the determinations in the *Uranometria Argentina* depend.

Dr. Gould seems to have been especially favoured in atmosphere, for he says that there is no doubt, in the most favourable nights, stars of 7·0 mag. are easily seen at Cordoba by ordinarily good vision. His aim was to determine the magnitudes of all stars within two or three tenths of a unit below 7·0 mag. To determine a standard for these faint stars, Dr. Gould made experiments by the method of limiting apertures, and finally adopted results which showed that an increase of 0·01 inch in the diameter of the aperture corresponds approximately to an additional 0·1 magnitude for stars of that order of brightness. But his standards of 7·0 mag. were practically determined by continuing downwards the series of differences of 0·1 mag. between 5·0 and 6·0 mag. With regard to the accuracy with which magnitudes can be determined, Dr. Gould makes the rather surprising statement that he has little doubt that "for stars of the seventh order gradations of light corresponding to no more than the *thirtieth* of a unit may be unhesitatingly recognised without more optical aid than a common opera-glass." Observations were made of nearly 1,800 stars for standards, and only those were adopted upon which the accordance of the four observers was absolute; these amounted to 722. He has great confidence in the accuracy of the results, for he says that "There have been instances in which, by common consent, the adopted magnitude for a star has been changed by 0·2 or 0·3, but I am strongly of opinion that in the great majority of such cases the change has been in the star itself rather than in the appreciations of its magnitude."

The Catalogue of 722 Standard Magnitudes is given and compared with the *Durchmusterung*, Lalande, Bessel, *Uranometria Nova*, Heis, and Dr. Gould's own determinations in 1858, at Albany, U.S.; it is followed by many valuable notes.

The important and difficult question of the boundaries of the constellations has received careful attention and modification. The following are the main principles which have guided the author in dealing with the southern constellations:—

1. The constellations of Ptolemy and Hevelius, together with those of Lacaille, are retained.

2. The Latin form is employed for the names.

3. The boundaries are arranged so that the constellations shall include all stars designated by Greek letters by their authors. The boundary lines to be formed, whenever possible, by meridians of right ascension and parallels of declination for the mean equinox of 1875·0. The result of this is necessarily the transference of several stars to other constellations than they belong to in Argelander's *Uranometria Nova*. Appended to these is a careful specification of each constellation and its position in the celestial sphere.

An idea of the extensive work accomplished by Dr. Gould may be formed from the fact, that in three years the magnitudes of

10,650 stars were determined, involving more than 46,000 independent estimates of magnitude.

The *Uranometria Argentina* is contained in a Catalogue of the positions and magnitudes of 7,730 stars, all above 7·1 mag. Approximate places are given for the equinox of 1875·0, the stars being arranged in order of R.A. according to constellations. A somewhat inconvenient method of arranging the constellations has been adopted. "Beginning with *Octans*, which includes the South Pole, they follow the order of the polar distances of their southern limits, proceeding spirally around the celestial sphere in the direction of right ascensions." The result is an anomalous arrangement, which places *Canis Major* next to *Ophiuchus*, *Lepus* next to *Aquarius*, and *Pisces* next to *Leo*. Not the least important part of this splendid volume are the notes to the Catalogue, which occupy over 100 pages, giving details of the colour and variability of stars, with their range of brightness.

The Catalogue is accompanied by an atlas of thirteen charts, together with an index-map. The first is a polar chart, extending to a radius of 32½° round the South Pole. The six which follow contain the belt between the parallels of 72½° and 27½° South Decl., the six next contain the equatoreal belt between 32½° South and 12½° North Decl. The charts are constructed on the stereographic projection, corresponding to a sphere of which the radius is one metre. Being reproduced by photolithography, they were drawn upon the scale of a sphere 104 centimetres in radius, to permit of a slight reduction. Stars are represented by circular dots, of which the areas are proportional to the amount of light. Names of stars are omitted, and the lines of R.A. and Decl. being faintly drawn, the result is, perhaps, the most beautiful and perfect representation of the heavens yet published, and is a fitting atlas for so valuable a Uranometry as Dr. Gould has presented to Astronomy. This work concludes with a discussion of the distribution of the stars according to the magnitudes assigned by Gould, Argelander, and Heis; the following being the principal conclusions arrived at by Dr. Gould:—

"1. There is in the sky a girdle of bright stars, the medial line of which differs but little from a circle, inclined to the galactic circle by a little less than 20°.

"2. The grouping of the fixed stars brighter than 4·1 mag. is more symmetric relatively to that medial line than to the galactic circle; and the abundance of bright stars in any region of the sky is greater as its distance therefrom is less.

"3. The known tendency to aggregation of faint stars towards the Milky Way is according to a ratio which increases rapidly as their magnitudes decrease, and the law of which is such that the corresponding aggregation would be scarcely, if at all, perceptible for the bright stars.

"4. These facts, together with others which have been stated, indicate the existence of a small cluster, within which our system is eccentrically situated, but which is itself not far from the middle

plane of the Galaxy. This cluster appears to be of a flattened shape, somewhat bifid, and to consist of somewhat more than 400 stars of magnitudes from the first to the seventh, their average magnitude being about 3·6 or 3·7.

"5. The general distribution of the fixed stars according to magnitude does not appear capable of being well represented by any simple algebraic expression. Yet, by adopting the data of the preceding paragraph and supposing the several magnitudes of the stars in the cluster to follow the Law of Probabilities, we obtain for each class of magnitudes a number which being subtracted from the observed number in the sky leaves a system of distribution which may be represented by the expression $\Sigma_m = ab^m$ within the limits of errors of observation.

"6. The accordance thus obtained holds good for the stars of both hemispheres down to the lowest limits of magnitude for which trustworthy enumerations exist.

"7. The form of the expression $\Sigma_m = ab^m$ is that which corresponds to the hypotheses that in general the stars are distributed at approximately equal distances from one another and are of approximately equal intrinsic brilliancy. It is, however, not requisite for its application that their distribution be equable in all directions, but only that their number be proportional to the volume of the spherical shell within which they are contained.

"8. Each of the authorities (Argelander, Heis, and Gould) and each hemisphere, affords data from which result essentially the same value for the ratio b, the differences in the data being in every case represented by differences in the coefficient a. The value thus obtained for b corresponds to the light ratio 0·4028 for descending, or 2·4827 for ascending magnitudes."

The Dunsink Observations.

During the past year Part III. of the *Astronomical Observations and Researches at Dunsink* has been published. The part contains four papers, two of which are by Dr. Brunnow and two by Dr. Ball.

Dr. Brunnow has compared the declination of the planetary nebula H. IV. 37 with that of a tenth magnitude star north preceding. Measurements were made on 33 nights, and the resulting value for the parallax is

$$+0''{\cdot}047 \pm 0''{\cdot}030.$$

Dr. Brunnow also gives a series of measurements of 82 double stars.

Dr. Ball has observed on 35 nights the difference of declination between the preceding star of 61 *Cygni* and a star of 9–10 mag. following. The resulting value of the parallax is

$$+0''{\cdot}4654 \pm 0''{\cdot}0497.$$

A first instalment is also given of the results of a search in the hope of finding stars with appreciable parallax. The present paper contains observations of 42 stars in which a negative result has been arrived at. Many of the stars in this list are taken from Schjellerup's Catalogue of Red Stars.

Astronomical Bibliography.

Under the title of "Répertoire des Constantes de l'Astronomie" M. Houzeau, the Director of the Royal Observatory at Brussels, has published, in vol. i. of the new series of the *Annals* of that institution, a very valuable contribution to Astronomical Bibliography.

Every practical astronomer feels keenly the difficulty of ascertaining the results that previous astronomers have obtained in any particular branch of the science, and those who have endeavoured to smooth these difficulties by bibliographical researches are deserving of his gratitude. To this end M. Houzeau has laboured in the preparation of a work which, dealing with the principal elements of practical astronomy, affords not only references to the publications wherein those elements may be found, but also in numerous cases furnishes the very details the astronomer requires for his researches.

The "Répertoire des Constantes de l'Astronomie" consists of nineteen chapters, each containing many subdivisions; each subdivision is prefaced by a paragraph of a general historical nature. Chapter 1, on Spherical Astronomy, has subdivisions dealing with the Obliquity of the Ecliptic, Precession, Nutation, Aberration, and Refraction; the values of these several constants as determined by astronomers from the earliest period to the present day being arranged in chronological order, with the authors' names and references to the works in which they are to be found. For example, 102 determinations of the obliquity of the Ecliptic are given for dates ranging from Tcheou-Kong, B.C. 1100, to Airy, 1868.

Chapter 2 is devoted to the elements of the principal planets as found in ancient tables. Separate chapters, 3–13, are devoted to the Sun, Planets, Earth, Moon, and Asteroids. Subdivisions give, in chronological order, determinations of the Elements of the Orbit, Diameter, Mass, Rotation, Brightness, and Physical Condition of those bodies, together with corresponding details of the satellites, with full references to authors' names, dates of publication, and values obtained. In fact each chapter forms a fairly complete history of the physical astronomy of each member of the solar system. The remaining chapters are devoted to Comets; Meteors; The Solar System generally; Catalogues of Stars; Physical investigations of Stars, such as Photometry, Spectroscopy, Parallax, Proper Motion of Stars; Double Stars

and Nebulæ; the Zodiacal Light; the subdivisions of each chapter containing very succinct details conveniently arranged in a clear and intelligible form. M. Houzeau's work will be found a very useful table-book for daily reference; but it is to be regretted that it was not made a separate publication, so that it might be more widely available to working astronomers.

Hartwig's Paper on the Diameters of Venus and Mars.

In *Publication XV. der Astronomischen Gesellschaft*, Mr. E. Hartwig has published the results of heliometric measurements of the diameters of *Venus* and *Mars*, made at Strasburg by means of several of the small heliometers which were employed in some of the expeditions for observing the last transit of *Venus*, and he has also collected and discussed, and partly reduced, the older series of observations. An examination of the different determinations of the diameter of *Venus* shows that almost all the measurements are affected by comparatively large constant errors, the sources of which can only be partially traced. Rejecting as untrustworthy the observations, made with wire micrometers and with double-image micrometers, the optical errors of which have not been investigated, Hartwig considers that only three series of measurements, namely those made by Main of Oxford, by Kaiser of Leiden, and by himself, can claim to have contributed to a knowledge of the diameter of the planet, and he deduces from them $17''\cdot55$ as the value of the diameter of *Venus* seen by reflected sunlight at a distance equal to the mean distance of the Earth from the Sun. This value is about $0''\cdot6$ greater than that derived from the measurements of the black disk of *Venus* made during the last transit by Auwers and by Col. Tennant.

In the case of *Mars* Hartwig combines finally the results of the observations of Bessel, Kaiser, Main, and himself, and deduces $9''\cdot352$ as the value of the diameter (at the distance 1), which is only a little greater than the value $9''\cdot328$ resulting from Bessel's old measures alone.

It is to be hoped that Hartwig's valuable paper, by exhibiting the present state of our knowledge of the diameters of the two planets, may help to induce capable observers to undertake such series of measurements as will be really creditable to the art of observing.

Lord Rosse's Observations of Nebulæ and Clusters.

In the *Philosophical Transactions* for 1844, 1850, and 1861, the late Earl of Rosse published extracts from the observations of Nebulæ and Clusters made with his 3-foot and 6-foot Reflectors. These observations have been, with a few interruptions, carried

on from 1848 up to the middle of 1878, and the present Earl has now published, in the *Transactions of the Royal Dublin Society* (New Series, vol. ii.), a very complete continuation of the earlier papers. As the latter only contained a selection from the observations, and in many cases gave very little information about new objects found at Birr Castle, it was considered desirable, not only to publish the observations made since 1860, but also to give at greater length the earlier ones. To facilitate the preparation of the work for publication, it was divided into three parts, the two first of which, comprising the first fourteen hours of R.A., were published last autumn, while the third part, containing the last ten hours, is in the press and will follow in a few months.

The observations made during the last six or seven years before the series was finished contain a great many micrometric measures of groups of nebulæ or of nebulæ and neighbouring stars, made to a great extent in order to verify the new objects found during previous years. For obvious reasons the absolute positions of the objects examined cannot be directly obtained, and this circumstance, combined with the very condensed form of the earlier publications, made it impossible for Sir John Herschel to avoid making many mistakes in entering the "R. Novæ" in his General Catalogue. Many of these mistakes were pointed out with more or less certainty by D'Arrest. In the new publication particular attention has been paid to the identification of the objects and the finding of exact places for the new nebulæ, and Mr. Dreyer, by whom most of the work of preparing and arranging the observations for publication has been performed, has, in brackets, added a considerable number of notes and remarks, chiefly relating to questions of identification. The brighter and more striking nebulæ having been frequently delineated, and drawings of them published in the former papers, there has therefore remained in recent times less scope for the pencil. The engravings in the *Philosophical Transactions* have not been reproduced in the new paper, but are only referred to in notes; while four lithographic plates contain drawings of nebulæ which either were not figured in the earlier papers or which appeared to have been less successfully represented by the earlier observers. To this latter class belong two of the most remarkable objects in the heavens, the so-called "Crab Nebula" (M. 1), and the great Spiral in the *Canes Venatici* (M. 51). Of both these carefully executed drawings are given, founded on a great number of observations, and believed to be faithful representations of the objects as they appear in the 6-foot Reflector. Diagrams, executed by the pantagraph process, are introduced in the text to show the relative positions of groups of nebulæ or of nebulæ and stars. In some cases they represent the appearance of objects of which only rough drawings had been made by the telescope.

The observations are given in the observer's own words, and none have been omitted, except such as were clearly inferior,

through weather or other causes, to others of the same object made under manifestly better conditions, and mere notes, only stating that an object had been viewed, without giving any details of description. While the first fourteen hours of R.A. in the paper of 1861 only occupied 34 pages, they extend over 130 pages in the present publication; and Lord Rosse is to be congratulated on the completion of this valuable series of observations.

Investigations in Optics, with special reference to the Spectroscope. By Lord Rayleigh.

Lord Rayleigh has published during the past year a series of four papers in the *Philosophical Magazine*,* in which he discusses several questions of interest to spectroscopic observers. He shows that, with beams of rectangular section, such as are usually made use of in both grating and prism spectroscopes, the diffraction phenomena are independent of the vertical aperture or height of the prisms, and that a double line in a spectrum cannot fairly be resolved unless its components subtend an angle exceeding that subtended by the wave-length of light at a distance equal to the horizontal aperture. For the best definition the angular width of the slit should not exceed a moderate fraction of the above angle.

It has long been known that the resolving power of a telescope may be increased by blocking out the rays from the centre of the object-glass, and Lord Rayleigh shows theoretically (and has also experimentally proved) that a similar increase of resolving power of a spectroscope may be obtained by blocking out vertical sections of the rectangular beam. In the second and third papers the influence of aberration and the amount of accuracy required in optical surfaces is investigated, and in the number for January 1880 the general design of spectroscopes is discussed.

The Selenographical Society.

This Society was founded at the beginning of 1878 by several Fellows of the Royal Astronomical Society interested in lunar work. The first annual report of the Society shows that it then consisted of 60 members, a number which has since been largely increased, and it now includes many astronomers in Australia, North and South America, and the principal States of Europe, from whom the most cordial support and encouragement has been received. Although only some two years old, as regards the number of its members, the progress in its special work, and its financial state, it may be considered to be in an established and satisfactory condition.

* *Phil. Mag.* October 1879 to January 1880.

The Society, in addition to the obvious advantages which may be expected from the combined efforts of its members, has certain special objects in view, such as the revision of the present, or the initiation of a standard, nomenclature, the production of a good map, and the exact delineation of as much as possible of the lunar surface. The last is a work of some importance, as it is likely to lead to some definite result on the question of lunar changes.

Since April 1878 a journal has been regularly published every lunar month (appearing on the day of new moon) treating of lunar matters, and giving amongst other things a list, with notes, of objects favourably situated during the next coming lunation, and from time to time outline maps, instructions and directions to observers, drawings, and photographs.

A collection of books, drawings, maps, photographs, &c. has already been formed by the Society.

Report on the Progress of Meteoric Astronomy in the Year 1879–80, by Professor Alexander Herschel.

Papers and essays on meteoric subjects, and occasional notes of observations relating to aërolites, meteors, and meteor showers were published during the past year, or were communicated to the Luminous Meteor Committee of the British Association pretty frequently, with about their usual abundance. Owing partly to the increasing range and progress in theory and practice of the subject of meteor-registration, producing as a natural consequence of its development a more varied and scattered form of its materials; and partly owing to losses in efficiency and organisation which the Meteor Committee of the British Association sustained last year by the death of one, and by the retirement of another of the active members of its staff, sufficient progress has not yet been made in reviewing writers' memoirs, and in selecting and extracting from details of observations the chief points of valuable and important interest which they contain, to enable the chief meteor events of the past year to be clearly described, and to be satisfactorily and adequately represented in a brief report.

There is no very pressing or urgent need to refer in this year's Report to the more insignificant phenomena of last year's meteoric displays; and it is to be hoped that the Council's Report will not suffer any material loss of scientific consequence if discussion of the phenomena of the smaller meteors is deferred to the more convenient season of another year.

The following list contains particulars of recent falls of aërolites, and a collection of results obtained from combinations and comparisons with each other of observations of several considerable fireballs of whose real paths determinations have recently been made.

Real Paths of Large Meteors, and Falls of Aërolites recently recorded.

Date and Hour of Appearance, G.M.T. (or Local Time); and general Locality.	Place of Fall; or Height (in English miles, m.) and Locality at Appearance and Disappearance.	Radiant-point, Length of Path, and Velocity.	General Description; Reference; and Comparison with known Radiant-points.
1864, Nov. 11 ($5^h 35^m$ p.m.) South of France; Ryde and Kent, England.	From 70 or 80 m. over a point midway between Lyons and Clermont, to 50 or 60 m. over a point midway between Cahors and Montauban.	$85°+35°$ ($\pm 10°$), near *θ Aurigæ*; about N.E., alt. 5° or 10°. 150 or 200 m. in about 5 seconds; 35 miles p. sec. Parabolic speed 32½ m. p. sec.	At dusk, in full moonlight, which it eclipsed in Arriège. White, with a voluminous white tail, and persistent streak behind it; the latter visible on its whole course for several minutes. No appearance or sound of an explosion. (*British Assoc. Reports*, vol. for 1865, pp. 78, 120; with an additional observation at Ryde, *Ib.*, vol. for 1879, p. 108, where the date is in error). Corder 1876, Nov. 8, 9, 10, a special radiant-point of large meteors at $86°+36°$.
1868, Sept. 5 ($8^h \cdot 5^m$ p.m. Berne Time). Switzerland, Italy, and France; and Germany.	From 250 m. over Belgrade (beginning perhaps even earlier) to 85 or 100 m. (? 70, or ? 115, not very certain) over Ozaine, near Tours, France.	$14°-2°$, near 20 *Ceti*. 1200 m. (or ? even more) in 43 seconds (average estimated duration); about 28 m. p. sec. (and perhaps swifter?). Parabolic speed 26 m. p. sec.	A splendid, long-pathed fireball, widely and pretty carefully observed. Nucleus with little colour, tail, or enduring light-streak on its course; disappeared rather suddenly, without sound or appearance of explosion. The real path (with omission here of about a third part of its computed height and calculated extent at its commencement) by G. von Niessl (*Verhandlungen des Naturforschenden Vereins in Brünn*, Bd. xvii.); Schmidt, Sept. 1–10, $17°+9°$; and Denning 19 (1876).
1878, July 15 ($1^h 45^m$ p.m.) Stonefall in Moldavia.	Tieschitz (one stone only found). The meteor passed over Daubrawic and Sloup.	$68°+40°$; from about azimuth 108°, alt. 40°.	A stone of 27·5 kgm. fell from a small dark cloud, with loud noise, 100 paces from some people who discovered it while still warm. It is roughly pyramidal, with a black crust, and resembles internally and externally the Pultusk stones (Jan. 30, 1868), but is finer than them in texture, and chondritic (*Denkschrifte der Naturwissenschaftlichen Classe, Akad. d. Wissenschaften*, Wien, Bd. xxxix., Nov. 1878).
1878, Dec. 30 (about $6^h 57^m$ p.m.) Ohio and Pennsylvania, U.S.	From 72 m. over Columbiana Co. to 17 or 18 m. over Tuscarawas Co., Ohio (a disruption), and proceeding farther to 12 or 13 m. above the earth.	$90°+55°$ ($\pm 10°$), near *δ Aurigæ*. The first part of the flight 85 m. in about 2 seconds (determinations not very certain).	Nucleus elongated, as wide apparently as the Moon's disk, and many times longer; greenish, bursting into red pieces near the end of its course without detonation (D. Kirkwood, *American Phil. Soc. Proc.*, May 2, 1879, p. 241). Denning, Dec. 31, 1872, and Dec. 30, 1878, $92°+57°$. Several meteors from this radiant-point, and one of them as bright as *Sirius*.
1879, Jan. 12 ($7^h 25^m$ p.m. Berlin Time). Prague, and the centre of Bohemia.	From 40 m. over the Sudetengebirge (in the N.E. of Bohemia, beginning perhaps earlier), to 9 m. over Rakonitz, 25 m. W. from Prague, passing about 18 m. over Prague.	$133°+19°$ ($\pm 3°$), near *δ Cancri*; about E.N.E., alt. 14°. 124 m. in 3–5 seconds (6 estimates); 18 m. p. sec. Parabolic speed 23 m. p. sec.	A very large detonating meteor, shaking houses in Prague and Rakonitz by its concussion. Distinct from the next one, though the meteors appeared within a few minutes of each other. $130°+20°$, Dec. 21—Jan. 5, 1876–7, Denning; ♌ 1680, Dec. 26, $132°+21°·5$; fireball, Jan. 19, 1877, Ireland, $135°·5+22°$ (von Niessl), doubtful agreements (?). Calculations by G. von Niessl, Vienna Academy *Sitzungsberichte*, vol. lxxix., May 8, 1879.
1879, Jan. 12 ($7^h 32^m$ p.m. Berlin Time). Western part of Bohemia and Saxony.	From 78 m. over Pibram, in Western Bohemia, to 23 m. over Grosshain, near Dresden.	$52°-10°$ ($\pm 5°$), near *γ Eridani*, nearly from S. to N. 60 m. in 5 seconds, or 124 m. in 10 seconds (2 estimates); 12½ m. p. sec. Parabolic speed 11 m. p. sec.	A fireball similar in appearance to the last, but less large and brilliant, and not apparently producing any detonation. $57°-12°$, Jan. 4–20, 1877, Denning; and fireball of Jan. 7, 1877, England, $48°-11°$. G. von Niessl, *Ibid.*

1879, Jan. 28 ($2^h 28^m$ a.m.) Wisconsin and Michigan, U.S.	From near 100 m. over a point in N. lat. 44° 25′, long. 9° W. from Washington, to 26 m. (or ? lower) over Charlevoix, Michigan.	142°+14° (about, or from alt. 55°, S.W. by S.), between *Leo* and *Cancer*. About 124 m.; duration not noted, but the meteor's observed motion was rather slow.	An immense fireball; 4×the Moon's apparent width, casting off a perfect ring of fiery sparks, which followed it; and producing a violent explosion, like an earthquake's shock, in Traverse City, Michigan (D. Kirkwood, *American Phil. Soc. Proc.*, May 2, 1879, p. 243). Perhaps a Jan.–Feb. Cancrid; 133°+26°, Feb. 13, SZ. 32; ☌ 1833 ʊ Jan. 27, 135°+25°.
1879, Feb. 22 ($12^h 20^m$ a.m.) Essex, Suffolk, and Cambridgeshire.	50 or 75 m. over some point between Godstone and Guildford, Surrey, to 5 or 6 m. over a point between Haverhill and Newmarket, Cambridgeshire.	140°+5° (±7°), near the head of *Hydra*. A length of path of about 85 m. in 3–5 seconds may be gathered roughly from the observations.	A very vivid fireball; nucleus half the Moon's apparent diameter, white and green, then red; burst into fragments with a report like thunder heard in less than half a minute at Haverhill and at Saffron Walden, in Essex (*British Assoc. Reports*, vol. for 1879, pp. 87, 118). Stationary 4th mag. meteor at 145°+8°, Feb. 24, 1878, E. F. Sawyer; 141°−2°, Jan. 1—March 16, Greg No. 16, 1876.
1879, Feb. 24 ($2^h 53^m$ a.m.) York, and East Yorkshire. Seen also as far as Dundee, and Shoreham, near Brighton.	From about 60 m. above a point on the North Sea 28 m. N.E. from Whitby (first appearance probably still earlier) to 6 or 7 m. over a point halfway between Leeds and Selby.	310° (±15°)+55° (∓10°), near χ *Cephei*. A good average direction, 39° E. from N., alt. 32°, with rather more probability of the upper than of the lower-signed corrections of the place. About 102 m. in 6 or 8 seconds by several estimations; 14½ m. p. sec. Parabolic speed 18 m. p. sec.	A fireball with immense illumination over York. Disk ⅓ or ¼th of the Moon's apparent size, and light like daylight. White, or reddish with red tail at distant points of observation; sparks, and a disruption overhead at York, but no light-streak left upon its course, and the final disappearance rather sudden. A deafening report at York, with a concussion at Stockton like an earthquake's shock, a minute and a half after the appearance. No known radiant-point at the same date agrees with the observed position. Calculation of the real path by J. E. Clark, *British Assoc. Reports*, vol. for 1879, p. 89 and p. 118, where the time of appearance ($12^h 45^m$ a.m.) is in error; and *The Observatory*, vol. iii., p. 304 *et seq.*
1879, May 10 (5^h p.m.) A stonefall in Iowa, U.S., America.	The large mass fell at Esterville, Emmet Co., Iowa; a smaller one at 2 m. distance from it, and many small pieces fell in the vicinity.	The smaller mass penetrated the earth vertically to a depth of 4½ feet; the larger stone was dug out from a depth of 14½ feet in a stiff clay soil.	The masses, weighing respectively 500 lbs., 170 lbs., and a few ounces or pounds, fell from a meteor seen to explode in full daylight, with a noise like a long roll of thunder. They consist, in nearly equal portions, of stony matter and metallic nickel-iron (the latter apparently containing tin), which shows Widmanstättian structure. Some of the crystalline minerals included in the stony part are 2 inches thick. (*American Journal of Science*, vol. xviii., p. 77.)
1879, May 17 (about 4 p.m.) A stonefall in Silesia.	A stone fell near Gnadenfrei, and a rather smaller one 2 m. due N.E. from it in Schobergrund, villages between Reichenbach and Frankenstein, in Silesia.	The holes, 1 ft. and 8 in. deep, made by the stones were vertical. The explosive sounds came from S.E., and were heard in that direction from the place of fall much more generally and loudly than N.W. of it.	Two stones were found, 2 lbs. and 1½ lb. in weight. The former, seen to fall with a rushing noise about 70 seconds after a loud report (which was triple) and a rattling sound, was cold when found. The other, discovered accidentally a few hours later, was broken into many pieces by the finders. They are loose-textured, chondritic, with much interspersed nickel-iron. The sky was overcast, and no fireball was observed. Enquiries made on the spot by Prof. Galle, and the analyses by Prof. von Lasaulx. (*Monaths-bericht* of the Berlin Academy, July 31, 1879.)
1879, Sept. 23 ($8^h 10^m$ p.m.) Over the centre of Scotland.	From 70 or 80 m. above Ben Nevis, to a height of 20 or 25 m., between Loch Oich and Loch Laggan. A projected course of 10 m., a little E. of the Caledonian Canal.	288°+48° (±5°), between ι *Cygni* and π *Lyræ*. 50 or 55 m. in between 3 and 4 seconds (duration well observed); 15 or 16 m. p. sec. Parabolic speed 15½ m. p. sec.	A fireball several times as bright as *Venus*, and casting considerable light. Bluish, globular, without sparks or streak; disappeared rather suddenly. Seen at Aberdeen, Edinburgh, Silloth, near Carlisle, and at many places in Scotland. Radiant of the great fireball of Sept. 24, 1876, over the English Channel, 285°+35° (±5°), near δ *Lyræ*; and speed of that meteor's flight, relatively to the earth, 15 m. p. sec. G. & H. B., Aug. 2—Sept. 25, 285°+44°; Heis B., Sept. 1–15, and Schmidt, Sept. 1–10, 290°+58°, agree nearly with these fireballs' radiant-points. Calculation of the real track, from several observations of the meteor's course, by Prof. Herschel.

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Papers read before the Society from February 1879 *to February* 1880.

1879.

Mar. 14. Observations of Encke's comet, 1878. J. Tebbutt.

Les longueurs du pendule à séconde à Poulkova, à St. Pétersbourg, et aux différents points de la Russie occidentale, corrigées de l'influence produite par la flexion des supports du pendule construit par M. Repsold. Prof. A. Sawitsch.

A formula for reducing precession in right ascension and declination from Bessel's to Struve's constants. Prof. A. Krueger.

Observations of absorbing vapours upon the sun. E L. Trouvelot.

On the change in the errors of Hansen's lunar tables between 1848 and 1876. W. T. Lynn.

Note on *η Draconis*. W. T. Lynn.

Remarks on Mr. Sadler's paper in the January number of the *Monthly Notices*. C. G. Talmage.

Notes on a Persian MS. of Ulugh Beigh's catalogue of stars belonging to the Royal Astronomical Society. E. B. Knobel.

On the probable presence of oxygen in the solar chromosphere. A. Schuster.

On a new method of determining astronomical refractions. D. Gill.

Note on systematic errors in right ascension depending on magnitude. D. Gill.

On the computation of the heights of lunar mountains from photographic measurements. Prof. C. Pritchard.

Note on Gruithuisen's lunar crater "Schröter." J. Birmingham.

April 9. An apparently new variable star. J. L. E. Dreyer.

Ephemeris for physical observations of *Jupiter*, 1879. A. Marth.

Notes on large telescopes, with suggestions for mounting reflectors. A. A. Common.

On the desirability of photographing *Saturn* and *Mars* at the next conjunction. A. A. Common.

On some formulæ for expressing the value of the excentric anomaly &c. in functions of the mean anomaly. A. de Gasparis.

Note on the death of M. E. de Chazal. Lord Lindsay.

May 9. Observations of Brorsen's comet, February 1879. J. Tebbutt.

Meteor radiant points of April 9–12. W. F. Denning.

Catalogue of 222 stationary meteors. W. F. Denning.

Observations of Brorsen's comet, 1879. H. C. Russell.

Observations of Brorsen's comet. Lord Lindsay.

Two short and easy methods of correcting lunar distances. J. J. Astrand.

Note on Sir John Herschel's reference catalogue of double stars. Prof. C. Pritchard.

On the determination of the solar parallax from the parallactic inequalities in the longitude of the moon, and on the correction to Hansen's coefficient of the annual equation. E. Neison.

Order of publication of successive volumes of the *Observations de Poulkova* (extract from a letter to Mr. Downing). O. von Struve.

On the spectrum of Brorsen's comet, observed at the Royal Observatory, Greenwich. Astronomer Royal.

June 13. Observations of Brorsen's comet, February and March 1879. J. Tebbutt.

An important invention, giving perfectly uniform rotary motion in driving clocks. H. C. Russell.

Determination of the longitudes of Berlin, Munich, Leipzig, Vienna, Paris, and Pulkowa. T. Oppolzer.

On the values of the constants in the equation ${}_rA_r x^{(r)} + {}_rA_{r-1} x^{(r-1)} + \ldots + {}_rA_t x^{(t)} + \ldots + {}_rA_0 - y_x = 0$; obtained by the method of least squares, from the $n+1$ values of y_x when $x = 0, 1, 2, \ldots n$; n being greater than r. C. Carpmael.

On the value of the solar parallax derived from observations of *Mars* made at Ascension during the opposition of 1877. D. Gill.

On the knowledge of the zodiacal light possessed in the east from of old. J. W. Redhouse.

Spectroscopic observations of the motion of stars in the line of sight, made at the Temple Observatory, Rugby. G. M. Seabroke.

Note on the Cape catalogue of 12,450 stars (extract from a letter to the Astronomer Royal. E. J. Stone.

A new method of controlling the driving clock of an equatoreal. R. C. Johnson.

Preliminary note on the Babylonian astronomy. The Babylonian calendar. R. H. M. Bosanquet and A. H. Sayce.

On the coincidence of the bright lines of the oxygen spectrum with bright lines in the solar spectrum. H. Draper.

On the applicability of the mean refractions of Bessel's fundamenta to the Washington observations. A. M. W. Downing.

Note on the photographic semi-diameter of the moon. Prof. C. Pritchard.

Théorie analytique des mouvements des satellites de *Jupiter*. M. Souillart.

Note on the transit of the earth and moon across the sun's disk as seen from *Mars* on November 12, 1879, and on some kindred phenomena. A. Marth.

On a new method of mounting an equatoreal. Rev. J. Pearson.

Nov. 14. Ephemeris for physical observations of *Mars*, 1879. A. Marth.

Note on the difference of variation of gravity at Revel and St. Petersburg, and on Grischow's pendulum observations at other stations. Major J. Herschel.

On the physical configuration of *Mars*. W. Harkness.

On the change in the mean error of longitude of Hansen's lunar tables since 1876. W. T. Lynn.

Mean areas of umbræ, whole spots, and faculæ upon the sun's disk, as measured on photographs taken at the Royal Observatory, Greenwich, for each rotation of the sun from 1873, July 11. Astronomer Royal.

The nebula in the *Pleiades*. Maxwell Hall.

Observations of comets *Palisa* and *Hartwig* made with the Merz equatoreal of the *Collegio Romano*. P. Tacchini.

Conjunction of *Mars* and *Saturn* observed at the observatory, Melbourne. R. L. J. Ellery.

Comparison of Hansen's coefficients of the moon's latitude with those of Plana, Damoiseau, &c. R. Wilding.

The zodiacal light. Maxwell Hall.

Description of a self-registering micrometer. A. Bowden.

Observations of comet 1879 *d* (Palisa) made at Dun Echt observatory. Lord Lindsay.

Observations of the spectrum of comet 1879 *d* (Palisa). Lord Lindsay.

On a photograph of the solar spectrum showing dark lines of oxygen. J. C. Draper.

Note on the semi-diameter of the moon. E. Neison.

On the polarisation of the corona. A. Schuster.

A comparison between the right ascensions and north polar distances of the *Nautical Almanac* and the General Cape Catalogue for 1880. E. J. Stone.

On the evidence of a past connexion between four widely separated southern stars. E. J. Stone.

On the working of the speculum for the 37-inch reflector. G. Calver.

Note on two accompanying sketches of *Jupiter*. Captain W. Noble.

Note on the ellipticity of *Mars*, and its effect on the motion of the satellites. J. C. Adams.

Observations of the solar eclipse 1879, July 18, made at the University Observatory, Strasburg. A. Winnecke.

Dec. 12. Note on the correction to the mean longitude of Hansen's lunar tables. S. Newcomb.

Occultation of 64 *Aquarii* by *Jupiter*, observed at the Melbourne Observatory. R. L. J. Ellery.

The Babylonian astronomy, part II. R. H. M. Bosanquet and A. H. Sayce.

New double stars. S. W. Burnham.

Discovery of a gaseous nebula in *Cygnus*. Rev. T. W. Webb.

Sur la variation du demi grand' axe des orbites planétaires. A. de Gasparis.

Occultation of stars by the moon, and phenomena of *Jupiter's* satellites, observed at the Stonyhurst observatory. Rev. S. J. Perry.

The November meteors. Rev. S. J. Perry.

On recent changes in the mean error of longitude of Hansen's lunar tables. W. T. Lynn.

Note on the gaseous nebula in *Cygnus*. G. Knott.

On a form of reading microscope which appears to combine most of the advantages of the German and English forms. E. J. Stone.

Note on the north polar distances of the Greenwich seven-year catalogue for 1860. A. M. W. Downing.

Observations of the outer satellite of *Mars* made at Dun Echt observatory. Lord Lindsay.

Note on *General Catalogue* (Supplement) No. 6000. Lord Lindsay.

Observations of Mr. Baxendell's new star in *Canis Minor*. Lord Lindsay.

Note on the spectrum of the red spot on *Jupiter*. Lord Lindsay.

Note on the Rev. T. W. Webb's new nebula. Lord Lindsay.

On the correction for personality required by the observations of the moon made with the Greenwich transit-circle. E. Neison.

Observations of the satellites of *Mars*. A. A. Common.

Note on *Mimas* and *Hyperion*. A. A. Common.

Note on a new star in *Canis Minor*. G. Knott.

Ephemeris for finding the positions of the satellites of *Uranus*, 1880. A. Marth.

Note on Mr. Webb's new nebula. A. Winnecke.

1880.

Jan. 9. Observations of the phenomena of *Jupiter's* satellites &c., made at the observatory, Adelaide. C. Todd.

Meteor showers. W. F. Denning.

Observations of occultations of stars by the moon, and

of phenomena of *Jupiter's* satellites, made at the Royal Observatory, Greenwich, in the year 1879. Astronomer Royal.

Meteor showers, 1870–79. H. Corder.

On the rotation period of *Jupiter.* H. Pratt.

Note on the remeasurement of a lunar photograph, in reply to Mr. Neison's criticism. C. Pritchard.

Changes of relative brightness of *Jupiter's* satellites. C. E. Burton.

Notes on "a catalogue of 10,300 multiple and double stars" &c., forming volume xl. of the *Memoirs of the Royal Astronomical Society.* Hours o to vi. H. Sadler.

Note referring to observations and estimations of the brightness of *Mars* which ought to be made in February and March 1880. A. Marth.

Estimations of the time of transits of the red mark on *Jupiter* across the central meridian. T. W. Backhouse.

On the systematic errors of the Greenwich north polar distances. W. H. M. Christie.

Mean area of spots and faculæ upon the sun's disk, as measured on photographs for 1878 and 1879. Astronomer Royal.

Observations of the outer satellite of *Mars* made at the Royal Observatory, Greenwich. Astronomer Royal.

On the effects of personality on the tabular errors of the moon. E. Dunkin.

The Greenwich standard right ascensions. A. M. W. Downing.

On some changes in the markings of *Mars.* N. E. Green.

List of Public Institutions and of Persons who have contributed to the Library &c. since the last Anniversary.

Her Majesty's Government in Australia.
Her Majesty's Government in India.
The Lords Commissioners of the Admiralty.
British Association for the Advancement of Science.
British Horological Institute.
Geological Society of London.
Institute of Actuaries.
Institute of Mechanical Engineers.
Meteorological Office.
Meteorological Society.
Photographic Society of Great Britain.
Physical Society of London.
Royal Asiatic Society.
Royal Geographical Society.
Royal Institution.
Royal Observatory, Greenwich.
Royal Society.
Royal United Service Institution.
Selenographical Society.
Society of Arts.
University College, London.
Zoological Society of London.
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Dunsink Observatory.
Edinburgh, Royal Society.
Glasgow Philosophical Society.
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Berne, Meteorological Institute.
Bologna, Academy of Sciences.
Bordeaux, Society of Sciences.
Breslau, Royal Observatory.
Brussels, Royal Observatory.
Cherbourg, Society of Sciences.
Christiania, Norwegian Meteorological Institute.
Collegio Romano, Observatory.
Copenhagen, Society of Sciences.
Genoa, Society of Literature and Science.
Göttingen, Royal Society of Sciences.
Helsingfors, Finnish Society of Sciences.
Kazan, Imperial University.
Leipzig, Astronomical Society.
Leipzig, Royal Saxon Academy of Sciences.
Madrid Observatory.
Mannheim Observatory.
Milan, Royal Observatory.
Moncalieri Observatory.
Montpellier, Academy of Sciences.
Moscow Observatory.
Munich, Royal Bavarian Academy of Sciences.
Neuchatel, Society of Sciences.
Palermo, Royal Observatory.
Italian Spectroscopic Society.
Paris, Academy of Sciences.
Bureau des Longitudes.
Comité International de Poids et Mesures.
Dépôt de la Guerre.
Dépôt Général de la Marine.
École Polytechnique.
Société Mathématique de France.
Société Philomathique de France.
Prague Observatory.
Pulkowa Observatory.
Rome, Pontifical Academy dei Lincei.
Royal Academy dei Lincei.
St. Petersburg, Imperial Academy of Sciences.
San Fernando Observatory.
Tiflis Observatory.
Turin, Royal Academy of Sciences.
Vienna, Imperial Academy of Sciences.
Adelaide Observatory.
Philosophical Society.
Batavia Observatory.
Royal Scientific Society.
Boston, American Academy of Arts and Sciences.
Scientific Society.
Bombay Asiatic Society.
Calcutta, Asiatic Society of Bengal.

Cordoba (Argentine Republic) Observatory.
Georgetown College, U.S.
Harvard College Astronomical Observatory.
Melbourne Observatory.
Royal Society of Victoria.
Philadelphia, American Philosophical Society.
Franklin Institute.
Sydney, Government Observatory.
Tasmania, Royal Society.
Toronto, Canadian Institute.
Washington, American Nautical Almanac Office.
Smithsonian Institution.
United States Chief Signal Office.
United States Naval Observatory.
Editor of the American Journal of Science and Arts.
Editor of the American Journal of Mathematics.
Editor of the Analyst.
Editors of the Annalen der Physik.
Editor of the Astronomical Register.
Editor of the Astronomische Nachrichten.
Editor of the Athenæum.
Editors of the Bibliothèque Universelle.
Editors of the Bulletin des Sciences Mathématiques &c.
Editor of Design and Work.
Editor of the Inventors' Record.
Editor of the English Mechanic.
Editor of the Monthly Journal of Science.
Editor of the Naturforscher.
Editor of the Observatory.
Editor of the Revue Scientifique.
Editor of the Science Index.
Editor of Sirius.
Editor of the Telegraphic Journal.

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The Meeting then proceeded to the election of the Officers and Council for the ensuing year, when the following Fellows were elected:—

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J. R. Hind, Esq., F.R.S., Superintendent of the *Nautical Almanac.*

Vice-Presidents.

J. C. Adams, Esq., M.A., LL.D., F.R.S., Lowndean Professor of Astronomy, Cambridge.
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E. J. Stone, Esq., M.A., F.R.S., Radcliffe Observer.
Major G. L. Tupman, R.M.A.

Erratum in the *Monthly Notices*, vol. xxxix. p. 506, line 12: *for* 1843 *read* 1841.

MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. XL. MARCH 12, 1880. No. 5.

J. R. HIND, Esq., F.R.S., President in the Chair.

The Rev. C. S. Harris, M.A., Mepal Rectory, Ely; and
The Rev. J. J. M. Perry, M.A., St. Paul's Vicarage, Alnwick;

were balloted for and duly elected Fellows of the Society.

Sur la Variation du demi-grand Axe des Orbites planétaires.

By M. A. De Gasparis.

Mon seul but dans cette communication est de montrer qu'en appliquant les résultats contenus dans ma Note (*Astron. Nach.*, N° 2270), on arrive à des valeurs connues.

À la fin du temps t,

$$x_1\, y_1\, z_1, \quad x_1'\, y_1'\, z_1$$

sont les coordonnées, et leurs dérivées par rapport au temps, de la masse troublée m_1. Après le temps dt, x_1 aura une variation (dx_1) (elliptique) plus une correction c_{x_1} due à l'intervalle dt après le temps t, et par l'influence de la masse troublante m_2. De même x_1' aura une variation $(dx_1') + c_{x_1'}$.

L'on voit donc qu'à la fin du temps $t + dt$

$$x_1 \text{ croît de } (dx_1) + (c_{x_1})_0''\, dt^2,$$

$$x_1' \text{ croît de } (dx_1') + (c_{x_1})_0''\, dt.$$

Ainsi pour y et z.

Or, on a

$$\frac{1}{a_1} = \frac{2}{r_1} - \frac{dx_1^2 + dy_1^2 + dz_1^2}{k^2 (1 + m_1)\, dt^2};$$

par la différentiation on trouve

$$\frac{da_1}{2a_1^2 dt} = \frac{r_1 dr_1}{r_1^3 dt} + \frac{dx_1 dx_1' + dy_1 dy_1' + dz_1 dz_1'}{k^2 (1+m_1) dt^2},$$

c'est-à-dire,

$$\frac{da_1}{2a_1^2 dt} = \frac{x_1 dx_1 + y_1 dy_1 + z_1 dz_1}{r_1^3 dt} + \frac{dx_1 dx_1' + dy_1 dy_1' + dz_1 dz_1'}{k^2 (1+m_1) dt^2},$$

dans laquelle au lieu de $dx_1\, dx_1'$ il faut mettre $(dx_1) + (c_{x_1})_0''\, dt^2$, et $(dx_1') + (c_{x_1})_0''\, dt$, ainsi pour y et z.

Après quelques réductions qui se présentent en tenant compte de ce qui est dû au mouvement elliptique, et par l'évanouissement des termes de second ordre, l'on trouve

$$\frac{da_1}{2a_1^2 dt} = \frac{dx_1 (c_{x_1})_0'' + dy_1 (c_{y_1})_0'' + dz_1 (c_{z_1})_0''}{k^2 (1+m_1) dt};$$

mais l'on a

$$(c_{x_1})_0'' = m_2 k^2 \left(\frac{x_2 - x_1}{\rho_{12}^3} - \frac{x_2}{r_2^3}\right), \text{ etc.}$$

Donc

$$\frac{da_1}{2a_1^2 dt} = \frac{m_2}{1+m_1} \left\{ \frac{dx_1 (x_2 - x_1) + dy_1 (y_2 - y_1) + dz_1 (z_2 - z_1)}{\rho_{12}^3 dt} - \frac{dx_1 . x_2 + dy_1 . y_2 + dz_1 . z_2}{r_2^3 dt} \right\}$$

$$= \frac{m_2}{1+m_1} . \frac{d}{dt} \left\{ \frac{1}{\rho_{12}} - \frac{x_1 x_2 + y_1 y_2 + z_1 z_2}{r_2^3} \right\},$$

en faisant varier dans cette dernière seulement les coordonnées de m_1.

(Voyez l'excellent mémoire de M. Lespiault, *Théorie géométrique de la Variation des Éléments des Planètes*, 1868, Paris, Gauthier-Villars.)

On a Form of Reading Microscope which appears to combine most of the Advantages of the German and English Forms. By E. J. Stone, M.A., F.R.S.

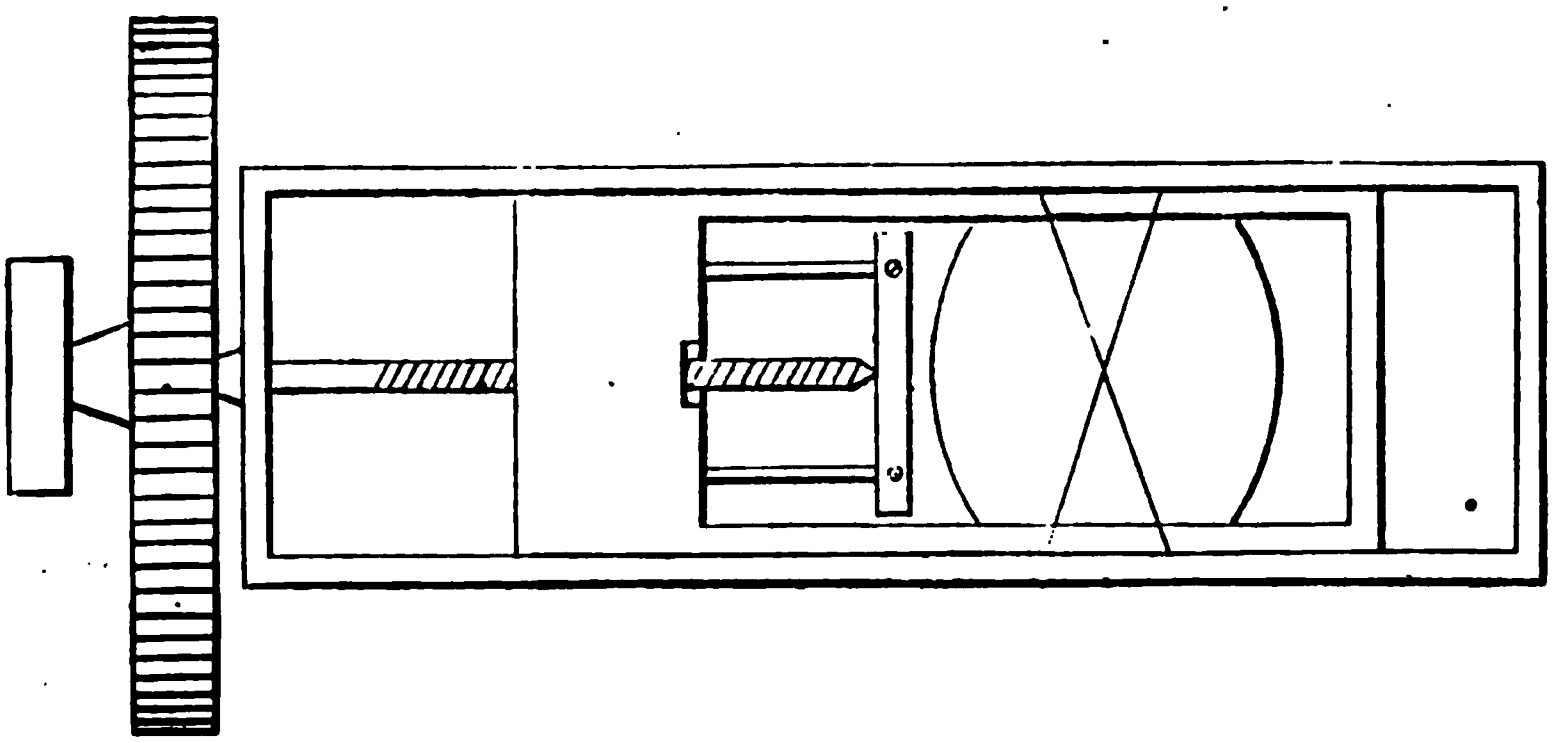
The chief advantage of the English form appears to consist in the sensible coincidence of the axis of the screw of measurement with the line along which the measures are made.

The chief defects appear to be connected with the nature of the bearing from which measurements are made. This bearing is much exposed, and any alteration of its state during the observations directly affects to its full extent the readings.

The chief advantages of the German form appear to arise from the nature of the bearing against which the screw abuts, and from which the measures are taken, and the protection of this bearing from dirt by its inclosure within the microscope.

The chief defect of this form appears to be that the axis of the measuring screw is far removed from the line of measurement, and that any play in the position of the screw produces very sensible errors along the line of measurement.

If these views are correct, there is no great difficulty in making a microscope, as shown in the figure, which shall combine the advantages of both systems.



I have furnished Mr. Simms with a plan which appeared to satisfy the necessary requirements, and he has kindly had a Microscope constructed for exhibition at the Meeting.* I should be glad to hear of objections to the form suggested, with a view to improvements.

* The Microscope was exhibited at the January Meeting.—Ed.

Observations of the Satellites of Mars. By Professor A. Hall.

(*Communicated by Rear-Admiral John Rodgers, U. S. N., Superintendent U. S. Naval Observatory.*)

PHOBOS.

Date.		Wash. M. T.		p	No. Comp.	Wt.	Wash. M. T.		s	No. Comp.	Wt.	Remarks.
		h	m	°			h	m	″			
1879 Oct.	12	13	3.5	52.12	4	3	13	9.5	24.24	2	3	
	15	13	53.5	234.53	2	1	...	...	...	...	...	Extremely faint, sky murky.
	16	12	32.0	235.71	4	4	12	37.0	25.46	2	4	Very bright.
	16	12	59.0	231.45	4	4	13	5.0	24.42	2	4	
	16	13	33.5	225.61	3	3	13	38.5	17.87	2	3	Faint.
	19	13	14.2	54.36	4	2	13	20.7	25.92	2	2	
	19	14	4.7	46.61	4	2	14	11.2	19.49	2	2	Very faint.
	20	12	10.3	55.35	4	2	12	15.3	25.54	2	2	Faint, sky murky.
	20	12	41.3	50.38	4	2	12	47.8	23.77	2	2	
	23	12	47.5	234.54	4	2	12	56.0	25.83	2	2	
	24	11	54.1	234.97	4	2	11	59.6	26.22	2	2	Faint.
	25	11	1.6	231.58	4	3	11	6.1	26.21	2	3	
	26	9	52.2	233.81	4	3	9	57.2	26.72	2	3	Visible with Mars in field.
	26	10	26.7	228.16	4	3	10	32.2	22.92	2	3	
	2	13	56.1	54.74	4	2	14	3.1	27.09	2	2	Faint, moonlight.
	3	12	54.1	53.88	4	4	12	58.6	26.34	2	4	
	3	13	15.1	49.77	4	4	13	18.6	25.55	2	4	
	3	13	44.1	44.76	4	4	13	49.1	21.49	2	4	
	4	11	57.2	52.13	4	4	12	1.7	26.52	2	3	

Date.		Wash. M. T.		p	No. Comp.	Wt.	Wash. M. T.		s	No. Comp.	Wt.	Remarks.
		h	m	°			h	m	″			
Nov.	4	12	52·7	41·48	4	4	12	58·7	18·03	2	4	
	6	10	15·8	48·33	4	2	10	21·3	24·13	2	2	
	6	13	48·3	232·37	4	3	13	53·8	26·52	2	3	
	7	8	57·8	51·00	4	2	9	3·8	25·83	2	2	
	7	12	56·3	229·11	4	3	13	1·3	25·67	2	3	
	12	11	24·6	50·80	4	2	11	30·1	25·21	2	2	Windy.
	13	10	4·1	54·05	4	3	10	8·1	25·89	2	3	
	14	9	2·1	53·19	4	3	9	6·6	25·93	2	3	
	14	12	19·3	238·93	4	2	12	27·1	23·91	2	2	Clouds.
	14	13	6·6	231·28	4	3	13	13·1	25·86	2	3	
	15	11	48·4	233·42	4	4	11	53·4	26·43	2	4	
	16	11	0·8	232·13	4	4	11	5·3	25·91	2	4	Visible with Mars in field.
	20	10	38·5	51·29	4	3	10	44·0	24·76	2	3	
	21	9	25·5	53·72	4	3	9	29·5	25·15	2	3	
	21	13	21·5	232·26	4	3	13	27·0	25·17	2	3	No illumination.
	22	8	5·6	56·95	4	3	8	10·6	24·37	2	3	
	23	11	27·7	229·21	4	3	11	32·7	24·57	2	3	
	24	6	41·3	47·46	4	2	6	46·3	23·52	2	2	
	24	10	11·3	231·84	4	4	10	16·8	24·85	2	4	
	30	7	59·7	48·06	4	3	8	5·2	23·07	2	3	
Dec.	1	6	59·8	48·84	4	3	7	4·8	22·86	2	3	Faint.
	1	10	46·8	228·91	4	3	10	50·8	22·94	2	3	Faint.

Deimos.

Date.	Wash. M. T.	p	No. Comp.	Wt.	Wash. M. T.		No. Comp.	Wt.	Remarks.
	h m	°			h m	″			
1879 Oct. 13	12 10·8	57·11	4	2	12 17·3	59·47	2	2	Very faint, haze.
13	13 37·8	53·85	4	2	13 47·3	63·45	2	2	Very faint, haze.
15	10 36·0	235·78	4	2	10 41·5	63·93	2	2	Faint.
15	11 52·5	232·26	4	2	12 0·0	62·81	2	2	Visible with Mars in field.
15	13 26·5	228·66	4	2	13 32·5	56·90	2	2	Faint, haze.
16	12 45·0	249·30	4	4	12 52·0	37·55	2	4	
16	13 19·0	246·03	4	3	13 25·3	42·93	2	3	Very faint, haze.
20	11 5·8	236·74	4	3	11 9·3	63·29	2	3	
20	12 54·3	232·76	4	3	12 59·3	64·74	2	3	
23	13 6·5	60·17	4	2	13 13·5	54·61	2	2	
24	11 40·6	219·64	4	2	11 45·6	40·08	2	2	Faint.
25	10 52·6	240·46	4	3	10 57·6	57·60	2	3	
25	13 1·6	234·13	4	3	13 7·1	66·81	2	3	
29	10 9·8	229·59	4	1	10 18·8	62·83	2	1	Extremely faint.
Nov. 2	14 28·1	66·60	4	2	14 42·4	51·48	2	2	Faint, clouds and moonlight.
3	10 22·6	230·48	4	4	10 27·1	65·98	2	4	Just visible with Mars in field.
3	13 3·1	220·62	4	4	13 8·1	49·33	2	4	
3	14 1·1	215·59	4	4	14 7·6	39·20	2	4	

Date.	Wash. M. T.		p	No. Comp.	Wt.	Wash. M. T.		s	No. Comp.	Wt.	Remarks.
	h	m	°			h	m	″			
Nov. 4	11	43·2	246·44	4	3	11	49·7	48·86	2	3	
4	13	4·7	240·69	4	4	13	11·2	59·68	2	4	
6	10	1·8	62·76	4	2	10	6·8	55·54	2	2	
6	12	4·8	55·65	4	3	12	10·3	65·27	2	3	
6	13	34·3	51·56	4	3	13	39·8	65·88	2	3	
7	9	10·8	210·32	4	2	9	15·8	36·65	2	2	Faint.
10	9	17·5	48·55	4	2	9	22·0	63·13	2	2	Clock fails and acts badly.
12	7	54·9	223·29	4	2	8	12·1	55·12	2	2	Clouds, clouds.
12	10	15·6	209·29	4	2	...	...	...	...	...	Very faint.
13	7	42·6	246·92	4	2	7	50·6	49·22	2	2	
13	10	13·1	236·62	4	4	10	18·1	64·65	2	4	
14	12	36·6	258·08	4	2	12	55·3	38·14	2	2	Clouds.
15	9	19·9	50·82	4	3	9	25·9	65·30	2	3	Visible with Mars in field.
15	11	35·9	43·35	4	4	11	41·9	55·96	2	4	
16	11	13·3	68·73	4	4	11	18·3	46·93	2	4	
18	11	42·4	235·21	4	2	...	...	...	...	...	Faint, cloudy.
20	8	6·0	58·70	4	2	8	12·0	60·42	2	2	Windy.
20	10	29·0	50·59	4	3	10	33·5	63·36	2	3	
21	13	10·5	64·59	4	3	13	15·5	52·67	2	3	No illumination, lamp fails.

Date	Wash. M. T. h m	p	No. Comp.	Wt.	Wash. M. T. h m	s	No. Comp.	Wt.	Remarks.
Nov. 22	7 52·1	230°·43	4	4	7 57·6	62″·71	2	4	
23	11 7·7	241·70	4	2	11 22·2	57·17	2	2	Clouds.
24	6 51·8	44·67	4	3	6 55·8	56·92	2	3	
29	6 49·1	48·29	4	3	6 55·1	58·50	2	3	Faint.
30	9 53·2	61·23	4	3	9 58·2	53·70	2	3	
Dec. 1	7 11·8	214·59	4	3	7 17·8	41·59	2	3	Very faint.
12	11 47·8	232·56	4	3	11 55·8	51·46	1	3	Faint.
16	7 37·2	228·55	4	2	7 44·2	50·37	1	2	Very faint.
18	5 55·2	44·77	4	3	5 59·7	47·68	2	3	Faint, but distinctly seen.

The outer satellite was found immediately on the first night it was looked for, October 10, and very near the position computed from my elements. The sky soon became cloudy, and no observation could be made on that night. On the next clear night, October 12, I had the pleasure of finding the inner satellite, also near the computed position. This satellite was decidedly brighter than I expected to see it. After this day the sky became covered with a thin haze, which made the observations of the satellites difficult, but on October 16, the sky cleared up and the satellites came out remarkably bright. On this day Professor Holden, who was observing the inner satellites of Saturn, made the following note:—

"Phobos easily seen; it is much brighter than I have ever seen Mimas at elongation, and I should say almost, if not quite, as bright as Enceladus is at elongation."

The observations were made in the following manner. In order to avoid sliding the eye-piece, as was done in 1877, a piece of coloured glass, covering one half the field of view, was inserted in the forward end of the eye-piece, near the micrometer wires. It might be better to silver one half the forward lens of the eye-piece, but an attempt to do this did not give a good result. In making the observations, the planet was placed behind the coloured glass, through which the wire could be seen, and, both objects being kept near the centre of the field, the angle of position and the distance were measured by bisecting the disk of the planet and the satellite. In this way the observations were made in much less time than by sliding the eye-piece. Generally, as will be seen, each observation depends on four settings of the position circle, and four measures of the single distance, the No. of Comp. for the distance being divided by 2, to make them conform to those of 1877. The effect of the coloured glass was to give a slightly different adjustment of the focus for the two parts of the field, but this adjustment was always made for the satellite, or the free part of the field. The eye-piece used was one furnished by the makers of the telescope, and gives a magnifying power of about 400. This eye-piece is not quite achromatic, showing some colour near the edges of the field; and in order to compare the measures made with it and the coloured glass, with those made with the achromatic eye-piece used in 1877, and which has no coloured glass, I measured the distance of Mars from several stars which it passed, with both eye-pieces, the measures being so arranged that the motion of the planet was nearly eliminated for the mean of the times. These measures show no constant difference between the two eye-pieces, and I think the measures of last year will be found as accurate as those of 1877.

The observations have been reduced in the same manner as those of 1877, and the preceding values of p and s are corrected for differential refraction and the figure of the disk.

Before comparing the observations with the elements the perturbations of these satellites produced by the Sun were computed,

and for the secular perturbations, I found values agreeing with those given by Mr. Marth (*Astr. Nach.* No. 2280). If we suppose the perturbations to be really effective, the probability that the satellites at the time of their discovery should move in planes coincident with the equator of the planet is very slight. There can be but little doubt that the view taken by Professor Adams, (*Monthly Notices*, November 1879), and also by M. Tisserand, (*Comptes Rendus de l'Académie des Sciences*, Dec. 8, 1879), will prove to be correct: viz. that these satellites are held nearly in the plane of the equator by the slightly elliptical figure of the planet. For this reason I have neglected the secular perturbations. The coefficients of some of the periodical terms depending on the position of the satellite in its orbit may amount to nearly three minutes of arc; but these terms have very short periods and their effect on the observations is nearly insensible. I have therefore, for the present, neglected these also.

In the case of Phobos, the Besselian auxiliary quantities f, F, g, G, depending on the position of its orbit-plane, were computed from the elements found from the Washington observations of 1877. A preliminary calculation having shown that the periodic time of this satellite is nearly

$$T = 7^{h}\ 39^{m}\ 13^{s}{\cdot}996,$$

the values of u were computed for this value of the period. The observations of Deimos were compared directly with the elements.

The following tables give the results of these comparisons. The residuals Δp have been converted into errors of longitude Δu, and for these errors I assume that the equation of condition is of the form

$$x + by + n = 0,$$

in which $x = \Delta u$ for the epoch Nov. 5.0 Greenwich m.t.; y is the variation of Δu in ten days, and n is the value of the error in longitude resulting from the observation. Strictly, the weights for these equations should depend on the factor for converting the residuals Δp into errors of longitude. This factor varies from 2·9 to 6·2 in the case of Phobos; and from 1·1 to 5·6 for Deimos. But on the other hand, when this factor becomes small the observations were more difficult, and in the present work I have given the weight unity to all the observations, except that of Phobos on Oct. 15, which depends on two comparisons only, made when the satellite was hardly visible; and to this I have given a weight of one-fourth.

The quantities Δp and Δs are given in the sense, Calculated *minus* Observed place. In the last column are given the residuals found by substituting the resulting values of x and y in the equations of condition.

PHOBOS.

		Δp	Δs	Equations of Condition.	Residuals.
Oct.	12	$+1^\circ.31$	$+0''.54$	$x-2.32y-8^\circ.09=0$	$-7^\circ.93$
	15	-2.18	...	$\frac{1}{2}x-1.01y+6.52=0$	$+6.71$
	16	-0.88	-0.18	$x-1.93y+5.14=0$	$+5.59$
	16	-0.20	$+0.07$	$x-1.92y+1.13=0$	$+1.59$
	16	0.00	$+1.23$	$x-1.92y+0.00=0$	$+0.46$
	19	$+0.31$	-0.20	$x-1.62y-1.75=0$	$+1.06$
	19	$+0.78$	$+1.89$	$x-1.62y-3.25=0$	-2.56
	20	-0.41	$+0.18$	$x-1.53y+2.27=0$	$+3.02$
	20	$+0.23$	$+0.89$	$x-1.53y-1.21=0$	-0.46
	23	$+1.04$	$+0.17$	$x-1.23y-5.45=0$	-4.47
	24	-0.77	$+0.07$	$x-1.13y+4.11=0$	$+5.17$
	25	$+1.12$	$+0.16$	$x-1.03y-5.87=0$	-4.74
	26	-0.13	-0.22	$x-0.94y+0.68=0$	$+1.88$
	26	-0.01	$+0.94$	$x-0.93y+0.04=0$	$+1.25$
Nov.	2	-0.27	-0.32	$x-0.22y+1.25=0$	$+2.99$
	3	$+0.44$	$+0.37$	$x-0.13y-2.01=0$	-0.20
	3	$+1.00$	$+0.94$	$x-0.12y-4.38=0$	-2.56
	3	$+0.54$	$+1.09$	$x-0.12y-1.79=0$	$+0.03$
	4	$+1.17$	$+0.31$	$x-0.03y-5.35=0$	-3.46
	4	$+0.32$	$+1.92$	$x-0.02y-0.85=0$	$+1.04$
	6	$+1.35$	$+1.81$	$x+0.16y-5.71=0$	-3.68
	6	-0.37	$+0.20$	$x+0.18y+1.64=0$	$+3.69$
	7	$+0.68$	$+0.79$	$x+0.26y-2.96=0$	-0.85
	7	$+0.96$	$+0.49$	$x+0.28y-4.07=0$	-1.95
	12	$+0.87$	$+1.09$	$x+0.77y-3.55=0$	-1.06
	13	$+0.60$	$+0.23$	$x+0.86y-2.37=0$	$+0.19$
	14	$+1.33$	$+0.19$	$x+0.96y-5.20=0$	-2.57
	14	$+3.19$	-0.88	$x+0.97y-9.18=0$	-6.54
	14	$+0.27$	$+0.29$	$x+0.98y-1.07=0$	$+1.58$
	15	$+1.18$	-0.44	$x+1.07y-4.55=0$	-1.83
	16	-0.58	$+0.11$	$x+1.17y+2.26=0$	$+5.05$
	20	$+0.43$	$+0.72$	$x+1.56y-1.60=0$	$+1.49$
	21	$+0.18$	$+0.14$	$x+1.66y-0.66=0$	$+2.50$
	21	$+0.67$	$+0.31$	$x+1.68y-2.48=0$	$+0.70$
	22	$+0.92$	-0.29	$x+1.75y-3.00=0$	$+0.23$
	23	$+0.84$	$+0.15$	$x+1.87y-3.00=0$	$+0.32$
	24	$+1.27$	$+0.70$	$x+1.95y-4.39=0$	-1.01
	24	$+1.18$	$+0.04$	$x+1.96y-4.23=0$	-0.84

	Δp	Δs	Equations of Condition.	Residuals.
Nov. 30	+1°·76	+0″·36	$x+2·55y-6°·53=0$	−2°·69
Dec. 1	+0·38	+0·25	$x+2·65y-1·27=0$	+2·64
1	+0·92	+0·19	$x+2·67y-3·08=0$	+0·85

DEIMOS.

	Δp	Δs	Equations of Condition.	Residuals.
Oct. 13	+0°·24	+0″·48	$x-2·23y-1°·24=0$	−2°·23
13	+0·34	−0·08	$x-2·22y-1·92=0$	−2·91
15	−0·65	−1·04	$x-2·04y+3·58=0$	+2·58
15	+0·12	−0·82	$x-2·03y-0·63=0$	−1·63
15	−0·07	−1·57	$x-2·02y+0·30=0$	−0·70
16	−0·73	+1·35	$x-1·93y+1·46=0$	+0·46
16	−0·49	+1·24	$x-1·92y+1·28=0$	+0·28
20	−0·26	−0·06	$x-1·53y+1·31=0$	+0·30
20	−0·42	−0·83	$x-1·52y+2·16=0$	+1·15
23	+1·30	+1·52	$x-1·22y-4·71=0$	−5·74
24	−1·91	−1·87	$x-1·13y+3·30=0$	+2·27
25	−0·01	+1·03	$x-1·03y+0·04=0$	−1·00
25	+0·22	−0·84	$x-1·02y-1·09=0$	−2·13
29	−1·19	−2·59	$x-0·64y+4·68=0$	+3·63
Nov. 2	−1·12	+0·88	$x-0·22y+2·87=0$	+1·80
3	−0·37	−1·11	$x-0·14y+1·53=0$	+0·46
3	−0·35	+0·23	$x-0·12y+0·82=0$	−0·25
3	−1·91	−1·98	$x-0·12y+3·13=0$	+2·06
4	−0·42	+1·18	$x-0·03y+0·99=0$	−0·09
4	−0·35	+0·82	$x-0·03y+1·21=0$	+0·13
6	−0·65	+1·55	$x+0·16y+1·55=0$	+0·46
6	−0·28	+1·07	$x+0·17y+0·91=0$	−0·18
6	−0·43	+0·22	$x+0·18y-1·77=0$	+0·68
7	−1·16	−2·90	$x+0·26y+1·27=0$	+0·18
10	−0·62	−0·18	$x+0·56y+3·42=0$	+2·31
12	−0·20	−1·70	$x+0·75y+0·54=0$	−0·57
12	−2·15	...	$x+0·76y+2·27=0$	+1·16
13	−0·57	+1·69	$x+0·85y+1·28=0$	+0·17
13	−0·32	−0·02	$x+0·86y+1·18=0$	+0·06
14	−1·65	+1·90	$x+0·97y+2·16=0$	+1·04
15	+0·03	−0·56	$x+1·06y-0·11=0$	−1·23
15	−0·80	−1·83	$x+1·07y+2·10=0$	+0·97
16	−1·15	[illegible]1·79	$x+1·17y+2·37=0$	+1·24

	Δp	Δs	Equations of Condition.	Residuals.
	°	"	°	°
Nov. 18	−0·29	...	$x+1{\cdot}37y+1{\cdot}04=0$	−0·10
20	−0·23	+0·31	$x+1{\cdot}56y+0{\cdot}79=0$	−0·35
20	−0·22	−0·33	$x+1{\cdot}57y+1{\cdot}15=0$	0·00
21	−1·12	+1·34	$x+1{\cdot}68y+2{\cdot}86=0$	+1·71
22	−0·11	−0·48	$x+1{\cdot}75y+0{\cdot}38=0$	−0·77
23	−0·15	−0·29	$x+1{\cdot}87y-0{\cdot}42=0$	−1·58
24	−0·26	−1·26	$x+1{\cdot}95y+0{\cdot}72=0$	−0·44
29	+0·18	−0·50	$x+2{\cdot}45y-0{\cdot}57=0$	−1·75
30	−0·75	+1·04	$x+2{\cdot}56y+2{\cdot}11=0$	+0·90
Dec. 1	−1·09	+0·81	$x+2{\cdot}65y+1{\cdot}81=0$	+0·61
12	−0·46	+1·15	$x+3{\cdot}77y+1{\cdot}45=0$	+0·22
16	−0·02	−1·11	$x+4{\cdot}15y+0{\cdot}06=0$	−1·22
18	+0·26	−1·85	$x+4{\cdot}35y-0{\cdot}71=0$	−1·98

The normal equations for Phobos are

$$+40{\cdot}25x+\ 6{\cdot}16y-81{\cdot}60=0,$$
$$+\ 6{\cdot}16x+82{\cdot}83y-74{\cdot}28=0;$$

the solution gives

$$x=\Delta u=+\overset{\circ}{1}{\cdot}912\pm\overset{\circ}{0}{\cdot}3429.$$

The probable error of a single value of Δu is $\pm\ 2^{\circ}{\cdot}163$. We have therefore the value of the periodic time of this satellite

$$T=\overset{h}{7}\ \overset{m}{39}\ \overset{s}{13}{\cdot}9376\pm\overset{s}{0}{\cdot}01382.$$

The longitude for the epoch Nov. 5.0, 1879, is

$$u=68\overset{\circ}{\cdot}87\pm\overset{\circ}{0}{\cdot}343.$$

The normal equations for Deimos are

$$+46{\cdot}00x+\ 17{\cdot}24y+50{\cdot}45=0,$$
$$+17{\cdot}24x+137{\cdot}20y+24{\cdot}74=0;$$

the solution gives

$$x=\Delta u=-\overset{\circ}{1}{\cdot}080\pm\overset{\circ}{0}{\cdot}1671,$$

and the probable error for a single value of Δu is $\pm\ 1^{\circ}{\cdot}106$. Hence the periodic time of this satellite is

$$T=\overset{h}{30}\ \overset{m}{17}\ \overset{s}{54}{\cdot}377\pm\overset{s}{0}{\cdot}0949;$$

and the longitude for the epoch is

$$u = 322^\circ.91 \pm 0^\circ.167.$$

After December 1 the weather, which had been unusually favourable, became bad, and the image of the planet was so blazing and unsteady that no further observations of Phobos were made. Although the brighter of the satellites, Phobos is always so near the limb of the planet that it is much more affected by a poor image of Mars. This satellite was seen last on December 16. On December 18, Deimos could be observed with tolerable accuracy, and this satellite was seen last, with certainty, on December 26, but it was too faint to observe. When all the observations of these satellites are published, I hope to undertake their discussion and the determination of new elements.

Although these satellites were brighter than I expected to see them in the Opposition of 1879, still the formula

$$\text{Brightness} = \frac{C}{r^2\Delta^2}$$

will probably represent the brightness for different Oppositions with a fair approximation to the truth. Assuming this brightness to be unity on October 1, 1877, the following table gives the brightness for several dates in 1879 and in 1881:—

Date.	Brightness.
1879, Sept. 21	0·491
Nov. 5	0·730
Dec. 12	0·423
18	0·371
26	0·335
1881, Dec. 9	0·389
31	0·384

These satellites will therefore be visible for several weeks in December 1881, in the Washington 26-inch Refractor, the declination of the planet being + 26°. Unfortunately this Opposition occurs at a season of the year when the weather is generally very unfavourable for making observations at the northern observatories. If such a powerful telescope as Mr. Commons' reflector could be mounted at Malta or Madeira, perhaps a series of observations of these satellites might be secured in the Opposition of 1881. Such a position would also be favourable for observing the satellites of Saturn, whose oppositions will occur in the autumn, and which is coming into a good position for observations of its Ring and faint satellites.

As the Washington observations of Hyperion, the faint satellite of Saturn, have been referred to, I will say that I observed this satellite on 32 nights in 1878, and on 28 nights in 1879.

The Washington observations of the satellites of Saturn need revision and a complete reduction, and as I have not found time to do this, their publication is delayed; but anyone who needs my observations of Hyperion can have them.

1880, *January* 27.
Washington, U.S.

Observations of Mimas and of an Occultation of Rhea in 1879 *with the* 26-*inch Equatoreal at Washington.* By Prof. Edward S. Holden, U.S.N.

(*Communicated by Rear-Admiral John Rodgers, U.S.N., Superintendent U.S. Naval Observatory.*)

1879. *Sept.* 29.

W. m. t. p (estimated) 270°. Mag. Power 600 A; Wt. 4.

$10^h\ 27^m$. $p = 270°\cdot2$ (4).
$10^h\ 59^m$. $s = 28''\cdot62$ (4); satellite very faint; moonlight.

1879. *Sept.* 30.

p (estimated) 285°. Mag. Power 600 A; Wt. 4.

$10^h\ 27^m$. $(s) = 24''\cdot14$ (4) distance measured in the line of major axis of the ring; nearly full moon about 10° distant.

1879. *Sept.* 30.

Mag. Power 800 A.

$10^h\ 50^m$. I do not think it is up yet, to *s. p.* the end of the ring, but it is very nearly up.
$10^h\ 54^m$. I suspect I have made a mistake and that it was a little past at $10^h\ 50^m$; it seems to be past now. The seeing is growing quite unsteady (Wt. 2) and *Mimas* is v. F.
$10^h\ 59^m$. *Mimas* not seen after this with eye-pieces 400 A and 600 A.

1879. *Sept.* 30.

Mag. Power 400 A; Wt. 2.

[Occultation (disappearance) of *Rhea*.

$11^h\ 3^m$. *Rhea* is still visible, but very close to the limb of ball.
$11^h\ 10^m$. I no longer see any dark space between the satellite and the ball, but the planet's limb is unsteady.
$11^h\ 13^m$. Satellite seen less than half the time.
$11^h\ 15^m_-$. Satellite nearly smooth with the limb. Seeing much better (Wt. 4).

W. m. t.

11^h 22 . Satellite seen perhaps a third of the time.

11^h 23^m. Seen for an instant.

11^h 24^m. " "

Rhea not seen after 11 24^m. Very satisfactory observation.

The satellite seemed to grow smaller and more intense white in colour as it came close to the ball of *Saturn*. The colour may have been due to contrast with that portion of the ball near which it impinged. This was the south polar belt, which was a very dark dull olive.

The satellite certainly appeared smaller than usual and smaller than *Dione*, which was near. It is usually larger than the latter satellite.]

1879. *Oct.* 5.

Mag. Power 800 A; Wt. 4.

9^h 41^m. *Mimas* is just visible approaching *s.f.* conjunction—and very faint.

10^h 11^m. The sky is hazy, so much so that *Enceladus* near elongation (E) is quite faint, and the larger satellites show that there is a fog or haze around them.

Under these conditions I am not absolutely sure that this is *Mimas*, but if it is, it is just past, or a little past *s.f.* conjunction at 10^h 11^m W. m. t.

1879. *Oct.* 11.

12^h 20^m. *Mimas* first seen at this time (on account of clear space through the thin clouds or fog) and it is past the *n. p.* conjunction with end of ring by several degrees.

1879. *Oct.* 12.

Mag. Power 800 A; Wt. 2.

10^h 27^m. *Mimas* not yet up to *n. p.* Conjunction with end of ring.

10^h 35^m. A little beyond the principal division. *Mimas* not seen again till 10^h 55^m, and then only for a moment.

10^h 56^m. It is probably past. Not seen again till

11^h 8^m. when it was certainly past. Only seen for a moment.

11^h 12^m. Certainly past (Hall).

Whole observation unsatisfactory, on account of unsteady atmosphere.

1879. *Oct.* 14.

Mag. Power 800 A; Wt. 3. Very hazy.

8^h 27^m. *Mimas* is, I fear, past [*n.p.*] conjunction a very little. This is the first I have seen of it. The planet is low

W. m. t.

and the sky hazy. *Mimas* cannot be seen with less than 800 mag. power, and 800 makes the images unsteady.

$8^h\ 32^m$. *Mimas* is certainly past, though I can only see it by glimpses. *Enceladus* near elongation is extremely faint.

$9^h\ 13^m$. *Mimas* is just barely seen with 800 A, and not with 600 A.

1879. *Oct.* 16.

p (estimated) 260°. Mag. Power 800 A; Wt. 3.

$10^h\ 27^m.5$. *Mimas* is pretty constantly seen, but is very faint. The sky is hazy.

$10^h\ 51^m.5$. *Mimas* about 4″ *west* of the meridian through the end of the ring.

$11^h\ 4^m.5$. Nearly up.

$11^h\ 13^m.5$. Up.

$11^h\ 17^m.5$. Up, or possibly just past.

Mimas not seen after $11^h\ 18^m$, and only seen four times between that and $10^h\ 50^m$. Sky very hazy and observation unsatisfactory in the highest degree.

The above observations were all that could be obtained during the Opposition, using diligence. They are copied literally from the observing books.

Observations of Saturn's Satellites, made at Mr. E. Crossley's Observatory, Bermerside, Halifax. By Mr. J. Gledhill, F.R.A.S.

The instrument with which the following observations were made is an Equatoreal having an aperture of $9\frac{1}{3}$ inches.

Nov. 28th, 1879. Good definition. Power 282. The web was kept bisecting the ball.

Rhea in conjunction with the centre of the ball.

h. m.		
10 40	G.M.T.	Conjunction has not yet occurred.
48		In conjunction.
11 0		Certainly past conjunction.

Clouds prevented the observation of Tethys and Enceladus.

Nov. 29th, 1879. Good definition. Power 282. The web was set perpendicular to the plane of the ring.

Tethys in conjunction with west end of ring: *np*.

	h. m.	
(*a*)	7 20	Not yet in conjunction.
(*b*)	25	In conjunction.
(*c*)	30	Certainly just past.

(*a*), (*c*) good; (*b*) probably the best observation.

Dec. 6th. *Dione* in conjunction with end of ring: *nf*.

h. m.	
9 30	Not up yet.
35	In conjunction.
40	Past.

Power 282. Pretty good definition.

Notes on the Physical Appearance of Jupiter: being part of the Paper "Observations of Eclipses, Occultations and Phenomena of Jupiter's Satellites made with the 8-inch Equatoreal (Cooke) at the Adelaide Observatory, South Australia" (*ante*, pp. 170–176). By C. Todd, Esq., Director of the Observatory.

1878, July 5, 10.25 P.M.—North of the equatoreal belt was a well-defined streak of dark salmon colour; the dark belt on the south of the equator was covered with scattered white-looking clouds, to the south of which was a bright belt, having a remarkable bright spot, like a cumulus cloud, on the southern edge; on the meridian, to the east of which, at some distance, was a well-defined rectangular indentation.—(T.)

July 21, 9.45 P.M.—The appearance of the salmon-coloured streak was as if viewed through a thin veil of cloud. On the south edge of the equatoreal cloud belt was an oval brilliantly white cloud, nearly on the equator and exactly mid planet.—(T.)

July 26, 8.35 P.M.—At three minutes after observation No. 23, the shadow was about its own diameter inside the limb of planet, and immediately following the satellite—in fact, touching it to all appearance. The satellite entered on the dark band north of the equator, the central portions of which were of a bright sienna, or a nearly rose colour. The equatoreal cloud belt was generally diffused, but its central parts were very dense, having the appearance and shape of cumulus clouds. The dark belts have still the appearance as if seen through hazy cloud or fog. The satellite when it first entered planet appeared large and bright, but became gradually smaller—still bright—and the shadow seemed larger than the satellite.—(R.)

July 29, 8.35 P.M.—The equatoreal belt is not so broad or so well defined as usual, and consists of comparatively narrow streaks of broken cloud on the southern edge; the northern dark belt is not at all well marked, and is of a rose salmon colour, especially on the north side; the southern dark belt is broken by streaks of white cloud, and is not so well marked or defined, although broader than the northern belt, and is less highly coloured.—(T.)

July 29, 10.50 P.M.—No. I satellite and its shadow traversing the north portion of the equatoreal belt, and almost touching each other; the shadow

a little to the north of satellite, the south pole of the shadow being in line with the equator of the satellite. The equatoreal belt is broader than before, and the north brown belt darker; the southern dark belt is broken by a white streak, stretching nearly across the planet, and is hazy, and not so red or brown as the north belt.—(T.)

August 9, 7.50 P.M.—*Jupiter* well defined. The long, oval, dark reddish patch south of the southern belt, noticed on the 4th, still visible, but I think slightly longer. The bright patch on bright belt not distinguishable.—(T.)

August 13, 10.0 P.M.—The oval red space on southern bright belt, noted on the 4th, still visible, but not the bright spot on the equatoreal cloud belt, but a bright spot very like it precedes it several Jovian hours, being near the western edge of the planet when the red space was rather more than half visible on the opposite or eastern side of the planet.—(T.)

October 1, 10.0 P.M.—The equatoreal cloud belt was dense, and the occultation was complete at limb. It may be here remarked, as a general note, that the satellites, as a rule, are not visible through the dense white cloud belts, but are seen at occultation through the broken cloud belts and brown belts.—(T.)

October 2, 10.0 P.M.—The southern dark belt this year has generally been covered with a thin film of white cloud, and has seldom the decided salmon tint noticed in previous years, occasionally assuming a greyish green. The equatoreal bright belt to-night resembled a bank of cumulus cloud with its base toward the north and its crest toward the south. The northern dark belt is narrow, and of a salmon tint, varied by dark streaks and patches.—(T.)

Phenomena of Jupiter's Satellites observed at Mr. E. Crossley's Observatory, Bermerside, Halifax. By Mr. J. Gledhill, F.R.A.S.

The instrument with which the following observations were made is an Equatoreal having an aperture of $9\frac{1}{3}$ inches. The power used was almost invariably 240: occasionally powers 330 and 470 were applied:—

Day of Obsn.	Satellite	Phenomenon		G.M.T. of Obsn. h.	m.	s.	Time from N.A. h.	m.	s.	Remarks
1879										
Aug. 30	I.	Oc. R.	first seen	10	6					
			half out		7		10	8		very bad definition
			just off		8	30				
31	III.	Tr. E.	first contact	11	11					
			half off		15		11	19		very bad definition
			just off		19					
Sept. 14	I.	Tr. I.	first contact	8	29					
			bisection		30	30	8	29		bad definition
			second contact		32					
14	I.	Sh. I.	bisected	8	52	30	8	51		bad definition
			fully on disk		55					
14	II.	Oc. D.	first contact	10	14		10	15		bad definition
			disappearance		18					

Day of Obsn.		Satellite	Phenomenon		G.M.T. of Obsn. h. m. s.	Time from N.A. h. m. s.	Remarks
1879							
Sept.	29	I.	Oc. D.	first contact	9 13	9 15	very fair obsn.
				bisection	15		very fair obsn.
				disappearance	17		very good
	29	I.	Ec. R.	first seen	12 15 50	12 15 8	fair
Oct.	6	I.	Oc. D.	first contact	11 1	11 2	
				disappearance	11 3		
	9	II.	Oc. D.	disappeared	6 5	6 15	bad definition
	14	I.	Tr. I.	first contact	9 59	9 59	fair observation
				bisection	10 1		fair observation
				second contact	10 2		fair observation
	14	I.	Sh. I.	fully on disk	11 3	10 59	very bad defn.
	15	I.	Ec. R.	first seen	10 34 43	10 35 8	good obsn.
				fully out	36		uncertain
	20	III.	Tr. I.	first contact	7 32 48	7 32	much boiling
				bisection	36		
		III.	Tr. E.	inner contact	11 0	11 7	fair observation
				bisection	3		uncertain
				external contact	8		fair
	25	II.	Tr. I.	just on disk	6 24	6 3	very bad defn.
	25	II.	Sh. I.	first seen	8 23 30	8 22	
				half on	24		
				fully on	25		good obsn.
	25	II.	Tr. E.	inner contact	8 48 30	8 55	fair observation
				bisection	50 30		fair observation
				external contact	53		fair observation
	25	IV.	Ec. D.	began to fade	9 25	9 34 29	misty, but still
				gone	27		
	25	II	Sh. E.	off disk	11 7	11 14	
Nov.	1	I.	Sh. E.	just off	6 7	6 6	very bad defn.
	1	II.	Tr. I.	just within	8 33	8 32	very bad defn.
	1	II.	Sh. I.	just within	11 3	11 1	very bad defn.
	1	II.	Tr. E.	just off	11 25	11 24	very bad defn.
	2	IV	Tr. I.	certainly on disk	8 20	8 30	
	7	I.	Oc. D.	first contact		7 17	fair observation
				bisection	15 30		fair observation
				just gone	17		good obsn.
	12	II.	Sh. E.	first contact	5 35	5 48	very bad defn.

Day of Obsn.	Satellite	Phenomenon		G.M.T. of Obsn.	Time from N.A.	Remarks
1879				h. m. s.	h. m. s.	
Nov. 14	III.	Oc. D.	first contact	8 32	8 38	bad definition
			bisection	35		
			just gone	39		
14	I.	Oc. D.	first contact	9 8	9 11	good obsn.
			bisection	10		not good
			just gone	11		good
15	I.	Tr. I.	first contact	6 19	6 19	good obsn.
			bisection	21		good?
			just within	22		good
15	I.	Sh. I.	just within	7 40	7 39	
15	I.	Tr. E.	inner contact	8 36	8 38	uncertain obsn.
			bisection	38		good
			outer contact	40		good
23	I.	Ec. R.	first seen	9 9 53	9 11 19	good obsn.
*			full orb	11 30		good obsn.
24	I.	Tr. E.	outer contact	5 0	5 1	cloudy
Dec. 3	II.	Tr. I.	first contact	8 10	8 8	pretty good definition
			bisection	12		
			just within	14		
6	III.	Ec. R.	first seen	5 28	5 30 42	bad definition
			full orb	31		
6	IV.	Sh. I.	just within	8 58	8 56	very bad sky
8	I.	Tr. I.	just within	6 38	6 35	good defn.
8	I.	Sh. I.	just within	7 56	7 55	
8	I.	Tr. E.	inner contact	8 51	8 54	pretty good sky
			external contact	56		
12	II.	Oc. D.	first contact	4 58 30	4 59	good sky
			half gone	5 0 30		
			just gone	2 30		
21	II.	Sh. I.	just within	5 29	5 25	very bad defn.
21	II.	Tr. E.	inner contact	5 45	5 46	
			bisection	47		
			outer contact	50		

* See foot-note, p. 291.

A few phenomena observed with the same instrument in the years 1875, 1876, and 1877 are subjoined.

Day of Obsn.		Satellite	Phenomenon		G.M.T. of Obsn.	Time from N.A.	Remarks
1875					h. m. s.	h. m. s.	
April	13	I.	Ec. D.	began to fade	9 17 53		good
				half gone	18 18	9 19 11·6	not so good
				disappeared	19 18		good
		I.	Oc. R.	first seen	11 29 30		bad definition
				half out	31	11 34	bad definition
				just clear	33		bad definition
	19	I.	Tr. I.	first contact	13 46 30		not good
				bisection	48	13 49	not good
				just within	49 30		pretty fair
	19	I.	Sh. I.	fully on disk	13 54	13 52	uncertain
	28	I.	Tr. I.	first contact	9 57 57		uncertain
				bisection	59 27	9 59	uncertain
				just within	10 1 27		uncertain
	28	I.	Sh. I.	bisected	10 16	10 15	defn. very bad
	28	I.	Tr. E.	first contact	12 6 27		definition bad
				bisection	8 27	12 11	
				just clear	11 27		the best obsn.
May	14	I.	Tr. E.	first contact	10 4 4		good
				bisection	5 4	10 8	fair
				just off	7 34		good
		I.	Sh. E.	first contact	10 42 34		fair
				bisection	44 34	10 46	fair
				last contact	46 34		fair
	22	I.	Ec. R.	first seen	9 53 49		good
				half out	54 28	9 54 3·6	uncertain
				full orb	55 40		good
June	13	I.	Tr. I.	first contact	9 33 28		pretty good
				bisection	35 58	9 35	uncertain
				last contact	37 58		pretty good
	13	I.	Sh. I.	bisection	10 40	10 42	bad sky
				just within	43		bad sky
	14	I.	Ec. R.	first seen	10 5 39	10 5 48·5	good
	14	II.	Ec. R.	first seen	10 49 14	10 52 0·6	good
				full orb	52		good

Day of Obsn.		Satellite	Phenomenon		G.M.T. of Obsn.			Time from N.A.			Remarks
1876					h.	m.	s.	h.	m.	s.	
April	7	I.	Sh. I.	inner contact	12	57		12	56		bad definition
	7	I.	Tr. I.	inner contact	13	49		13	48		bad definition
	7	I.	Sh. E.	just off	15	9		15	8		bad definition
	7	I.	Tr. E.	just off	16	0		15	59		bad definition
	15	I.	Ec. D.	just gone	12	12		12	19	1·5	not good
	26	III.	Ec. D.	just gone	11	50		11	42	3.6	
	26	III.	Ec. R.	first seen	13	24		13	23	45·9	probably some seconds late
	26	III.	Oc. D.	just gone	13	50		13	48		
	30	I.	Sh. I.	inner contact	13	7		13	5		bad definition
	30	I.	Tr. I.	inner contact	13	31		13	30		bad definition
	30	I.	Sh. E.	just off	15	19		15	17		bad definition
May	2	I.	Sh. E.	just off	9	48		9	46		fair definition
	2	I.	Tr. E.	just off	10	9		10	7		
	7	I.	Sh. I.	just within	15	0		14	59		very bad sky
	7	I.	Tr. I.	just within	15	15		15	14		very bad sky
	8	I.	Ec. D.	first seen	12	23		12	21	55·9	very bad sky
	9	I.	Sh. I.	inner contact	9	29		9	27		very bad sky
	9	I.	Tr. I.	inner contact	9	41		9	39		very bad sky
	9	I.	Sh. E.	just off	11	42		11	40		very bad sky
	9	I.	Tr. E.	just off	11	53		11	51		very bad sky
June	1	I.	Sh. I.	inner contact	9	40		9	39		bad definition
	1	I.	Tr. E.	inner contact	11	26		11	29		bad definition
				outer contact		29					bad definition
	1	I.	Sh. E.	just off	11	51		11	51		bad definition
	16	I.	Oc. D.	first contact	10	4		10	7		uncertain
				gone		8	46				pretty good
July	2	I.	Ec. R.	first seen	11	15	13	11	15	17·7	good
				full orb			7				

Day of Obsn.		Satellite	Phenomenon		G.M.T. of Obsn.			Time from N.A.			Remarks
1877					h.	m.	s.	h.	m.	s.	
July	1	II.	Tr. I.	first contact	10	12		10	14		bad definition
	*			bisection		14					
				inner contact		16					
	26	II.	Tr. E.	just off	9	1		9	3		very bad sky

* During the observations marked thus * Mr. Crossley was at the telescope and I watched the chronometer.

Day of Obsn.	Satellite	Phenomenon	G.M.T. of Obsn.	Time from N.A.	Remarks
1878			h. m. s.	h. m. s.	
Sept. 10	III.	Oc. D. first contact	9 0	9 4	a little uncertain
		half gone	4		very fair
		gone	8		good
	I.	Ec. R. first seen	10 39 1	10 38 33	good
		half out	39 49		uncertain
		fully out	40 49		uncertain
Oct. 26	I.	Oc. D. first contact	7 29	7 31	bad sky

Observations of the exterior Satellite of Mars at the Oxford University Observatory. By Prof. C Pritchard.

The accompanying observations of the exterior satellite of Mars were made with the Grubb Equatoreal of 12¼ inches' aperture, the power employed 125. During the observations the planet was nearly hidden behind a bar which could be turned with the Position Circle. As no illumination of the field could be permitted, the position angles were determined by turning the bar till it was at right angles to the line joining the satellite and the centre of the planet, the judgment being assisted by the small segment of the planet left uncovered. It was thought that this angle could be judged with some degree of accuracy. The distances are estimations, assisted by a knowledge of the width of the bar and of the diameter of the planet.

Oxford Sidereal Time.	Position Angle	Distance.	Remarks.
h m	°	″	
Nov. 5 0 57·2	58	62	The satellite kept pretty steadily in view. The error of the position angle probably within 5°. Definition very favourable. Some moonlight.
1 0·4	55	60	
1 45·5	55	65	
Nov. 11 22 27	223	55	False scattered light beyond the limb of the planet very troublesome. The definition of the planet not so good as on the 5th inst., and the observation less satisfactory.
23 0	225	50	
23 15	225	48	
Nov. 15 3 29	60	60	It was thought at times that there was an object on the other side of the planet at about half of this distance.
4 58	57	62	

These observations were made by Mr. W. Plummer.

Oxford,
1880, *March* 11.

Note on Mr. Maxwell Hall's Paper on the Nebula in the Pleiades. By Dr. C. Wolf.

Je trouve dans le dernier No. des *Monthly Notices*, qui vient de m'arriver, une note de M. Maxwell Hall sur la nébuleuse des Pléiades, et la requête qu'il adresse en terminant aux astronomes de vouloir bien s'occuper de nouveau de cet intéressant objet. Je crois que la réponse complète à la demande de M. Hall est donnée dans le mémoire, 'Description du Groupe des Pléiades,' que j'ai publié en 1877 dans les *Annales de l'Observatoire de Paris*, et dont j'ai offert un exemplaire à la Société Royale Astronomique.

J'ai résumé dans ce mémoire l'historique complet de la nébuleuse en question, et montré pourquoi les observateurs ont tant différé d'opinion sur sa forme et sur sa visibilité. Comme le montre ma carte des Pléiades, la Nébuleuse, telle que Goldschmidt l'a vue le premier dès 1864, *entoure complètement* le groupe de ces étoiles, qui brillent sur le fond noir du ciel, comme dans un trou percé au milieu de la nébulosité. Elle est médiocrement brillante, et l'absence de contours bien tranchés, sauf dans la région de Mérope, la rend difficile à voir même par un ciel très pur. Le moyen qui m'a le mieux réussi consiste à regarder les fils d'araignée du réticule, en même temps qu'on déplace peu à peu la lunette. Sur le fond noir du ciel, au milieu du groupe des Pléiades, les fils disparaissent complètement; ils se détachent en noir dès qu'une portion du champ est envahie par la nébulosité. On comprend donc comment, avec un puissant instrument dont le champ est toujours très restreint, cette nébulosité échappe aisément à la vue. La plus légère brume suffit d'ailleurs à égaliser l'éclairement du fond du ciel et à faire disparaître toute trace de la nébuleuse, lors même qu'elle ne voile pas les plus faibles étoiles de 13me et de 14me grandeur. Toutes mes observations ont été faites avec un objectif de 33 centimètres d'ouverture.

Les diverses portions de la nébuleuse sont très inégalement brillantes. La plus facile à voir est le grand éventail dont Mérope occupe le sommet: c'est la région vue par Tempel et la plupart des observateurs. Elle se prolonge vers Electre et au-delà de Cœleno, comme l'a dit Schiaparelli. Dès 1768, Teaurat paraît en avoir aperçu un autre lambeau, qui forme un second maximum auprès des deux anonymes 31 et 32 de Bessel.

Les différences d'aspect de cette nébuleuse suivant l'état du ciel et l'étendue du champ de la lunette employée, ont fait penser qu'elle pouvait être variable. J'avais moi-même à l'origine énoncé cette opinion. Il est aujourd'hui certain pour moi qu'elle n'a pas varié depuis 1864.

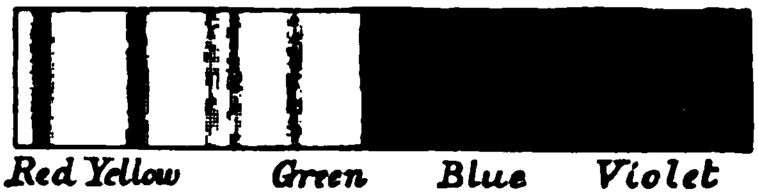
Paris,
1880, 11 *Janvier.*

Note on the Spectrum of Mr. Baxendell's new Star in Canis Minor. By Dr. H. C. Vogel.

(*Extract from a letter addressed to the Foreign Secretary.*)

I permit myself to communicate to you my observations concerning the new star found by Mr. Baxendell.

The new star was observed by Dr. Müller and me on Dec. 16, 1879, and on Feb. 7, 1880. The colour is reddish yellow, magnitude on Feb. $7 = 9^{m}\cdot 3$. The spectrum is highly interesting. The blue and violet are strongly absorbed, so that at the first glance only half the spectrum seems to exist. In the blue and violet three dark, broad bands are plainly recognised. In the green and red there appear also to be bands, but they are too faint to be certainly distinguished. On Feb. 7 I examined also the spectrum of the preceding star B.D. + 8°, 1846, which has a yellow colour, and found that this star has a similar spectrum to that of the new star, except that it is not so clearly developed.



The star B.D. + 8°, 1848, gives a continuous spectrum without bars or bands.

I am under the necessity of supposing that in the spectroscopical observation Mr. Lohse has made a mistake, and in place of the new star has observed No. 1848.

Astrophysical Observatory, Potsdam,
1880, *Feb.* 10.

Supplementary Note to the Paper of Dr. Vogel. By Lord Lindsay.

Baxendell's Nova has been repeatedly observed at Dunecht with the spectroscope. It was not considered an interesting object.

Herr G. L. Lohse measured the limits of the spectrum on March 7, to be—

	Spectrum begins	Spectrum ends
Nova	647^{mmm}	456^{mmm}

with a narrow slit. Again, with a wide slit, on March 9,

Nova	660	432

In the blue, near F, is one broad darkish band.

On March 9 the magnitude was estimated at M. 9·5, certainly not brighter.

Observations of the Great Southern Comet 1880, I. *made at Monte Video.* By Lieut. B. Gwynne.

(*Communicated by the Secretaries.*)

The accompanying diagrams* are intended to show the appearance and relative position of the tail of a large comet with reference to some fixed stars near which it was situated, observed at Monte Video (Rio de la Plata) on the 1st, 2nd, 3rd, 4th and 8th February, the overcast state of the sky preventing my observing its position on the intermediate evenings (4th to 8th).

On the 1st February, after a long twilight, the comet's tail was observed in a S.S.W. direction, having an altitude of about 20° above the horizon.

Subsequent observations show the comet to have an apparent motion of 2° to the westward in twenty-four hours.

The brilliancy of the comet's tail was, on the 1st, about that of the Milky Way on a clear frosty night, and each succeeding day showed a diminution in brilliancy until the 8th, when it was difficult to trace the direction of the tail, although the night was favourably dark.

No vestige of a nucleus could be detected with a ship's spyglass (sufficiently powerful to see *Jupiter's* satellites with).

Therefore I am unable, for want of space and time, to give a clearer idea of the comet's position and apparent motion than with a few diagrams.

I am of opinion that the comet was in or near perihelion on the 1st of February and that the tail preceded the nucleus in its motion through space. Doubtless, observations of value have been made at the Cape, Australia and Cordoba (South America), and other Observatories.

H.M.S. "Garnet," Monte Video,
1880, *February* 9.

Observations of the Great Southern Comet 1880, I. *from Feb.* 1 *to Feb.* 7, *made at Monte Video.* By the Rev. S. S. O. Morris.

(*Communicated by the Astronomer Royal.*)

The tail of a comet was observed from H.M.S. 'Garnet,' at about 8 P.M. on the evening of Sunday the 1st Feb., 1880. The 'Garnet' was at the time lying in the outer roads at Monte Video. The nucleus of the comet was at the time beneath the horizon. The visible part of the tail cut the horizon at about the S.S.W. point nearly at right angles, and the altitude of the extremity of the tail visible to the naked eye was about 30°. The tail lay

* The diagrams were exhibited at the Meeting.—Ed.

across the constellations of *Grus* and *Piscis Australis* in direction parallel to the line of direction of the Milky Way. The R.A. of the nucleus conjectured from the direction of the tail and the position of the Sun was at 8 P.M. on the 1st Feb. about 21h. Two small stars of *Piscis Australis* were visible through the tail. These two small stars lying close together were noted as a mark whereby the motion of the comet could be observed on future nights. There was a slight arch in the tail concave towards the south pole, convex towards Fomalhaut, which lay to the west of

the comet. The tail was again observed on the evening of the 2nd; the nucleus was not visible. The position of the tail at 8 P.M. was altered from the previous evening; it had moved from the two bright small stars in *Pisc. Austr.* and had approached Fomalhaut; at the same time its extremity had a greater altitude than on the night of the 1st Feb. We watched carefully for the first appearance of the comet on the evening of the 3rd, but, although the tail, now of immense dimensions, soon became visible, the nucleus had not yet become visible. The position of the tail on the night of the 3rd was to the right and west of its

position on the night of the 2nd. The successive positions of the tail on the nights of the 1st, 2nd, and 3rd implied a motion in R.A. of the nucleus of about 10^m in 24 hours. The increase in South Declination of the nucleus would be very roughly about 4°. On this night the end of the tail was not far off Achernar (*α Eridani*). On the 4th the nucleus was not yet visible; the position of the tail on this night established the general path in the heavens of the nucleus—viz. about 10^m increase in R.A., 4° increase in S. Decl. An arc of a great circle passing through Canopus and Achernar would have very nearly coincided with the axis of the tail. The evening of the 5th was overcast, but the clouds broke sufficiently for it to be seen that the motion already conjectured had continued. The evening of the 6th was overcast. The evening of the 7th was cloudy, but in a break of the clouds the comet's tail was seen to the right of Achernar and now extending much beyond Achernar.

Note on the Great Southern Comet 1880, I. *as seen at Melbourne, Feb.* 2 *to Feb.* 5. By R. L. J. Ellery, Esq.

A large comet, low on the western horizon, was first seen here on the evening of February 2. Owing to its lowness and the fact of the horizon being very hazy every evening since its appearance no observation of the nucleus has yet been possible: indeed, the nucleus has not been seen at this Observatory at all, although it is stated to have been seen in other parts of the colony.

On Feb. 2 the tail was seen extending above the horizon reaching to *β Gruis*; it tended towards west, making an angle of 60° with the horizon. It has been observed every night since, and last night the tail reached a little beyond *ε Phœnicis*—the north-edge passing through it. It is much less bright than on Feb. 2, but much longer; and on Feb. 4 the tail could occasionally be traced nearly to the horizon without any appearance of condensation, while it extended upwards to the prolongation of the line joining *α* and *ε Phœnicis*.

Probably other observers in Australia have been more fortunate than we have, but this brief account from here may not be without interest.

The weather has been unusually clear for some weeks past and the horizon was carefully scanned on the evening of Feb. 1, but nothing was seen of the comet till the evening of the 2nd.

Observatory, Melbourne,
1880, February 6.

Observations of the Great Southern Comet 1880, I. *made at the Adelaide Observatory, New South Wales.* By Charles Todd, Esq., C.M.G.

(*Extract from a letter to the Astronomer Royal.*)

I just write a few lines to advise you of a fine comet that has unexpectedly come into view during the past week. I have not yet succeeded in seeing the nucleus, which is probably below the horizon, but the tail is of great length and is a most conspicuous object in the south west after dark. It appears to have been seen on Sunday evening, February 1. I first saw it on Monday evening between 8^h and 9^h, when in St. Vincent's Gulf on my way from the fine steamer *Orient*. It then appeared as a narrow whitish auroral streak extending from the horizon (North of S.W.) and passing nearly through the stars δ^1, δ^2, and β *Gruis*, the upper portion curving somewhat sharply to the south. Since then the tail has increased in altitude and swept over the sky in a northerly direction, as though turning about a centre a little below the horizon, the upper extremity having a more rapid motion than that near the horizon. The following notes will sufficiently indicate the position of the tail on each night since it was first seen.

February 2nd, 8^h 40^m.—The upper part of tail passes through δ^1, δ^2, and β *Gruis*, curving towards ϵ *Gruis*. Could not trace it to horizon, but about S.W. by W.

February 3.—Sky rather cloudy in S.W., and view of comet interrupted, but first seen in strong twilight about 8^h; very bright later, when quite dark. The tail has moved to the north, the stars θ and ι *Gruis* lying nearly on the northern edge. Tail curved to southward, and approximately concentric with a line connecting δ^1, δ^2, and β *Gruis*, and near the horizon it must pass close to β *Piscis Australis*, which, however, is behind a low bank of cloud. Tail brighter to-night, and at 8^h 20^m could be traced to an altitude of about 27°.

February 4.—Could see the comet at 8^h; very bright later, when quite dark; certainly brighter than last night, being broadest and most luminous immediately below θ *Phœnicis*. Could not trace it to horizon; sky thick; β *Piscis Australis* at 8^h 20^m within the tail not far from its northern edge; θ *Phœnicis* within and close to northern edge, and the tail could be traced nearly up to and pointed almost directly towards η *Phœnicis*; could see nothing of nucleus, though I looked carefully for it from soon after sunset directly in course of tail.

February 5.—Tail not so bright, first seen at 8^h 10^m. It reached up to ζ *Phœnicis* and passed through or close by ϵ *Phœnicis*, less curved.

February 6.—The tail certainly fainter and less curved as the extremity sweeps over to the north. It now reaches up to χ *Eridani*; air thick on horizon.

February 7, 9^h P.M.—The tail is much fainter and is reduced to a thin nebulous streak extending from Fomalhaut (over which it passes) up to a little north of γ *Phœnicis*, passing directly over α *Phœnicis*; sky thick below Fomalhaut.

We have had a great many meteors—one extraordinary one on Monday morning, February 2, about 10^h A.M., which was seen at different places over 200 miles apart. One correspondent describes it as a large ball of fire, as large as the sun, which descended towards the S.W. from an altitude of 60°.

Observatory, Adelaide,
1880, February 7.

Mr. L. A. Eddie's Observations of the Great Southern Comet 1880, I. at Graham's Town.

The following is an account of Mr. L. A. Eddie's observations at Graham's Town, derived partly from a letter to the Astronomer Royal and partly from particulars published in the *Eastern Star* newspaper:—

Mr. Eddie first saw the tail on Feb. 2, when it was about 1° in breadth, stretching upwards from the constellation *Piscis Australis* to β *Gruis*, 20° above the horizon. It had a decided curvature concave to the south, and shone with a light of a pale straw colour about equal in brightness to that of the Milky Way, and far more brilliant than on any subsequent evening.

Feb. 3 was cloudy and on Feb. 4 the comet's tail was found to have moved eastward about 20°. It was now 40° in length, passing over β *Piscis Australis*, and θ *Phœnicis*, and terminating near a small star about 5° below ζ *Phœnicis*. It had completely lost its curvature. On account of cloud low down, Mr. Eddie was unable to ascertain whether the nucleus was above the horizon.

On Feb. 5, the sky being very clear, Mr. Eddie discovered about 8^h p.m. with a 3-inch refractor, using a power of 80, a faint nucleus about equal in size to the annular nebula in *Lyra*, and resembling 47 *Toucani*, Lacaille's globular cluster, when viewed with a power too low to resolve it. It was of a pale yellow colour, condensed at the centre, and situated about 3° *sp* from Fomalhaut. The tail proceeded from an arc of the nucleus equal to a sixth of the circumference, and spread out as a fan till it reached a breadth of about 1° at a distance of about 20° from the nucleus, and preserved this breadth with but a small, if any, increase to about 50° from the nucleus, where it faded away from sight. The outline throughout the whole length was tolerably well defined, and to the naked eye that part of the tail appeared brightest which lay between 5° and 10° from the nucleus. The tail passed over γ *Piscis Australis*, ϵ and λ *Phœnicis*, and terminated at ρ *Phœnicis*, though by a side glance it could be traced as far as q^1 and q^2 *Eridani*.

On Feb. 6 clouds hid the nucleus. The tail passed over β *Sculptoris*, κ, μ, β and δ *Phœnicis* and χ *Eridani*, terminating near ϕ *Eridani*.

Feb. 7.—Nucleus so close to γ *Sculptoris* that to the naked eye it appeared as if this small star constituted the nucleus. In the 3-inch refractor, with a power of 50 or 60, the nucleus at $8^h\ 30^m$ was *nf* the star at a distance of a little less than half the field. At $9^h\ 15^m$ the nucleus was one-third of the field from the star, and was observed till $9^h\ 20^m$ when it was apparent that it would not transit over γ *Sculptoris* but would pass a little to the east. The coma was not distinguishable from the nucleus, but was condensed at the centre. The tail extended from γ *Sculptoris* nearly to κ *Eridani*, passing north of α *Phœnicis*, over ν and another small star near χ *Phœnicis*.

Feb. 8.—The comet, on account of increasing faintness, was not visible till $7^h\ 50^m$, and the nucleus was only found, after much search, at $8^h\ 40^m$, about 5° E. by N. of γ *Sculptoris*, in R.A. $23^h\ 30^m$, Dec. 35° S. approximately. The difficulty in finding it arose from the circumstance that the tail appeared to start from a point a little above the nucleus, the faint light of the comet being obscured near the horizon. The tail passed over ϵ and γ *Phœnicis*, and, passing a little to the right of κ *Eridani*, extended right across the constellation *Microscopium*. The light had now become so faint that the comet might easily escape observation.

Feb. 9.—Cloudy. Comet not seen.

Feb. 10 —Evening still cloudy. Comet visible for short intervals. The nucleus was seen in the telescope almost in the same field with θ *Sculptoris*. The tail passed over ι, θ and ϵ *Eridani*, and across the constellation *Horologium*.

Feb. 11.—Cloudy. Comet not seen.

Feb. 12.—Cloudy. A slight hazy light to be seen now and then through openings in the clouds.

The tail after the first evening was perfectly straight and colourless.

Observations of the Great Southern Comet 1880, I. *made at the Cape of Good Hope, Feb.* 2 *to Feb.* 15. By David Gill, Esq.

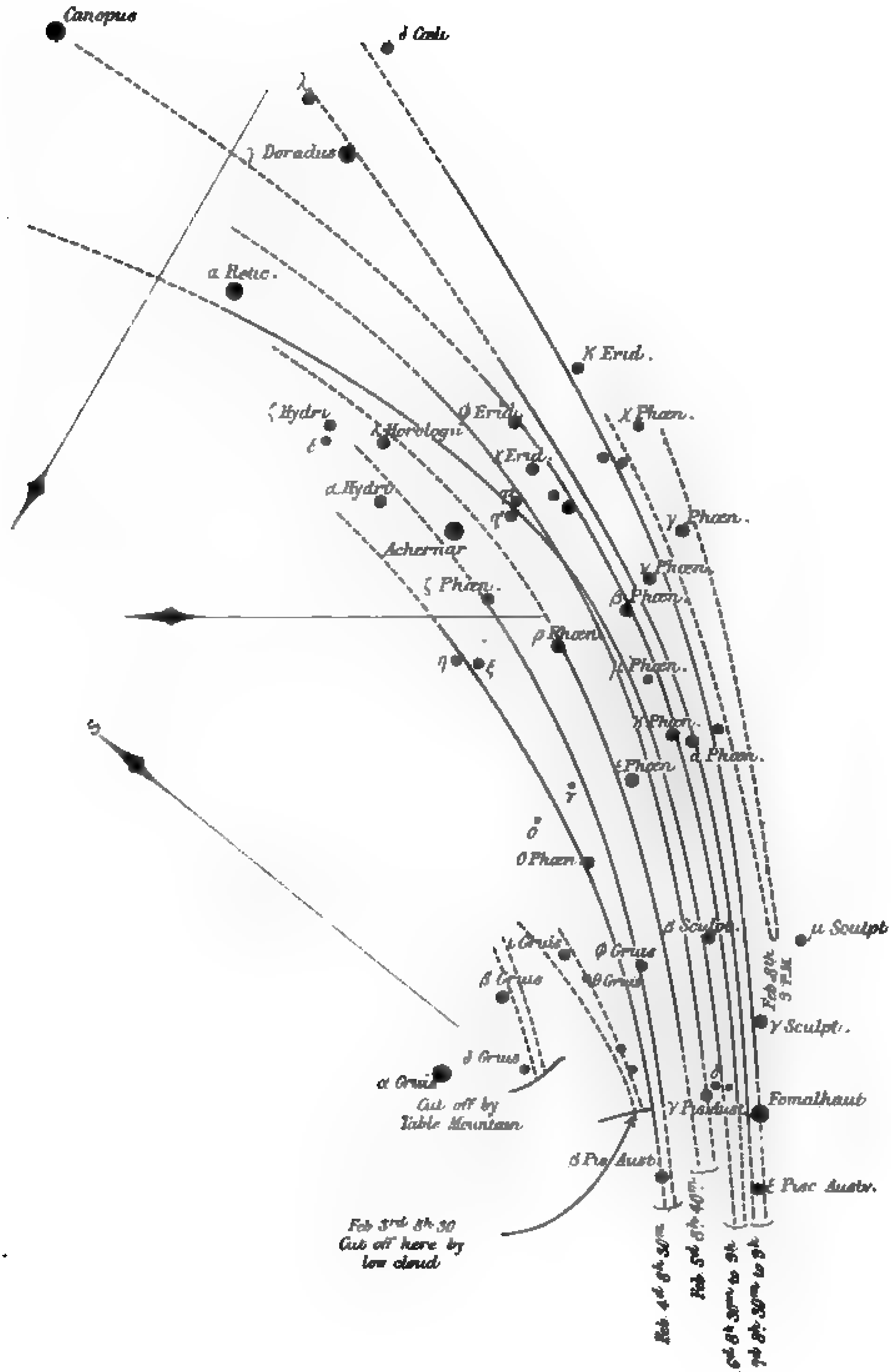
(*Extracts from letters to the Astronomer Royal.*)

By last mail I wrote to tell you that we have a comet by the tail, and I am sorry to say that we only have him by the tail still.

To get any observations at all it was necessary to go to Sea Point, to command the sea horizon to the S.W. of Table Mountain. I selected Mr. Henry Solomon's Garden as the best site, and the site where Maclear observed Donati's Comet in 1858.

I took the Dollond 10-inch altazimuth and a couple of chronometers there on the evening of Tuesday the 3rd inst., but the horizon was somewhat hazy and β *Piscis Australis* could hardly be made out, and the tail could not be traced so far.

(See p. 284.)



Mr. Gill — The Great Southern Comet

February 4 was a very fine night, except for a low bank of cloud on horizon. I could not make quite certain as to how the comet's tail passed β *Piscis Australis.* The rest of the drawing is very accurate.

February 5 and 6.—Very fine. On the latter day the tail had a curious curve. Drawings very accurate, except where outline is dotted.

February 7.—Less fine, and comet fainter. Some uncertainty about limit of comet. I certainly saw no nucleus, but others up country say it was seen.

February 8 was a cloudy night. In a break of cloud I picked up with an opera-glass a nebulous nucleus, with no well-defined point to the south of μ *Sculptoris*; but before I could bring the altazimuth to bear, the cloud had formed over the spot.

February 9.—From the faintness of the nucleus it seemed quite useless to attempt further observations with a portable telescope, so I remained here last night. We had only a glimpse of the nucleus with an opera-glass for a few seconds, and afterwards saw the tail portion by portion; but even before the nucleus could be got into the field of the finder the cloud had obscured it. The nucleus set, at about 9 o'clock last night, behind the highest peak of Table Mountain (the Devil's Peak). To-night I hope we shall have better luck, but the nucleus does not look like an object permitting very accurate observation. The drift of the comet seems to be southwards; I did not, therefore, think it necessary to incur the cost of a telegram to you, but I should be glad to have your opinion for my future guidance.

I do very sincerely trust we shall get three good observations sufficient to give an orbit exact enough for the period during which I drew the tail. The boundary of the tail was exceedingly well marked. I send a rough tracing from my drawings.

Royal Observatory, Cape of Good Hope,
1880, *February* 10.

We have now secured observations of the comet on February 10, 11, 12, 13, 14, 15, with the following approximate results.

Comet I., 1880.

	Cape M.T.	R.A.	N.P.D.
	h	h m	° ′
February 10	9¼	0 4	123 43
11	8¾	0 21	123 31
12	9	0 37	123 11
13	8½	0 52	122 44
14	8¾	1 6	122 10
15	8½	1 20	121 34

On February 11, 13, and 15, we have comparison stars whose places are well determined.

Royal Observatory, Cape of Good Hope,
1880, *February* 17.

On the Determination of the Personal Equation of Lunar Observers. By E. Neison.

In compliance with the request in Mr. Dunkin's Note in the *Monthly Notices* for January 1880 (page 166), I give a more detailed account of the method employed by me for determining the personal equations given in my Note in the previous number of the *Monthly Notices.*

Let the different observers be denoted by the Roman numerals I, II, III, ... and their personal equations by $e', e'', e''', \ldots$ respectively, and as these personal equations are assumed to be liable to variations from year to year, let their values for any year $1860+k$ be denoted by $e_k', e_k'', e_k''', \ldots$ respectively.

Next let H_k denote the mean error for the year $1860+k$ of the theory embodied in Hansen's lunar tables, and let the actual error at any epoch t be denoted by $H_k + h_t$ so that the quantity h_t will be one which quickly changes in sign and magnitude as the time varies. Further, let a_t denote the accidental error involved in the observation made at the epoch t. Then any observation made during the year $1860+k$ at the epoch t by the observer I will differ from the tables by the quantity

$$H_k + h_t + a_t + e_k'.$$

Consider the mean of a number n of such observations

$$H_k + \frac{1}{n}\Sigma_n h_t + \frac{1}{n}\Sigma_n a_t + e_k'.$$

The quantities denoted by a_t being purely accidental and as likely to be positive as negative, the mean of any number n of these quantities will tend to become smaller as n, their number, increases. The mean may therefore be denoted by $an^{-\frac{1}{2}}$. Similarly, if the observations be impartially distributed over the period, the quantities denoted by h_t will also be of opposite signs and will tend to neutralise each other, so that the mean of n of these quantities will tend to become smaller as n, their number, increases, and may be denoted by $hn^{-\frac{1}{2}}$. If the observations be not impartially distributed, but there be a tendency for them to occur at some particular part of the lunation or year, then they will not so tend to destroy each other, and the mean will not necessarily decrease with an increase in the number of observations. In general, however, there is little tendency to such a systematic error, and if the tabular errors in the parallactic equation, apparent annual equation, and variation be corrected, there will be no such tendency.

From these considerations the mean of n observations may be written in the form

$$H_k + \frac{a+h}{\sqrt{n}} + \mathrm{E},$$

if E be put for the effect of the personal equations.

Suppose during any year 1860 + k there were n observations made, n' by observer I, n'' by observer II, n''' by observer III, and n^{iv} by observer IV. Then the mean error for the year would be

$$H_k + \frac{a+h}{\sqrt{n}} + \frac{n'}{n}e_k' + \frac{n''}{n}e_k'' + \frac{n'''}{n}e_k''' + \frac{n^{iv}}{n}e_k^{iv}. \tag{1}$$

Similarly, the mean of the n' observations made by the observer I would be

$$H + \frac{a+h}{\sqrt{n'}} + e_k'.$$

Therefore, calling the difference between these two the apparent personal error of the observer, its value is

$$E_k' = e_k' - \left(\frac{n'}{n}e' + \frac{n''}{n}e_k'' + \frac{n'''}{n}e_k''' + \frac{n^{iv}}{n}e^{i}\right) + (a+h)\sqrt{\frac{n-n'}{nn'}}. \tag{2}$$

For each observer there will be a similar equation, so that there will be four equations to determine the four unknown quantities e_k', e_k'', e_k''', e_k^{iv}. If these equations were all independent it would be possible to completely determine each of these quantities in terms of the four known quantities E_k', E_k'', E_k''', E_k^{iv}, and the one unknown factor $(a + h)$, the sum of the mean values of the accidental errors of observation and the outstanding errors of theory. Owing to the presence of this unknown factor, it would be impossible to determine the actual personal equations with absolute accuracy.

One of these four equations, however, will be found to depend on the three others, owing to its having been used to eliminate H_k from the system. To render it possible, therefore, to determine the values of the personal equations, some arbitrary condition must be assumed, so as to obtain a fourth equation. This is equivalent to assuming some standard to which to refer the various observations, as, not knowing the absolute error of the tables, it is necessary to determine this error from the observations by referring them to some standard which is supposed to give the true place of the Moon. This condition I have taken to be that

$$e_k' + e_k'' + e_k''' + e_k^{iv} = 0. \tag{3}$$

Now, as the observations are supposed nearly equally divided amongst the four observers, it may be assumed that

$$\begin{aligned} n' &= \frac{1}{4}n - m', \\ n'' &= \frac{1}{4}n - m'', \\ n''' &= \frac{1}{4}n - m''', \\ n^{iv} &= \frac{1}{4}n - m^{iv}, \end{aligned}$$

when m', m'', n''', m^{iv}, will be four small integers, either positive or negative, and usually less than one-twentieth of the value of n. Substituting these values, then to determine e_k', e_k'', e_k''', e_k^{iv}, there will be the four independent equations

$$\left.\begin{array}{l} E_k' = e_k' + \frac{1}{n}\left\{ m'e_k' + m''e_k'' + m'''e_k''' + m^{iv}e_k^{iv} \right\} + (a+h)\sqrt{\frac{n-n'}{nn'}}, \\ \cdot \\ \cdot \\ E_k^{iv} = e_k^{iv} + \frac{1}{n}\left\{ m'e_k' + m''e_k'' + m'''e_k''' + m^{iv}e_k^{iv} \right\} + (a+h)\sqrt{\frac{n-n^{iv}}{nn^{iv}}}. \end{array}\right\} \quad (4)$$

In this manner it is possible to completely determine $e_k' \ldots e_k^{iv}$ in terms of known quantities and the small unknown factor $(a + h)$, this last factor rendering their exact values slightly uncertain.

Suppose in this manner the values of $e' \ldots$ are obtained for several successive years, and that they can be represented by

$$\begin{array}{lcl} e_k' = e_o' + \Delta e_k', & \ldots\ldots & e_k^{iv} = e_o^{iv} + \Delta e_k^{iv}, \\ e_{k+1}' = e_o' + \Delta e_k', & & e_{k+1}^{iv} = e_o^{iv} + \Delta e_{k+1}^{iv}. \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & \ldots\ldots & \cdot \end{array}$$

If the quantities denoted by $\Delta e_k' \ldots \Delta e_k^{iv} \ldots$ are small quantities, much smaller than the probable error of $e_o' \ldots$ and irregular in sign and value, it is obvious that each of these may be fairly assigned to the effect of the small error introduced by the small unknown quantity depending on $(a + h)$ and arising from the outstanding errors of observation and theory. For if there were any real variation in the personal equations, then these quantities $\Delta e_k' \ldots \Delta e_k^{iv} \ldots$ ought to reveal its existence by systematic variation in value, by regularly increasing and decreasing. If nothing of this kind occurs, then it may be fairly assumed that the personal equations have remained sensibly constant year after year.

Now, in the cases discussed in my paper this is actually the case. Thus, for instance, for the observers distinguished as III and IV the personal equations between 1863 and 1869 are

$$\begin{array}{ll} & \quad\ \ \mathrm{s} \quad\quad\ \mathrm{s} & \quad\quad\ \mathrm{s} \quad\quad\ \mathrm{s} \\ e'''_3 = +{\cdot}057 - {\cdot}002, & e^{iv}_3 = -{\cdot}082 - {\cdot}014, \\ e'''_4 = +{\cdot}057 + {\cdot}006, & e^{iv}_4 = -{\cdot}082 + {\cdot}027, \\ e'''_5 = +{\cdot}057 - {\cdot}004, & e^{iv}_5 = -{\cdot}082 - {\cdot}008, \\ e'''_6 = +{\cdot}057 - {\cdot}010, & e^{iv}_6 = -{\cdot}082 - {\cdot}009, \\ e'''_7 = +{\cdot}057 + {\cdot}013, & e^{iv}_7 = -{\cdot}082 - {\cdot}008, \\ e'''_8 = +{\cdot}057 + {\cdot}004, & e^{iv}_8 = -{\cdot}082 - {\cdot}003, \\ e'''_9 = +{\cdot}057 - {\cdot}008, & e^{iv}_9 = -{\cdot}082 + {\cdot}006. \end{array}$$

In finding then the personal equations of these observers, it

may justly be assumed that their personal equations remained constant throughout this period, and all seven years may be grouped together to determine them. Moreover, as the personal equations have remained constant, the standard to which they have been referred must have remained constant, as it is derived from the condition

$$e_o' + e_o'' + e_o''' + e_o^{iv} = 0, \tag{5}$$

As long as the observations are made by the same four observers, the preceding method can be adopted without alteration. Now, suppose in the year 1860 + j, one of the four observers, say I, is replaced by a new observer, say V; then throughout the quantity e' will be replaced by e^{v}. The method of determining the personal equations will remain unaltered, but they will now be referred to the standard derived from the condition

$$e_j'' + e_j''' + e_j^{iv} + e_j^{v} = 0, \tag{6}$$

and not, as before, from the condition that

$$c_o' + c_o'' + e_o''' + e_o^{iv} = 0.$$

It will therefore be necessary to take into account the effect of this change in the standard to which the observations are referred.

Now, suppose these two standards differ by a small quantity c, and that (e_j'') (e_j''') (e_j^{iv}) . . . represent the personal equations as determined by the new standard (6); then each of these will be greater than its true value referred to the original standard by the quantity c. Therefore c can be determined by the equation

$$e_o'' + e_j''' + e_o''' = (e'') + (e_j''') + (e_j^{iv}) + 3c,$$

$$c = \frac{1}{3}\,\frac{(e_o'') + e_o''' + e_o^{iv}}{(e_j'') + (e_j''') + (e_j^{iv})}, \tag{7}$$

provided it be assumed that no change has occurred during the year in the personal equations of these three observers. This assumption is reasonable enough. If they have been constant for several years previous to this, it is unlikely that they should change at this very time that a new observer is introduced, as the mere change of observers cannot give rise to any such variation. It is of course possible that one observer may have chanced to change his personal equation at this very epoch; but, even if this be so, only one-third of this change will be thrown on the determination of the difference between the two standards: this risk is unavoidable. Even, however, if one observer had changed his personal equation by the small increment x, then c would undergo a fictitious increase of $\frac{1}{3}x$; and when the personal equations came to be compared with those of previous years, whilst two would seem to have decreased by $\frac{1}{3}x$, the third, which had really varied, would seem to have increased by $\frac{2}{3}x$.

If, therefore, only one had varied, this would serve to indicate which with some degree of probability. For it is much more probable that one should have varied than that two should have done so and in the same direction.

It has been already shown that it is possible to determine with some certainty whether the personal errors of the observers have changed during any group of years $k, k+1, \dots j-1$, through which period the observers have remained the same. Suppose a break to occur at the beginning of the year j, through one observer being replaced by another, and that this fresh arrangement remains unaltered for the group of years $j, j+1, \dots l-1$; then it can be ascertained in exactly the same manner whether the personal equations of the observers have remained unchanged for the period between the years j and l. Suppose during both these groups they remain unaltered, then the difference between the two standards can be determined with considerable approximation by the comparison of the mean results for each group of years in the manner already detailed. The difference—denoted by a in my paper—between these two standards having been determined, it is possible to reduce the personal equations for the entire period covered by both groups to any standard which may be assumed.

The same process may then be extended to a third group of years commencing with the year l in which yet another old observer is replaced by a new observer, and in this manner a third set of personal equations determined and reduced to the same standard as before.

It is in this manner that the four quantities a_1, a_2, a_3, a_4, were determined which were given in my former paper, and by means of which I was able to determine the personal equations of the different observers, *without* being obliged to assume either that these personal equations remained constant, or even that the personal equations of any one observer must be assumed to remain constant. It so happens that I find the personal equations of the older observers to be constant, so that their effect can be accurately computed for the years between 1862 and 1873. The later observers seem to have personal equations liable to variation, or through less experience are liable to greater accidental errors of observation. In my former paper, having practically only two groups, each of two years' duration, from which to determine two new personal equations, the results were less accurate than could be desired. When, however, the observations for 1877, 1878 and 1879 are available it will be possible to arrive at more definite results on this most important matter.

I think the detailed account given above will remove any doubts which Mr. Dunkin may have entertained as to the soundness of the method by which were determined the personal equations given in my paper. It is obvious that, admitting that Mr. Thackeray's personal equation may be liable to vary or its value may be founded on too few observations, these things can have

no influence in vitiating the results arrived at by me, as the results for the other observers do not depend in the least on the value assigned by me to Mr. Thackeray's personal equation. At the same time I would remark that in my paper I point out that the results for Mr. Downing, and more especially for Mr. Thackeray, rest on so few observations that they must be considered uncertain.

Mr. Dunkin, in the concluding paragraph of his Note, gives the results of his determination of the personal equations of Messrs. Lynn, Downing and Thackeray, as compared with Mr. Criswick for the three years 1877 and 1878, derived from data inaccessible to me. Assuming Mr. Criswick's error to have remained unaltered, these results transformed into my own standard show that the personal equations of Mr. Downing and Mr. Thackeray have changed since 1876. The correction to be applied to the mean of the observations of both limbs, in order to reduce Mr. Downing's and Mr. Thackeray's observations to my standard is given in the following table together with those derived from the data supplied by Mr. Dunkin.

	1875	1876	1877	1878
	s	s	s	s
Mr. Downing =	−0·086	−0·097	+0·033	−0·017
Mr. Thackeray =	−0·108	−0·130	+0·033	−0·098
Mean effect on year	−0·057	−0·054	+0·020	−0·020

The mean effect on the year is derived by referring the personal equations to the standard adopted by me and taking into account the number of observations made by each observer. The above results show that the change in the personal equations will produce an apparent diminution of the tabular error of the Moon amounting to about 1″·2 in 1877 and 0″·6 in 1878. Now, this is exactly what is shown by the observations, the apparent decrease in the tabular errors for these years being 1″·3 for 1877 and 0″·9 for 1878, taking into account the effect of the apparent decrease in the tabular error due to the change in the nature of the systematic errors of long and short period. So far, therefore, from the results given by Mr. Dunkin invalidating the conclusions arrived at in my paper they tend to strongly confirm them.

Supplementary Note on the Changes in the Error of Hansen's Lunar Tables. By W. T. Lynn, B.A.

This Note is simply a supplement to the paper printed at page 82 of the present volume (*Monthly Notices* for December 1879), giving the completion of the mean errors in longitude up to the end of last year, as deduced from the Altazimuth observations. And, at the close of my superintendence of that instru-

ment, I have to thank the Astronomer Royal for the permission to communicate these results.

In the paper just referred to, I gave the mean error in longitude for the first three quarters of last year, and I now add that for the last quarter, putting the four together, as follows:—

	Mean Error in Longitude.	No. of Observations.
1879, January to March	$+8''\cdot25$	28
April to June	$+8\cdot69$	42
July to September	$+9\cdot48$	47
October to December	$+9\cdot48$	38

The mean result for the year, therefore, is $+ 9''\cdot04$ from 155 observations.

The following Table exhibits the mean error for each successive year of the last four years:—

Year.	Mean Error in Longitude.	No. of Observations.
1876	$+9''\cdot31$	171
1877	$+8\cdot25$	182
1878	$+7\cdot48$	161
1879	$+9\cdot04$	155

The diminution to a turning-point, and subsequent return to an increase, are sufficiently obvious. In consequence of my long illness in 1878, I did not observe after the month of April in that year, and only occasionally in 1879. A considerable number of observations was therefore made in those years by the computers, so that the observers were more numerous than usual, and the mean results are likely to be more than usually independent of personality. I may remark that had the rate of increase to which I called attention in my first paper on the subject continued, the mean error in longitude in 1879 would have been about 12″, differing from the actual mean error by a larger quantity than can well be accounted for by personality in observing.

Blackheath,
1880, *February* 2.

On the Longitude of the Observatory, Windsor, New South Wales.
By J. Tebbutt, Esq., Director of the Observatory.

The longitude $10^h\ 3^m\ 15^s\cdot7$ E. hitherto adopted for my Observatory depends on that derived for the Sydney Observatory by the Rev. W. Scott from corresponding moon-culminations observed at Greenwich and Sydney and the Cape of Good Hope and Sydney, the difference of longitude between Sydney and Windsor

having been approximately determined in 1865 by telegraph as $1^m\ 30^s{\cdot}0$. In the middle of 1878 I discussed twelve occultations of well-determined stars observed by me at Windsor and presented the results to the Royal Society of N. S. Wales. The Moon's tabular places were corrected by observations made at Greenwich about the dates of the occultations. It appeared from the discussion that the longitude of both the Sydney and Windsor Observatories, *east* longitude being regarded as *positive*, required a correction of $+6^s{\cdot}84$. I have since extended my investigation to eight additional occultations, and have now much pleasure in communicating the results to the Society. The geographical latitude of my Observatory as deduced from ninety-three prime vertical observations of well-determined stars culminating within 39′ of the zenith is $-33°\ 36'\ 28''{\cdot}9$. Adopting Bessel's ratio of the Earth's axis, the corresponding geocentric latitude and logarithm of the Earth's radius will be $-33°\ 25'\ 53''{\cdot}0$ and 9·9995576, and these with the assumed longitude $10^h\ 3^m\ 15^s{\cdot}7$ E. are the constants employed in the reductions.

The mean places of the occulted stars have been derived from numerous modern Catalogues. The Moon's places have been interpolated with second differences from the hourly Ephemeris of the *Nautical Almanac* and corrections based on observations at Greenwich, Radcliffe, and Washington applied. The corrections for the occultations in 1876 depend on Greenwich alone, and were kindly communicated to me in MS. by the Astronomer Royal. The accompanying table exhibits the dates and local mean times of the occultations, the adopted corrections of the Moon's tabular places and the resulting corrections to the assumed longitude. No corrections have been applied to the Moon's tabular parallax and semidiameter. I may state that the occultations of *m* Tauri, 3 Sagittarii, and both phases of the occultation of Antares on July 3, 1876, were very unfavourable for longitude determinations. The mean of the twenty corrections is, however, $+6^s{\cdot}15$, which gives for the longitude $10^h\ 3^m\ 21^s{\cdot}85$ E. Fifty-three transits of the Moon, comprising fifty-one of the first limb and two of the second, compared with the hourly Ephemeris of the *Nautical Almanac* corrected by means of the Washington transit-circle observations give $10^h\ 3^m\ 21^s{\cdot}58$ E. for the longitude. It will now be interesting to compare the occultation and moon-culmination results with the longitude of my Observatory as deduced from the longitudes of the Sydney and Melbourne Observatories combined with telegraphic differences. The difference of longitude of the Sydney and Windsor Observatories has been recently found by careful telegraphic determinations to be $1^m\ 28^s{\cdot}83$, no allowance, however, having been made for personal equation. The longitude of the Melbourne Observatory from a large number of moon-culminations is $9^h\ 39^m\ 54^s{\cdot}80$ E., and that of the Sydney Observatory as given by Mr. Russell in a recent paper read before the Royal Society of N. S. Wales, and also derived from moon-culminations, is $10^h\ 4^m\ 50^s{\cdot}81$ E. The tele-

graphic difference of longitude between the Sydney and Melbourne Observatories (see Mr. Russell's paper just cited) is $24^m\ 55^s\!\cdot\!77$; consequently, we have the corresponding results for my Observatory as follows:—

	h m s
Longitude of the Sydney Observatory	10 4 50·81 E.
Telegraphic difference of longitude between Sydney and Windsor	− 1 28·83
Longitude of my Observatory	10 3 21·98 E.
Longitude of the Melbourne Observatory	9 39 54·80 E.
Telegraphic difference between Melbourne and Sydney . .	+ 24 55·77
„ „ „ Sydney and Windsor . .	− 1 28·83
Longitude of my Observatory	10 3 21·74 E.

We have thus four determinations, the mean of which is $10^h\ 3^m\ 21^s\!\cdot\!8$ E., and this value I propose to adopt for the future as the longitude of my Observatory.

Date of Occultation.	Star occulted.	Windsor Mean Time of Observation.	Phase of Occultation.	Adopted Corrections to Moon's Tabular Place. R.A.	N.P.D.	Resulting Correction to Longitude.
		h m s		s	″	s
April 18, 1866	B.A.C. 1468	6 14 36·2	D	+0·02	+0·1	+ 7·55
Oct. 15, 1866	„ 6267	7 21 22·3	D	−0·16	+2·0	+14·27
Feb. 27, 1868	μ Piscium	7 48 46·9	D	−0·06	−0·7	+10·04
Mar. 2, 1868	*m* Tauri	10 11 27·2	D	+0·14	+0·1	+17·42
Feb. 17, 1869	ξ′ Ceti	8 3 32·6	D	+0·01	−0·4	+12·09
Feb. 24, 1869	δ Cancri	8 0 49·3	D	−0·24	0.0	− 1·35
Feb. 11, 1870	ζ Tauri	8 18 1·5	D	−0·14	+0·5	+ 3·51
Nov. 30, 1870	ψ′ Aquarii	10 7 52·2	D	−0·38	−0·6	+11·72
Apr. 1, 1873	A′ Tauri	6 53 44·3	D	−0·42	+1.1	+ 5·45
Sept. 2, 1873	σ Sagittarii	13 27 17·2	D	−0·49	+0·8	+ 6·79
Feb. 27, 1874	ψ′ Cancri	10 53 58·7	D	−0·61	−2·7	+ 9·61
May 30, 1874	δ Scorpii	11 36 34·9	D	−0·48	−3·8	+ 5·64
Apr. 15, 1875	η Leonis	9 16 47·9	D	−0·56	−2·8	+ 3·06
May 22, 1875	3 Sagittarii	11 19 37·3	D	−0·52	−1·4	+ 3·10
Sept. 7, 1875	Antares	7 11 17·7	R	−0·29	−0·6	+ 2·41
Sept. 9, 1875	B.A.C, 6220	8 24 3·7	D	−0·29	−0·6	+ 7·81
June 2, 1876	Spica	12 34 7·2	D	−0·68	−5·8	+ 3·06
June 2, 1876	Spica	13 39 5·2	R	−0·68	−5·8	− 3·54
July 3, 1876	Antares	12 14 41·0	D	−0·76	−3·2	+ 6·07
July 3, 1876	Antares	13 7 32·9	R	−0·76	−3·2	− 1·79
				Mean Correction		+ 6·15

Windsor, N. S. Wales,
1880, *January* 6.

Mean Heliographic Latitude of Sun Spots for the Years 1874 to 1879, deduced from Photographs taken at the Royal Observatory, Greenwich.

(*Communicated by the Astronomer Royal.*)

The numbers given in the accompanying table have been formed as follows :—

The Heliographic Latitude of each spot for each day of observation has been multiplied by its area for the day, and the sum of the products for Spots North of the Sun's Equator has been divided by the sum of the corresponding areas to form Mean Hel. Latitude of Spotted Area North of Equator. Similarly for Spots South of the Equator. In forming the Mean Hel. Lat. of entire Spotted Area the algebraic sum of the products for spots north and south of the equator has been divided by the sum of the areas; and for the Mean Distance from the Equator for all spots, the numerical sum of the products without regard to the sign of the latitude has been similarly divided.

The mean areas have been formed by dividing the sum of the daily areas by the number of days of observation.

All these means have been formed for each rotation from 1874, April 16, to 1879, Dec. 27, and from these quantities the annual means given in the table have been deduced. The detailed results for each rotation will be printed in the Greenwich Observations for 1879.

The table shows that the outburst of spots which commenced in the latter half of 1879 was marked by an increase in the mean latitude N. and S. of the spotted area. This is in accordance with Dr. R. Wolf's observations that the mean distance from the equator is least at the epoch of minimum and greatest at that of maximum of sun-spots.

Year.	Spots North of the Equator.		Spots South of the Equator.		Mean. Hel. Lat. of entire Spotted Area.	Mean Distance from Equator of all Spots.
	Mean Area.	Mean Hel. Lat.	Mean Area.	Mean Hel. Lat.		
		° ′		° ′	° ′	° ′
1874	245	+ 9 3	326	−12 9	−2 57	10 48
1875	125	+11 12	127	− 9.50	+1 49	10 35
1876	43	+12 31	84	−10 55	−4 18	11 22
1877	32	+ 9 10	60	− 9 41	−4 17	9 32
1878	21	+ 7 13	3	− 7 40	+5 40	7 19
1879	11	+23 54	34	−22 39	−9 11	23 1

The Mean Areas, &c., for the year 1874 refer to the period of nine months from 1874, April 16, to 1875, Jan. 20.

Royal Observatory, Greenwich,
1880, *March* 11.

On the Systematic Errors of the Greenwich North Polar Distances. By W. H. M. Christie, Esq.

(*Abstract.*)

The long series of observations of N.P.D. made at the Royal Observatory, Greenwich, under the superintendence of Sir G. B. Airy from 1836 to 1878 is discussed in this paper with reference to the systematic errors in the flexure and R – D corrections, in the refractions and in the adopted latitude. It is pointed out that the coefficient of refraction cannot be determined with sufficient accuracy from circumpolar stars, since refraction produces on stars at different distances from the pole only a differential effect which is comparatively small as far as Z.D. 75°, at which point the law of refraction begins to be somewhat uncertain; and that this effect is masked by uncertainty in the flexure and by the probable errors of the observations. On this view the determination of refraction must depend mainly on observations of southern stars made at Greenwich and at southern observatories, for which the effects of refraction are additive.

In the series of observations 1836-1878 the instrumental conditions have been twice changed, involving changes in the horizontal flexure and R – D correction, and from this it is inferred that the R – D correction is probably connected in some way with flexure, and not with any abnormal refraction.* The observations may be divided into three periods: (1) 1836–1849 made chiefly with the Troughton Mural Circle, for which there was no flexure or R – D correction; (2) 1851–1865, with the Transit-Circle before the central cube was perforated: the flexure correction was + and the R – D correction – for south stars; (3) 1866–1878, with the Transit-Circle after the central cube was perforated: the flexure correction was –, and the R – D correction + for south stars. After the piercing of the cube the observations of the collimators in the determination of flexure were made through the cube, instead of with the Transit-Circle raised. A table is given of the annual results for co-latitude and corrections to the position of the ecliptic: and in order to make these comparable they are corrected so as to reduce them to the same refractions (Bessel's *Tabulæ Regiomontanæ*) throughout; and in the case of the result for the ecliptic to the same co-latitude 38° 31′ 21″·90 and to Le Verrier's value of the obliquity.

The latitude determinations from circumpolar stars are discussed in four groups:—(1) 1840–1847, with the Troughton Circle; (2) 1851–1861, with the Transit-Circle, when the old law $a + b \sin z$ for R – D was used; (3) 1862–1865, with the Transit-Circle, when the new law $a + b \sin z \cos^2 z$ was used;

* In a paper on the "Discordance of Direct and Reflexion Observations" (*Memoirs*, vol. xxxii.) the Astronomer Royal concluded from a discussion of the latitude determinations 1836–1860 that the R – D correction probably depended on the shutter opening; but this was written before the piercing of the Transit-Circle cube.

(4) 1868–1876, with the Transit-Circle after the piercing of the cube, when the diminution in Bessel's refractions (·00531 × refraction) proposed by Mr. Stone was adopted. Tables and curves are given showing the excess of N.P.D. above the Pole, or twice the error of assumed co-latitude, from groups of stars at different distances from the Pole for these four periods, with Bessel's refractions and the old and new laws for R—D respectively, and with Mr. Stone's diminished refractions and the new law for R—D. It is pointed out that for the first three periods, 1840–1847, 1851–1861, and 1862–1865, Bessel's refractions and the old law for R—D on the whole give better results at any rate as far as N.P.D. 40°. But as Mr. Stone had based his diminution of the refractions on the observations 1857–1865, these are discussed separately, giving residuals for this particular period which, though differing somewhat from those given by Mr. Stone, would be decidedly improved by a diminution of the refractions. The amount of this diminution depends to some extent on the adoption of the new law for R—D: with the old law it would be only one-half that found by Mr. Stone. But the strength of this evidence in favour of diminished refractions is shaken somewhat by the results for the longer period 1851–1861, and still more by those with the Troughton Circle, 1840–1847, which are unaffected by flexure and R—D corrections. Further, it is pointed out that Mr. Main's exhaustive discussion of the observations 1836–1854 (*Memoirs*, vol. xxvi.) shows that as far as Z.D. 82° Bessel's refractions unaltered represent satisfactorily the observations of northern as well as of southern stars, and that down to this limit no correction of Bessel's refractions of the *Tabulæ Regiomontanæ* is admissible for these observations.

The results for the period 1868–1876 indicate such a large error at N.P.D. 20° (where the curve runs up almost to a cusp) that no certain conclusion can be drawn from them as regards the refractions. Slightly more accordant results are given with Bessel's refractions unaltered and the new law for R—D. If the observations above Pole at N.P.D. 20° (where the R—D correction is well determined) are assumed to be free from error, a correction of nearly + 1″ would be required at Z.D. 60° N. to the observed zenith distance (numerical).

A comparison is next made of the co-latitudes deduced from observations of circumpolar stars and of the Sun, the latter being inferred from the ecliptic investigation in which the correction to the mean of the observed N.P.D.'s of the Sun throughout the year is given. The following are the results corrected for error of thermometer:—

	With Bessel's Refractions. Co-latitude		With Stone's Refractions. Co-latitude	
	from stars.	from Sun.	from stars.	from Sun.
	″	″	″	″
1836–1849	21·85	21·90	21·61	22·40
1851–1865	21·87	21.27	21·63	21·77
1866–1878	21.83	21·84	21·59	22·34

The balance of evidence is by a majority of 2 to 1 against any diminution of the refractions. But in the period 1851–1865 there is a large discordance in the results from the Sun, which seems to indicate an error at Z.D. 60° S. before the piercing of the cube, analogous to that at Z.D. 60° N. shown by circumpolar stars after the piercing.

A table is given exhibiting the comparison of the various Greenwich Catalogues with the Cape (1860) and Melbourne (1870) Catalogues in extension of that given by Mr. Downing (*Monthly Notices*, vol. xxxix. p. 133) for the first Seven Year Catalogue. It is pointed out that the use of Mr. Stone's diminished refractions in the Nine Year Catalogue has produced intolerable discordances, which are removed by the substitution of Bessel's (with Main's corrections below Z.D. 82°), and that the comparisons of the Greenwich first Seven Year Catalogue (1854–1860) with the Cape and Melbourne catalogues seem to show that a negative correction is required to the Z.D.'s south of the zenith, such as appeared to be indicated by the observations of the Sun 1851–1861.

The question of the obliquity of the ecliptic is discussed with special reference to the value of the secular variation. Bessel found the value $-0''.457\,t$ from observations 1755–1815, whilst Le Verrier adopted $-0''.476\,t$ based entirely on his theory of the Sun, rejecting the value $-0''.4576\,t$ which he had previously found from observations 1755–1846. Under these circumstances the following comparison has been made of Le Verrier's and Bessel's formulæ respectively with the Greenwich results:—

	With Old Law for R–D. Corrections to		With New Law for R–D. Corrections to	
	Le Verrier.	Bessel.	Le Verrier.	Bessel.
	″	″	″	″
1836–1849	+0.21	+0.22	+0.21	+0.22
1851–1865	+0.16	−0.11	+0.32	+0.05
1866–1878	+0.20	−0.40	+0.04	−0.56

It appears from these quantities that the Greenwich observations are well represented by Le Verrier's value of the secular variation and not by Bessel's, and that the old law for R—D, $b \sin z$ gives more accordant results than the new law, $b \sin z \cos^2 z$. It is pointed out that the constant correction $+0''.19$ to Le Verrier's obliquity given by the Greenwich observations represents very nearly the effect of the difference ($+0''.26$) between the refractions of the *Fundamenta* (which Le Verrier appears to have used) and those of the *Tabulæ Regiomontanæ* now used at Greenwich.

In conclusion it is urged that every effort should be made to obtain observations of stars by reflexion as near the horizon as practicable in order that the R—D correction may be determined for large zenith distances, and that the determination of horizontal flexure should be checked by other methods, since the adop-

tion of the old law, $b \sin z$ for the R—D correction, virtually neutralising the flexure correction, would imply that the latter was erroneous. As regards the question of refraction, a comparison is given between the temperatures of the air at different altitudes as observed by Mr. Glaisher in his balloon ascents and those assumed by Laplace, Bessel and Lubbock in their theories of refraction; and the inference is drawn that Bessel's refractions may be too large for large zenith distances, through his assumption of an erroneous law for the constitution of the atmosphere, though they may be sensibly correct as far as about Z.D. 80°.

Description of a proposed new Uniform Pressure Clock. By T. Buckney, Esq.

The object of the present paper is to submit to the consideration of the Royal Astronomical Society a means of preventing the changes in the density of the atmosphere from reaching a clock and affecting its rate, by enclosing the clock in an air-tight case in which a uniform pressure may be maintained.

The late Mr. Carrington endeavoured to attain the same end by having a case of copper made for a clock of the ordinary construction; but he does not appear to have been entirely successful. The necessity of winding the clock with a winder passing through a stuffing-box seemed to be a source of frequent trouble, and the repeated breaking of the plate glass covering the dial was also an annoyance. I venture to think the method I am about to describe promises a better result.

I propose, in the first place, to mount the clock movement and hang the pendulum on a massive bracket projecting like a shelf from the wall to which it is fixed; covering both with a bell-glass like the receiver of an air-pump.

To the under side of the shelf I propose to fix a similar bell-glass, but one so elongated as to take in and enclose the pendulum. The upper and under surfaces of this shelf, which would form the seat-plate of the clock, would be worked to a true plane, and the bell-glass covering the movement would have a ground edge, so that the joint made here would be air-tight. It would probably be very difficult to grind the edge of the lower glass on account of its depth, therefore it would be better to fasten to it, by a suitable cement, a metal ring previously worked to a true surface, and thus obtain an equally good joint with the under side of the seat-plate. The space thus enclosed by these glass vessels would form the clock-case, and would be placed in connection with an air-pump by a tube screwed into the seat-plate. Now, setting aside for the moment the necessity of winding, and assuming the above provisions to be successfully carried out, we should have a clock going in an air-tight case in which the pressure may be

reduced to and maintained at any desired point. A barometer tube and a thermometer placed in the case would enable the pressure and temperature to be readily seen.

I will now show how the clock may be kept wound without interfering with the state of things thus far produced. Almost all Observatory clocks, such as this would be, are provided with some means of making galvanic contact every second, this seconds' contact being used for the purpose of pricking on a chronograph barrel, working electric dials, or controlling other clocks—in fact, for distributing the time shown by the standard clock to other parts of the building. At the Royal Observatory, Greenwich, this is done through the agency of a "relay" which, itself worked by a feeble current sent through the clock-springs, sets in motion, simultaneously, three other distinct and more powerful currents, which convey the time to the distant instruments. I propose, therefore, that the clock we are considering shall, in like manner, make a seconds' contact, and work a similar relay: the wires connecting the relay with the clock springs passing through the seat-plate. Now, one of the currents emanating from the relay I propose to take back into the clock-case, and allot to it the task of winding the clock. This it would do by means of a small electro-magnet attracting, every time the current passed—that is, every second—an armature provided with a click which would act upon a ratchet wheel connected with the winding work. Thus the clock would not be an electric clock which any failure of the current would stop and render useless: it would be a clock actuated by a weight as usual; but this weight would be wound up every second by the current set in motion by the clock itself. It would be automatic, assuming the current to pass with regularity: but in the event of the current ceasing, the clock would continue to go until the weight had run down; and even then it might be wound from the outside by means of a hand "contact-breaker" introduced into the circuit which does the winding. But let us suppose that a failure has taken place in the current of the primary circuit, as we may call the one by which the relay is worked, that being the only one with which we need deal, seeing that any defect in the subsidiary ones could be remedied outside and independently of the clock; and let us consider what amount of inconvenience would be caused by any such failure. Now, setting aside as improbable such an accident as a severance or disconnection of the wires, a failure must, I conceive, arise from one of two causes: either the battery must have ceased to generate a current, in which case a new one would have to be substituted for it at no greater inconvenience than is experienced under existing circumstances; or there must be a stoppage of the current by the oxidation and fouling of the contact-springs, which would then require cleaning, for which purpose access to the clock must be had, and the uniformity of the pressure disturbed. But even from this no great harm would, I think, arise. It is assumed that the normal pressure in the clock-case would be but a little less than the lowest

atmospheric pressure of the locality, so that the danger of leakage might be reduced to a minimum. To remove the contact-springs from the clock all that would be necessary would be to let in the air; lift off the upper bell-glass; pull out the slide on which the springs are mounted; do what might be found necessary in the way of cleaning; replace them in their position; put on the bell-glass cover again, and reduce the pressure to its normal point. As I have said, I cannot conceive that such an operation as this should necessarily cause any great or permanent disturbance in the clock's rate. But in any case the contact-springs would require cleaning only at long intervals. The great trouble with galvanic contacts has arisen from the fusing and oxidation of the platinum points by the spark of the induced current produced at the breaking of the circuit. A simple means of entirely preventing this spark is now known, and there is no longer this difficulty. I believe the galvanic contact-springs of the Greenwich Standard Sidereal Clock have been working with the greatest regularity for five months, and it has not yet been found necessary to clean them: what is done at Greenwich in this way may, it is obvious, be done elsewhere with equal success.

We will now return to the winding, which, it will be remembered, is to be done every second. Now, in order that this constant winding may in no way interfere with the good going of the clock, but that the force acting on the clock train may be constantly uniform, I propose to use the arrangement known as the "endless chain," although in this instance it would probably be an endless line of silk of uniform thickness, which I think would answer perfectly. Sufficient fall for the weight should be allowed for the clock to go for two days after a cessation of the winding.

In such a clock any desired pendulum or escapement may be used. The clock should be fixed in such a position that a uniform temperature should be maintained, if possible; and, in order to facilitate this, I would envelop the glass vessels in an outer casing of thick wood lined with felt or other non-conducting material. Unless the air were entirely removed from the clock case—a state of things very difficult to maintain for any length of time—it is evident that changes of temperature would produce corresponding changes of pressure: and although these could, no doubt, be easily corrected by a proper allowance for them in the thermal compensation of the pendulum, as are the other more important temperature effects, still it would, I imagine, be better to avoid these errors. In any case, sudden changes of temperature ought to be carefully guarded against: they do a certain amount of mischief before the compensation is brought into operation.

I may state in conclusion, in order to avoid the charge of appropriating to myself and taking credit for the inventions of others, that the " endless chain "—the earliest form of maintaining power—was invented by the Dutch philosopher Huyghens in

the 17th century: and that the general plan of the air-tight case is almost identical with that used by Mr. Baily in his pendulum experiments; although I was not aware of this until quite recently. The method of winding is, I think, the only thing which is new: if, therefore, any merit attaches to the arrangement herein described, it is due rather to the aggregation of means conducing to the general result than to any special point of detail.

1880, *January*.

Notes on "A Catalogue of 10,300 Multiple and Double Stars &c.," forming Vol. XL of the Memoirs of the Royal Astronomical Society. By H. Sadler, Esq.

(HOURS O–VI.)

No. for Reference.

2. This is 316. Cephei. (Bode.)

3. Σ(2) c.g. This star is not entered in the *Catalogus Generalis* as a double star, for Struve expressly states on the *first* page of that Catalogue: "Stellæ, quibus nullum adjectum est epitheton, ut 6. γ Pegasi, sunt stellæ vere simplices, quantum hucusque compertum est." It is one of the "Stellæ Primariæ," of which a catalogue is given on pp. xxxxviii to l of the Introduction (cf. also the *Summa Catalogi Generalis*, on p. lxxx). The only claim that it has to be considered in any way a *double* star is that it is S.C.C. 2, but no reference is made to this.

8. The N.P.D. should be 79° 31′, not 79° 36′.

14. Σ.6. Add *Rej.* Rejected from the great catalogue because, being of Class iv. in distance, the companion is under the ninth magnitude.

30. Cf. No. 3. The only reason for including it in a Catalogue of Double Stars is that it is S.C.C. 6.

42. OΣ 3. This is identical with No. 62, Σ 19, there being an error of 2^m in the A.R. of the Catalogue of 1843. Cf. page 297 of *Catalogue revu et corrigé*, in the *Recueil de Mémoires*, &c.

57. Σ 18. In a note on No. 90. *h* 1018, Sir J. Herschel observes: "The N.P.D., description, and measures agree well with those of Σ 18, but the R.A. differs by 4^m. Original observation examined. All perfectly regular and clearly entered. It can hardly be the same double star" (p. 132). Σ 18 *is h* 1018, there being an error of 4^m of R.A. and 2′ of Decl. in the Dorpat Catalogue. Cf. Σ. *P.M.*, pp. xc and cxv.

67. Σ 21. Add *Rej.* Rejected in the *Mensuræ Micrometricæ* because the primary is less than the ninth magnitude.

87. This is also σ 6.

93. Σ 26. Add *Rej.* Cf. No. 14.

120. Single, and therefore rejected in the Catalogue of 1850.

128. Single, and therefore rejected in the Catalogue of 1850.

130. The R.A. is 1^m in error; it should be $0^h\ 19^m\ 51^s$. Cf. *P.M.* "Notæ Criticæ," pp. xciii, cxii; also pp. 173 and 295 (reference on).

No. for Reference.

150. This is also S.C.C. 12.

151. Rejected in the Catalogue of 1850, because the distance is 4′ instead of 4″, as printed in the Catalogue of 1843.

160. This is No. 163. OΣ's place is correct.

165. "There is no double star in this place; undoubtedly an erroneous observation of No. 211.' Burnham. A reference to its observation in Catalogue IV (1030) shows this at once.

172. Single. It has no claim whatever to be considered a double star. Cf. No. 3.

181*. *h* 3042. (Read *h* 3442.) "The R.A. has been corrected for an error in R.A. of 1^h. J. H." (Notes, p. 132). Notwithstanding this note of Sir J. Herschel's, this star appears again as No. 545.

195. This is also De. 2. It is also a double of Secchi's, who independently discovered the duplicity of A.

210. Single. Cf. No. 3.

211. This is also σ 10.

218. For OΣ 61, read OΣ 16.

225. Cf. No. 3. It is σ 11, but as Σ'(45) c. g. it is one of the "Stellæ Primariæ."

240. Σ 50. Add *Rej.* Cf. No. 14.

248. Cf. No. 3.

252. Σ 53. Add *Rej.* Cf. No. 14.

261. Cf. *P.M.* pp. 212, 295.

262. Σ 56. Add *Rej.* Cf. No. 14.

268. Σ 57. Add *Rej.* Cf. No. 67.

271. Σ 58. Add *Rej.* Rejected because the distance is greater than 32″.

276. Cf. No. 3. It is σ 12. P.O. 175.

278. Cf. No. 3. It is σ 13.

299. Σ 66. Add *Rej.* Cf. No. 14.

315. The R.A is 0^h 45^m 16^s. Cf. *P.M.* p. cxii.

318. It is Lacaille 240, B.A.C. 246.

348. Σ 81. Add *Rej.* Cf. No. 14.

355. This is 39 Andromedæ.

356. Σ 83. Add *Rej.* Cf. No 14.

364. Σ 85. Add *Rej.* Cf. No. 14.

379. Σ 89. Add *Rej.* Cf. No. 14.

398. Σ 92. Add *Rej.* Cf. No. 14.

405. Cf. No. 3. It is S.C.C. 43.

410. "Simplex." Σ. Cf. *P.M.* pp. 213, 295.

412. Cf. No. 819.

417. Single, and therefore rejected in the Catalogue of 1850.

418. This is identical with No. 420. It is 4 P. I Piscium.

No. for Reference.

419*. Σ 95 = *h* 324. "The R.A. and the position and distance of *h* 324 agree with those of Σ 95; and an error of 10′ in N.P.D. has been assumed in *h* 324, which gives 95° 52′ instead of 95° 42′. J.H." (Note, p. 132). The Decl. given in the *Catalogus Generalis* is really 10′ in error; and the N.P.D. should be 95° 52′, not 95° 42′ as printed on page 8 of vol. xl. Cf. *P.M.* p. xciii.

429*. "Notre étoile double n'est pas 34 Ceti, comme il avait été indiqué dans le Catalogue de 1843". OΣ. *Catalogue revu et corrigé*, p. 301.

431. Single, and therefore rejected in the Catalogue of 1850.

442. This is σ34.

450. Σ 105. 1st. } Cf. *P.M.* p. xciii. The places should therefore be
455. Σ 105. 2nd. } reversed.

453. Σ 102 = No. 452, *h* 2033. Cf. Σ *P.M.* p. xc. The place of Σ 102 in the *Catalogus Novus* should be increased by $2^{m}\cdot 1$, and the Decl. diminished by 2′. Cf. *P.M.* p. cxv. *M.M.* p. xxxvii.

461. Rejected in the Catalogue of 1850.

466. This is σ 36.

476. The N.P.D. should be 114° 29′.

481. Σ 116. Add *Rej.* Cf. No. 14.

494. The R.A. is 1^{h} too large.

495. Single. Cf. No. 3.

498. *h* 2044 = Σ 119 (No. 505) "indubié" Σ. Cf. also *P.M.* p. xciii.

511. Σ 123. Add *Rej.* Cf. No. 14.

523. Σ 126. Add *Rej.* Cf. No. 14.

527. Σ 128. Add *Rej.* Cf. No. 14.

539. The correct R.A. is $1^{h}\ 23^{m}\ 22^{s}$.

545. Cf. Note on No. 181.

594. For S.C.C. 72, read S.C.C. 62.

619. Σ 152. Rejected in the *Mensuræ Micrometricæ*.

620. Σ 151. Rejected in the *Mensuræ Micrometricæ*.

632. The R.A. should be $1^{h}\ 35^{m}\ 30^{s}$. Cf. *P.M.* p. xciii.

635. *h* 643 = No. 643, Σ 160. Cf. *P.M.* p. xc.

644. Σ 159. Add *Rej.* Cf. No. 14.

645. Σ 161. Add *Rej.* Cf. No. 14.

656. Σ 165. Add *Rej.* Cf. No. 14.

663. Σ 167. Add *Rej.* Cf. No. 14.

671. Σ 173. Add *Rej.* Cf. No. 14.

677. This is also R(2).

678. Σ 176. Add *Rej.* Cf. No. 14.

679. Single. Cf. No. 3.

681. Σ 177. Σ′ (162) e.g. is not Σ 177. Cf. *P.M.* p. xciii. The R.A. of Σ 177 is $0^{m}\cdot 6$ less than the one given.

No. for Reference.

692. "Étoile que je n'ai pas retrouvée." OΣ. It may be that some confusion has arisen between this star and the preceding one.

697. Σ 181. Add *Rej.* Cf. No. 14.

708. Σ 184. Add *Rej.* Cf. No. 14.

716. Σ 188. Add *Rej.* Cf. No. 14.

720. Σ 187. Add *Rej.* Cf. No. 14.

721. Σ 190. Add *Rej.* Cf. No. 14.

728. This is σ 50.

743. Σ 198. Add *Rej.* Cf. No. 271.

752. Σ 203. Add *Rej.* Cf. No. 67.

760. This is σ 52.

764. Σ 209. Add *Rej.* Cf. No. 271.

771. Σ 210. Add *Rej.* Cf. No. 14. (The R.A. given in the *Catalogus Generalis* is correct.)

772*. Σ 211. Add *Rej.* Cf. No. 14.

783. Cf. No. 3. It is S.C.C. 85.

792. This is S.C.C. 86.

798. Σ 220 is identical with No. 794, Σ 218. The place of the latter is the correct one. Cf. *P.M.* p. cxv, *M.M.* xxxiv.

809. This is Hh 55.

819. "η Arietis. Not double." Burnham. It is R(2), not R2.

822*. Single, and therefore rejected in the Catalogue of 1850.

833. This is identical with No. 860, Σ 246.

843. Single. Σ observes in the preface to his "Catalogus 795 Stellarum Duplicium ex diversorum astronomorum observationibus congestus, &c." (*Dorpat Obs.* III. p. xiv.):—"[Stellæ] quibus autem asteriscus appositus est, eæ seu in cœlo non ita inventæ sunt, seu duplices non apparuere, ulterioremque partim poscunt disquisitionem." The following stars contained in the first six hours of Vol. XL. fall under this category, not being included in other double star catalogues:—Nos. 520, 813, 1238, 1914, 2125 (Mayer), 2256, and 2325.

850. This star is single, and is the star 65 Arietis (Bode).

852. This star is not 65 Arietis (Bode), which is 4^s p and 6′ to the south. Cf. Σ. *P.M.* pp. 156, 174. It is incorrectly called 65 Arietis (Bode) in the *M.M.* p. 89.

855. Σ 242. Add *Rej.* Cf. No. 271.

856. Σ 243. This is identical with No. 889, Σ 256; there being an error of 5^m in the R.A. of Σ 243 in the *Catalogus Novus.* Cf. *M.M.* p. xxxiv, *P.M.* p. cxv.

864. Σ 247. Add *Rej.* Cf. No. 67.

882. Σ 253. This is identical with No. 897, Σ 258; there being an error of 2^m in the R.A. of Σ 253 in the *Catalogus Novus.* Cf. *M.M.* p. xxxiv, *P.M.* p. cxv.

886. Σ 255. Add *Rej.* Cf. No. 67.

No. for Reference.

887. This is Piazzi ii. 61, S.C.C. 96.

893. This star is single. It is entered in the *C.G.* as "vicina ad [Σ] 255." Cf. *P.M.* pp. xciv, 174.

896. This is identical with No. 917.

900. Σ 259. Rejected in the *M.M.*

912. Σ 267. Add *Rej.* Cf. No. 67. This star is, however, not Σ 267 *rej.* Cf. Note on p. 295 of the *P.M.* and p. 174. On examining its place in the heavens Σ found that Σ 267 was about 3′n and 8′f this star No. 912, which is C.G. 235. See *P.M.* p. xciv.

914. The A.R. may be 1^m in error. Cf. *P.M.* xciv., and p. 195. Σ remarks (p. 295) that the note on p. 195 makes it pretty certain that the star observed (*C.G.* 236) can hardly be Σ 264.

925. For Σ 239 read Σ 269, and also in the note on p. 133.

926. "13 Trianguli." This star is not double, and it is not 13 Trianguli. Cf. *P.M.* pp. 156, 215.

947. This is one of Struve's "Stellæ excludendæ, quia in cœlo non repertæ sunt."

964. This is also σ 75.

1007. The N.P.D. is 1° in error, it should be 47° 53′. Burnham states that this is identical with No. 1002; OΣ thinks it is not the same. Nos. 1000, 1002, and 1007 are in the cluster 34 Messier.

1016. Σ 298. Add *Rej.* Cf. No. 14.

1032. Σ 304. Add *Rej.* Cf. No. 14.

1036. Σ 305. The N.P.D. should be 71° 20′. Cf. *P.M.* p. xciv.

1042. Σ 339. Add *Rej.* Cf. No. 14.

1049. This is σ 83, = S.C.C. 117, = OΣ 47 *Rej.*

1057. The seconds of R.A. should be 33^s. Cf. *P.M.* p. xciv.

1075. Σ 319. Add *Rej.* Cf. No. 14.

1077. This is one of Struve's "Stellæ excludendæ, quia examine instituto non duplices sed simplices apparuere."

1082. Σ 327. Add *Rej.* Cf. No. 14.

1120. Cf. No. 3. It is S.C.C. 125, which is the only reason for including it in a Reference Catalogue of *Double* Stars.

1125*. Σ 340. "Not found by *h* in the place indicated. J. H." (Note, p. 133.) Σ 340 is identical with Σ 345, there being an error of 1° in the Decl. of Σ 340 as given in the *Catalogus Novus.* Cf. *M.M.* p. xxxiv, *P.M.* p. cxv.

1129. This is S.C.C. 126.

1130. This is identical with No. 1132, *h* 2172 being A of OΣ 50 and a third companion.

1135. Σ 348. Add *Rej.* Cf. No. 14.

1136. Σ 347. This is identical with No. 1128, Σ 343, the place of the latter being the correct one. Cf. *P.M.* p. xcv, &c.

1141. Cf. No. 3. It is S.C.C. 125 (the companion was discovered by Schröter), and up to a year or two ago this was the only ground on

No. for Reference.

which it could have been included in a *Double* Star Catalogue. It is one of Σ's "Stellæ Primariæ," and it is on this account that it appears in the *Catalogus Generalis* as No. 309.

1143. Σ 353. Add *Rej.* Cf. No. 67.

1145. Σ 354. Add *Rej.* Cf. No. 67.

1175. Σ 365. Add *Rej.* Cf. No. 67.

1180. Σ 366. Add *Rej.* Cf. No. 67. "Sine dubio in hujus stellæ singulâ R.A., a Preussio observata, error 1′ est commissus, et a C.G. minuto temporis est minuenda." *P.M.* xciv. The R.A. should be therefore $3^h\ 4^m\ 24^s$.

1197. Σ 373. Add *Rej.* Cf. No. 14.

1212. *h* 3565. The N.P.D. should be 109° 11′.

1217. No mention of Jacob's companion.

1218. Cf. No. 3. As a "double" star it is S.C.C. 131.

1237. Σ 387. Add *Rej.* Cf. No. 14.

1246. OΣ 55. Rejected in the Catalogue of 1850.

1256. OΣ 56. Rejected in the Catalogue of 1850.

1259*. Mäd. Dorp. xi. (1). This is identical with No. 1266.

1263. Σ 399. Add *Rej.* Cf. No. 14.

1272. Σ 402. Add *Rej.* Cf. No. 14.

1273. Σ 404. Add *Rej.* Cf. No. 14.

1274. Σ 405. Add *Rej.* Cf. No. 14.

1278. The R.A. should be $3^h\ 21^m\ 52^s$, the N.P.D. 101° 44′.

1283. Σ 409. Add *Rej.* Cf. No. 14.

1284. This is *h* 334, not Σ 410; which is $1^m\ 5^s$ f and 3′ 51″ s. Cf. Burnham in *English Mechanic*, August 20th, 1875, and Σ, *P.M.* p. xciv.

1286. Σ 411. Add *Rej.* Cf. No. 14.

1289. OΣ 58. Rejected in the Catalogue of 1850, because it cannot be found. It may be identical with Σ 414, with an error of 35′ in the Decl. Cf. *Catalogue revu et corrigé.*

1293. Σ 417. Add *Rej.* Cf. No. 14.

1294. Σ, from a comparison with the observation of this star in *h* V, finds an error of 1^m in the R.A. of the *Catalogus Generalis.* It should be $3^h\ 26^m\ 7^s$. Cf. *P.M.* p. xciv.

1295. Σ 416. Add *Rej.* Cf. No. 14.

1324. Σ 428. Add *Rej.* Cf. No. 14.

1325. Cf. No. 3. It is S.C.C. 135.

1326. Σ 429. Add *Rej.* Cf. No. 14.

1332. Σ 432. Add *Rej.* Cf. No. 14.

1333. Σ 433. Add *Rej.* Cf. No. 271.

1343. OΣ 61. Rejected in the Catalogue of 1850. It is, however, double.

1349. Σ 441. Add *Rej.* Cf. No. 14.

No. for Reference.

1354. Single.

1362*. 1363*. "σ 105 = S 437 and S.C.C. 140. A doubt exists of these being separate double stars." (Note, p. 133). S.C.C. 140 is a single star, 23 Pleiadum, and is used by Smyth as a pointer to σ 105 = S 437.

1376. Σ 451. Add *Rej.* Cf. No. 14.

1385. Σ 454 is identical with No. 1369, Σ 446; the place of the latter being the correct one. Cf. Σ, *M.M.* p. xxxiv., *P.M.* p. cxv.

1386. Σ 456. Add *Rej.* Cf. No. 14.

1411*. Σ 462 = *h* 2206. "The R.A. by *h* is 5^m less than that by Σ (in the *Catalogus Generalis*), and a note to the observation by *h* in the fourth series of observations of double stars states that it is possibly not the same double star." (Note, p. 133.) For "fourth series" read "fifth series." *h* 2206 *is*, however, Σ 462, and the R.A. of Σ in the *C.G.* is correct. Cf. *P.M.* p. xcv. *h* identifies his 2210½ with Σ 462, but the two stars cannot be the same, as is evident from the description of 2210½ in H's Fifth Catalogue.

1418. This is σ 109.

1431. Σ 467. "Non est." Σ, *P.M.* p. cxv., *M.M.* p. xxxv.

1434. Σ 468. This is identical with No. 1428, *h* 5458, there being an error of $0^m \cdot 9$ in the *Catalogus Novus.* Σ, *P.M.* cxv.

1475. Σ 488. Add *Rej.* Cf. No. 14.

1491. Σ 492. Add *Rej.* Cf. No. 67.

1503. Σ 496. Add *Rej.* Cf. No. 67.

1508. Σ 497 = *h* 672. In note, p. 133, for Σ 672 read *h* 672.

1509. Σ 499. This is *not* Σ 499. Σ's double is 20^s f and 4′ north of this star. Cf. *P.M.* p. ccliv.

1514. Σ 502. Add *Rej.* Cf. No. 67.

1524. Σ 507. This is identical with No. 1531, Σ 513, there being an error in the R.A. of the former in the *Catalogus Novus.*

1552. The R.A. is $4^h\ 8^m\ 12^s$. Cf. *P.M.* p. 153, and the final correction on p. xcv.

1553. This is the double companion of No. 1551.

1557. Σ 519. Add *Rej.* Cf. No. 14.

1567. This is σ 119.

1574. Σ 525. Add *Rej.* Cf. No. 67.

1596. Σ 532 is rejected in the *M.M.* Cf. No. 67. No. 1596 is *not* Σ 532, which is not in the *C.G.* but is "*vicina sequens ad* 532." Cf. *P.M.* p. xcv.

1606. Σ 539. Add *Rej.* Cf. No. 14.

1609*. Σ 541. Add *Rej.* Cf. No. 67. "This appears likely to be the same double star as No. 1608, H*h* 116, 65 Tauri; the place was incorrectly set down in the MS.; and so, perhaps, the probable identity escaped the author's notice." (Note, p. 133). Σ 541 cannot be identical with No. 1608; it is possibly identical with a little pair of De's between κ^1 and κ^2 Tauri. (*A.N.* No. 1979.)

No. for Reference.

1637. "Σ' (445) c.g." This is entered in the *C.G.* as "*vicina ad* 549," and is a single star.

1639. This is also Σ 10; App. I.

1641*. S 450. "This was entered in MS. in pencil, as if a doubt existed of its being really a double star." (Editors' note, p. 133.) The star is of course identical with No. 1645.

1650*. The correct R.A. is $4^h\ 19^m\ 18^s$.

1652. Σ 556 is rejected in the *M.M.* Cf. No. 14. No. 1652 is *not* Σ 556, which is not in the *C.G.*, being overlooked on account of its faintness. The star given is single. Cf. *P.M.* p. xcv.; see also note on p. 295.

1655. Σ 555. Add *Rej.* Cf. No. 14.

1660. Σ 557. The A.R. of the *Catalogus Generalis*, which is based on a single observation, is 1^m in error. The AR should be $4^h\ 22^m\ 3^s$. Cf. *P.M.* p. xcv.

1667. Mäd. Dorp. xi. 2. This is identical with No. 1656. *h* 679 = OΣ 84; Mädler's R.A. being $1^m\ 45^s$ in error.

1673. Σ 560. Add *Rej.* Cf. No. 14.

1679. Σ 561. Add *Rej.* Cf. No. 14.

1691. Σ 568. Add *Rej.* Cf. No. 14.

1706. Σ 573. Add *Rej.* Cf. No. 14.

1722. Σ 580. Add *Rej.* Cf. No. 14.

1724. Σ 581. Add *Rej.* Cf. No. 67.

1737*. Σ 586. "There is a doubt of the existence of this star: it may be really identical with Σ 587. J.H." (Note p. 134.) In *Memoirs*, vol. xxxv. p. 69, Sir J. Herschel remarks: "There are *not* two distinct double stars, Σ 586 and 587." Σ himself shows the identity of his 586 with 587 on p. xxxiv of the *M.M.*, there being an error of 1^m in the *R.A.* of 586 in the *Catalogus Novus*. Cf. also *P.M.* p. cxv.

1739. This is not Σ 585, the place in the *C.G.* being that of another star. Σ 585 is 14^s preceding and 5′ north of No. 1739, which is a single star. Cf. *P.M.* pp. xcv. ccliv. 144, and the reference on page 295.

1745. This is identical with Σ 587, No. 1742.

1750. Σ 591. Add *Rej.* Cf. No. 67.

1751. Σ 592. Add *Rej.* Cf. No. 14.

1755. This is identical with No. 1742, which is Σ 587. H's place is in error. See No. 1737* *supra*.

1758. Σ 593. Add *Rej.* Cf. No. 67.

1760. The star is single. Cf. No. 3.

1763. Σ 594. Add *Rej.* Cf. No. 67.

1766. Σ 597. Add *Rej.* Cf. No. 14.

1771*. *h* 684. } "Suspected to be the same double star." (Note p. 134.) There is no doubt of this being the case.

1772*. Σ 601. } Add *Rej.* One of Struve's "in acervis."

1773. Σ 600. Add *Rej.* Cf. No. 14.

No. for Reference.

1786. 1793. 1800*. Note to No. 1800. *h* 2240½. "The measures agree with those of Σ 609, but *h* has two observations closely agreeing in place." (p. 134). Σ 609 is *h* 2240½, which is also *h* 686 (No. 1786). No. 1793 is not Σ 609, which is not in the *Catalogus Generalis*, as the star given in the *C.G.* as Σ 609 is a single star of the 7th magnitude 1^m 3^s p and 6′ 29″ south of the true Σ 609. Cf. *P.M.* p. xcvi.

1789. Σ 605. Add *Rej.* Cf. No. 67.

1802. This is entered in the *C.G.* as "vicina ad 607," and is a single star. Σ 607 itself does not occur in the *C.G.*

1804. Σ 610. No mention of the duplicity of A discovered by De.

1807. Σ 611. Add *Rej.* Cf. No. 14.

1822. $Σ^1$ (487) c.g. Single. Cf. No. 3.

1830*. The R.A. is 4^h 46^m 29^s. The N.P.D. is 115°. (Cf. Note, p. 134.)

1841. $Σ^1$ (498) c.g. This is entered in the *C.G.* as "vicina ad 620." It is a single star.

1852. Σ 624 = *h* 30. "The minute of R.A. of *h* is 48. The observation is correctly reduced. J. H." (Note p. 134.) Σ's place is correct.

1859. Single. Cf. No. 3.

1860. Σ 626. Add *Rej.* Cf. No. 14.

1865. Σ 628. Add *Rej.* Cf. No. 14.

1877. H*h* 145 is entered twice over, as No. 1873 and here. The correct place is that of No. 1873. It is undoubtedly Σ 630.

1886. OΣ 94. Rejected in the Catalogue of 1850.

1890. $Σ^1$ (512) c.g. Single. Cf. No. 3.

1902. OΣ 96. Rejected in the Catalogue of 1850.

1913. Σ 641. Add *Rej.* Cf. No. 14.

1921. Σ 642. Add *Rej.* Cf. No. 14.

1932. Σ 647. 'Simplex.' Σ, *M.M.* p. xxxv. &c. It is also S. C. C. 183.

1934. Single.

1938. Σ 650. Add *Rej.* Cf. No. 14.

1958. This is probably identical with No. 1966, there being a mistake of 1^m in the R.A. of one of the two pairs.

1974. Σ 660. This is identical with No. 1972, Σ 657; the place of Σ 660 being in error. Cf. *M.M.* p. xxxiv. *P.M.* p. cxv.

1976. Σ 662. "The P.D. by the Dorpat Catalogue is 64° 15′, and the same N.P.D. is to be found for this star in the *Mensuræ Micrometricæ*, p. 110; but in the *Catalogus Generalis* it is given as 94° 15′ 54″! J. H. It is entered in the present Catalogue with N.P.D. 64° 16′." (Note p. 134.) The mistake in the *C.G.* is corrected on p. xcvi. of the *P.M.* The place of Σ 662 is 5^h 5^m 30^s N.P.D. 64° 15′.

1994. Σ 672. Add *Rej.* Cf. No. 14.

2018. Σ 682. Add *Rej.* Cf. No. 14.

2043. This is 22 Orionis.

No. for Reference.

2057*. Σ 699. "The N.P.D. of this star was set down in the MS. from the *Catalogus Generalis* as 52° 17′ 11″, but it is printed 52° 7′ on the authority of three observations of this star made at the Radcliffe Observatory." (Note p. 134.) The faulty decl. of the *C.G.* is corrected on p. xcvi. of the *P.M.*

2074. Σ 707. The R.A. should be increased by 1^m, and the N.P.D. diminished by 2′.

2080. Σ 703. Add *Rej.* Cf. No. 14.

2107. Σ 717. Add *Rej.* Cf. No. 14.

2119. Σ 722. Add *Rej.* Cf. No. 14.

2143. This is not Σ 733. Σ 733 precedes this star by 35^s. Cf. *P.M.* p. ccliv. No. 2143 is a single star.

2153. Single.

2162. This star is single. It is entered in the *C.G.* as "vicina ad 743."

2165. This is also R (3).

2167. This is identical with No. 2177.

2173. This is not 133 Orionis (Bode).

2176. Σ 746. Add *Rej.* Cf. No. 67.

2177. This is 133 Orionis (Bode).

2179. This is entered in the *C.G.* as "altera ad 743," and is a single star Cf. No. 2162.

2181. This is Σ 16; App. I.

2193. Σ 756. Add *Rej.* Cf. No. 14.

2194. "Σ 757." Σ 757 is 35^s f. and 2.5′ n. of No. 2194, which is *C.G.* 612, and is a single star. Σ 757 is 3^s p. Σ 758, on the same parallel of Decl. Cf. *P.M.* p. ccliv., the note on p. xcvi. being incorrect.

2206. Σ 760. Add *Rej.* Cf. No. 14.

2220. This is identical with No. 2201, Σ 758, there being an error of 2^m in the R.A. of Σ 765 in the *Catalogus Novus.* Cf. *P.M.* p. cxv. *M.M.* p. xxxiv. &c., and see No. 2194.

2224. Σ 767. This is identical with No. 2315, Σ 806, there being an error of 10^m in the R.A. of Σ 767 in the *Catalogus Novus.* Cf. *P.M.* p. cxv, *M.M.* p. xxxiv. &c.

2231. "Σ 773." This is not Σ 773, which is 25^s f. and 3′ south of this star, which is *C.G.* 621, and is a single star. Cf. *P.M.* p. ccliv., and note on page xcvi.

2233. This is Leporis 45 (Bode); the place being $5^h\ 31^m\ 47^s$. 107° 59′.

2247. This is not Σ 781, which is 24^s f. on the same parallel. No. 2247 is *C.G.* 628, and is a single star. Cf. *P.M.* p. ccliv.

2263. Σ 789. Add *Rej.* Cf. No. 14.

2274. Single. Cf. *P.M.* p. 198. It is entered as "Anonyma 7·7, praecedens 791."

2283. This is 133 Tauri.

2287. Σ 793. Add *Rej.* Cf. No. 14.

No. for Reference.

2294. Single.

2301. For N.P.D. 144° 34′ read 114° 34′.

2305. Does not exist. Cf. *M.M.* p. xxxv. *P.M.* p. cxv.

2307. Σ 801. Add *Rej.* Cf. No. 14.

2309. Σ 803. Add *Rej.* Cf. No. 67.

2310. Σ 804. Add *Rej.* Cf. No. 67.

2321. This is also De. 10.

2327. "Σ 813." This is not Σ 813, which is $0^m{\cdot}9$ *f.* No. 2327 is single. Cf. *P.M.* pp. xcvi. 158.

2330. N.P.D. 16° 1′.

2332. Σ 812. Add *Rej.* Cf. No. 14.

2336. Does not exist. Cf. *M.M.* p. xxxv. *P.M.* p. cxv.

2344. This is identical with No. 2346; the place of the latter star being the correct one.

2348. Σ¹ (656) c.g. One of the "Stellæ Primariæ." Cf. No. 3.

2360. Σ¹ (657) c.g. Cf. No. 2348.

2366. Σ 822. Add *Rej.* Cf. No. 14.

2389. Rejected in the Catalogue of 1850.

2393. Σ 827. Add *Rej.* Cf. No. 14.

2396. Σ 828. "Fortasse locus ita erroneus, ut sit eadem ac 849, errore 7′ in A.R. assumpto." Σ. *M.M.* p. xxxv. Cf. *P.M.* p. cxv. also p. 176. No. 2396 is a single star, and there is no doubt of the identity of Σ 828 with Σ 849.

2398. "Loco" Σ 828. Cf. *P.M.* p. 176. The star is single.

2405. Σ 832. Add *Rej.* Cf. No. 14.

2410*. Σ 833. Add *Rej.* Cf. No. 14. "This star was not seen by *h*, after repeated search for it. J. H." (Note p. 134.) No doubt some confusion has arisen between this object and No. 2415, and 2410 probably does not exist. R.A. of Σ 833 in *Catalogus Novus* is $5^h\ 53^m\ 5^s$. "Nisi A.R. = 54·5"?

2413. "Σ 835." This is not Σ 835, the R.A. of which is $5^h\ 53^m\ 40^s$. Cf. Σ *Cat. Novus* and *h V. Cat.* No. 2413 is *C.G.* 671, and is a single star. Cf. *P.M.* pp. xcvi. and 233.

2415. Σ 837. Add *Rej.* Cf. No. 14.

2416. Σ¹ (672) c.g. Entered in the *C.G.* as "vicina ad 835." The star is single.

2424. *h* 2292. This is identical with No. 2389, the place of the latter star being the correct one.

2434. The star does not exist. *M.M.* p. xxxv. *P.M.* p. cxv.

2435*. Σ 843. Add *Rej.* Cf. No. 67.

2436. Σ 842. Add *Rej.* Cf. No. 14.

2441. Σ¹ (676) c.g. Entered in the *C.G.* as "vicina ad 846;" it is, however, a single star.

2446. The N.P.D. should be 87° 51′. Cf. *P.M.* p. xcvii.

Note on the Phenomenon known as "Shadow Bands."
By Dr. R. Copeland.

I had read accounts of the "Shadow Bands" sometimes seen during solar eclipses, in the *Astronomische Nachrichten*, Nos. 1921 and 1922, and elsewhere, but had not the slightest idea that any kindred phenomenon could occur at sunset or sunrise until December 18, 1879, when an appearance of the same kind was most conspicuously visible on the inner wall of the great dome at Dunecht. The bands, or rather lines, were, generally speaking, about ¼ inch broad, almost exactly horizontal, and some 2½ inches apart. They were produced by the last rays of the Sun then setting behind the smooth horizontal ridge of the Hill of Fare, five miles distant. The lines ascended at the rate of a foot and a half in a second, and had a distinct quivering motion. The lines were seen by Herr Lohse (the assistant astronomer) and by the writer. An account of them was published in a recent No. of *Nature*.

The bands having been once seen were naturally looked for on subsequent occasions. Their occurrence proved to be very uncertain and variable. There are very fine sunsets which do not give a trace of them even on the whitest surface most completely screened from any but the direct light of the Sun. At other times the lines are short, crooked and disconnected, *descending* as well as ascending, forming and disappearing within a few seconds.

On January 13, 1880, when the Sun was setting behind the slope of Greymore (one of the summits of the Hill of Fare) the profile of which is inclined about 20° to the horizon, the northern end being the higher, Herr Lohse saw the dark lines running *downwards* for about 20 seconds. The apparently regular intervals were about half an inch broad, while the thickness of the lines was, roughly, ⅛ inch. The motion was about one foot in four or five seconds. The lines were not horizontal but parallel to the slope behind which the Sun was disappearing. The lines became visible on a sheet of white paper in a room with one closed plate-glass window when about two-thirds of the Sun's disk had disappeared. Prior to this for a short time there was an irregular ripple-like mixture of light and shade, which also had a decided downward motion.

On February 3, 1880, the writer saw the lines much broken and disconnected, some ascending, a greater number descending, while a few formed and hovered on the screen, disappearing in a few seconds.

One appearance may be mentioned, as it gives what seems to be a clue to the origin of the lines. Most people have noticed the small crescent-shaped images of the partially eclipsed Sun formed by its rays passing through foliage or other imperfect obstruction. In the same way when the disk of the setting Sun is reduced to a mere line and its light falls through the branches of a leafless tree the partial images so formed are reduced to straight lines

separated by straight dark bands parallel to the apparent horizon. This occurs whatever may be the average direction of the branches.

In the case of lines formed by the crest of a distant hill (or the Moon's limb) it is assumed that irregularities in the densities of the various strata of the intervening air form the requisite obstructions, and as these irregularities vary from instant to instant, so do the lines move or change. When the source of light subtends a very small angle we have scintillation. From this point of view the dark lines are a phenomenon of scintillation projected on a screen. This hypothesis readily explains the coloured fringes of the eclipse shadow bands seen by some observers.

As seen from the neighbourhood of the Observatory at Dunecht, the Sun sets behind one or other part of the treeless ridge of the Hill of Fare for about nine weeks before and after the winter solstice. For the rest of the year the Sun disappears or reappears behind a wooded horizon far less favourable to the production of a regular phenomenon of the character described. The Sun being now beyond the limits mentioned, these notes are laid before the Society in the hope that some member who resides in a favourable locality may be on the look-out for the bands. It is at least favourable, if not essential, to their visibility that the screen on which they are formed should be protected from side light, and that the observer should be very close to the screen.

A couple of white screens making a known angle with each other and with the horizon, exposed in a room with an open window but with partially closed shutters, would probably be a good arrangement.

On comparing the above with the account of the eclipse shadow bands at p. 47, *Mem. Royal Astronomical Society*, vol. xliv., it occurs to the writer that the apparent rotation of the bands about a distant centre there described was most probably an effect of perspective, just as furrows transverse to a line of railway appear to rotate before the eyes of a traveller. In future eclipses it may be well to see if the bands rotate about *two* opposite centres, or vanishing points.

Dunecht Observatory,
1880, *March* 6.

Note on the Photographic Diameter of the Moon. By E. Neison, Esq.

In his Note in the *Monthly Notices* for January 1880 (page 167) Prof. Pritchard adduces complete evidence of the accord between the measures made by himself and by the two assistants of the Oxford Observatory. It is with great satisfaction that I learn that the lunar photographs taken at Oxford will permit of such sharp and accordant measures being made. The difference 0″·56 between the separate results of Mr. Jenkins and Mr. Plummer is insignificant.

I would, however, point out that it is impracticable by means of measuring any single photograph to determine the error introduced through the irregularities on the Moon's limb; so that the evidence adduced by Prof. Pritchard fails to affect the justice of my criticism.

If these irregularities were of the nature of pointed peaks at intervals projecting in the sky, then Prof. Pritchard would be correct. But, in truth, they are great elevations extending along the limb for hundreds of miles at a time, so that it is indifferent from what point you measure: in every case you have a systematic error.

It is this systematic error which is so dangerous; the mere presence of peaks at intervals would give rise to errors which would disappear in the mean.

On some Changes in the Markings of Mars, since the Opposition of 1877. By N. E. Green, Esq.

The details of *Mars* were carefully scrutinised during the past Opposition, in order to identify as far as possible the forms observed at Madeira in 1877, and to detect any changes that might have occurred since that date.

The atmosphere of St. John's Wood has not been favourable, few evenings afforded good views, and even on favourable occasions, either the atmosphere of *Mars* or its brilliant reflection made definition especially trying to the eye. The best views were obtained on nights when *Saturn* was almost put out by fog, which, reducing the glare of *Mars*, served only to make its features more distinctly visible.

However, several drawings were made, the details of which confirm by far the greatest part of the map appended to the drawings of *Mars* in the Memoirs. The various markings have been identified point by point. Indeed, with the exception of a few minute forms, the whole has been re-observed, and where the London air has failed, Professor Niesten at Brussels, Mr. Burton near Dublin, and Mr. Denning at Southampton, have succeeded in drawing just those parts which required confirmation.

Professor Niesten has forwarded a very valuable series of 16 drawings made during the last Opposition, and Mr. Burton has been particularly fortunate in securing some very exact views of the neighbourhood of Terby Sea.

The intention of this Note is to point out a few changes in the surface markings, some of which have special interest, when the drawings of 1879 are compared with those of previous Oppositions. One of these changes is the appearance during November and December 1879 of a band of light in latitude 20° S. extending from longitude 260° to 360°, uniting in one long line of light Dreyer, Hirst and Phillips Islands. To the east of

Phillips Island this band of light turned towards the equator, and, passing between Dawes Forked Bay and Burton Bay, formed a connection with Beer Continent. The interest connected with this appearance is that it repeats what was seen by Beer and Mädler in 1830, by Lockyer in 1862, and Kaiten in 1864, whereas at Madeira in 1877 this portion of the planet's surface was more or less filled with half tint, on which the islands were but indistinctly seen. This was especially the case with Phillips Island, and the space between Dawes' Forked Bay and Burton Bay always appeared dark enough to continue the equatorial band as an unbroken effect of shade.

Near Noble Cape, during the last and some previous oppositions, a line of light has been occasionally seen, connecting this form with Webb Land; but here also there was the most definite filling up of half tint during the observations made at Madeira.

These temporary observations of the darker portions of the surface are amongst the most frequent phenomena of the planet, and will account for many discrepancies in the drawings of different observers.

One of the most remarkable instances of obscuration and reappearance is that of the dark mark to the north of Terby Sea known in Mr. Proctor's map as Dawes Sea. This form was drawn by Dawes, Lockyer and Kaiten, but in the Opposition of 1877 was certainly invisible. A most careful and persevering search was made for it at Madeira, with all powers, and on the best occasions, and, with the exception of a faint elongated streak of shade, nothing could be seen. This mark has returned in all its previous definite character during the Opposition just past.

With regard to the dark canals so sharply depicted by Professor Schiaparelli, both Mr. Burton and myself have seen traces of them, and in one instance there is very fair agreement in position, but it is hardly safe to regard them as belonging to the permanent markings, for if all the dark lines which have been seen by observers were filled in on a single map the greatest confusion would ensue.

It is possible that some of these lines may be the boundaries of faint tones of shade, so delicate that they escape the notice of any but a well-trained eye, or they may be spaces between veil-like masses of atmosphere; in either case their position would be variable.

These observations would lead us to regard the larger dark marks as the most permanent features of the planet, but subject to partial obliteration, or even long-continued disappearance, from the imposition of some lighter cloud-like material.

MONTHLY NOTICES

OF THE

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Mr. DUNKIN, F.R.S., Vice-President, in the Chair.

Lieut. T. Preston Battersby, R.A., Cromlyn, Rathowen, Westmeath, Ireland;

was balloted for and duly elected a Fellow of the Society.

On Mean Refractions. By E. J. Stone, M.A. F.R.S.

In the *Monthly Notices* for December 1867 there is a paper of mine on Bessel's Mean Refractions. The results there given were obtained from a discussion of the circumpolar observations made at Greenwich from 1857 to 1865. These observations were divided into five groups with N.P.D. $0^\circ-15^\circ$; $15^\circ-30^\circ$; $37^\circ-37^\circ\ 40'$; $37^\circ\ 40'-43^\circ$; $43^\circ-46^\circ\ 29'$; and, when the observations were reduced with Bessel's Refractions, the following values of excess of N.P.D. below pole over N.P.D. above pole were found for the five groups:—

	(1)	(2)	(3)	(4)	(5)
	$''$	$''$	$''$	$''$	$''$
A	+0·240	+0·196	+0·570	+1·054	+2·190.

The theoretical weights are

130	88	31	54	28.

Then assuming that the true co-latitude

$$= 38^\circ\ 31'\ 21''\cdot80+\tfrac{1}{2}x,$$

and True Refraction=Bessel's Refraction $(1-y)$, a notation which will be adopted throughout the present paper, the residuals A are reduced to the following, when $x=-0''\cdot41$ and $y=0\cdot0053$,

	(1)	(2)	(3)	(4)	(5)
	"	"	"	"	"
B	+0·15	−0·10	−0·14	−0·08	+0·10

The residuals A are out of all proportion to the theoretical probable errors of the groups; those of B are certainly well within them. The results of this paper were at the time accepted by the Astronomer Royal and used in the reduction of the observations 1868 to 1876.

It is, however, no great surprise to me to find, that the Astronomer Royal now proposes to abandon the use of these diminished Refractions and to revert to Bessel's unaltered Refractions. As a matter of convenience, pending further enquiry, I believe such a step may be expedient. But, while fully admitting such to be the case, I must call attention to the very slight evidence upon which the rejection of these diminished Refractions is based, and to some points of an unsatisfactory character in Bessel's Refractions.

When it was proposed, in 1868, to use the diminished Refractions it was, of course, known that the N.P.D.'s of the stars observed at Greenwich would systematically differ from the corresponding observations contained in the Catalogues for 1860 and 1864; such discordances do not show whether the old refractions or the new refractions are correct; they merely show that they are different.

With respect to the Sun observations the following are the values of z which were found 1857–1869 in the investigations of the positions of the plane of the ecliptic.

Year.	z.	
1857	−0″·640	
1858	−0·679	
1859	−0·683	
1860	−0·438	
1861	−0·600	Bessel's Refractions were used, and the Transit-Circle cube was unpierced, until 1865, August.
1862	−0·646	
1863	−0·689	
1864	−0·983	
1865	−0·510	
Mean	−0·641	
1866	−0·004	Bessel's Refractions were used, but the Transit-Circle cube was pierced.
1867	+0·118	
Mean	+0·057	
1868	+0·757	
1869	+0·487	
Mean	+0·622	Bessel's Refraction (1 − 0·0053).

The greatest change which could be introduced into the value of z by the change of refraction does not exceed $+0''\cdot7$. So far, therefore, as the observations 1857–1865 are concerned, these Sun observations do strongly confirm the apparent necessity of diminishing the refractions by $0\cdot0053 \times R$. But it is clear that the piercing of the cube, or the use of the new collimators, or some other instrumental changes introduced at the time of the piercing of the cube, have largely affected the values of z, and, before we can appeal to these results in proof of the accuracy of Bessel's Refractions, or of the refractions diminished in the proportion $0\cdot9947 : 1$, we must settle which of the two series of observations is the freer from systematic error.

And with respect to the comparison which is given in the Addendum to the Nine Year Catalogue between the results of that Catalogue and the Melbourne Catalogue of 1227 Stars for 1870, I must remark that the large systematic corrections which have been applied to the Melbourne results for supposed errors in Bessel's Refractions should be removed before this agreement is appealed to as a proof of the accuracy of Bessel's Refractions.

To show the importance of this point, I give the results of a comparison between the corrected Greenwich Catalogue before and after the corrections which have been applied to the Melbourne Catalogue have been removed.

The corrections applied to the Melbourne North Polar Distances were obtained from a discussion in my paper of 1867, December, of the Melbourne observations 1863, 1864, 1865. These observations were compared with the results of the Greenwich Catalogue 1860, and the corrections were deduced by forcing a close agreement between the results. It is therefore rather arguing in a circle to appeal to an agreement between the corrected Melbourne results and those of the Nine Year Catalogue, also corrected and made to agree with the Catalogue of 1860, as a proof of the accuracy of the corrections applied to the Greenwich Catalogue of 1872 or as a proof of the accuracy of Bessel's Refractions.

N.P.D.	M 1 Melbourne, with Stone's corrn. } – (Nine Year Cat. corrd. to 1860.)	M 2 Melbourne, uncorrd. } – (Nine Year Cat. corrd. to 1860)
°	"	"
48	+0·38	−2·32
52	+0·26	−1·66
56	+0·05	−1·65
60	−0·20	−1·35
64	−0·47	−1·40
68	−0·65	−1·41
72	−0·70	−1·33
76	−0·53	−1·06

N.P.D.	M 1 Melbourne, with Stone's corrn. } − (Nine Year Cat. corrd. to 1860.)	M 2 Melbourne, uncorrd. } − (Nine Year Cat. corrd. to 1860.)
°	"	"
80	−0·22	−0·66
84	+0·02	−0·34
88	−0·10	−0·39
92	−0·09	−0·33
96	−0·17	−0·35
100	−0·22	−0·35
104	−0·26	−0·35
108	−0·16	−0·20
112	−0·15	−0·15
116	−0·06	−0·02
120	+0·09	+0·17
122	+0·44	(+0·53)
124	−0·47	(−0·36)

The last two residuals contain observations which have been reduced with Bessel's Refractions diminished by 0·0023 × Tabular Refraction and 0·0049 × Tab. Refraction respectively, and are therefore enclosed in brackets.

The agreement M1 may well have been considered satisfactory; but it will be seen from the system M2 that Bessel's Refractions must be diminished to represent either the Melbourne or the Greenwich observations.

With respect to the Cape Catalogue of 1159 Stars for 1860, a comparison was made by me with the results of the Greenwich Catalogue for 1860, before the Cape Catalogue was committed to the press, and it was chiefly because I saw that there was a rather close agreement between its results and those of the Greenwich Catalogue that I adopted Bessel's Mean Refractions unaltered instead of attempting to apply any corrections to my results. It will be seen on reference to the Cape Catalogue, pages 9 and 10, that, from a discussion of the circumpolar observations, I obtained a diminution of Bessel's Refractions amounting to 0·0047 × Tabular Refraction; but on account of the Zenith Distance of the Pole at the Cape being more than 56°, it is very difficult to separate the errors of the Refraction tables adopted from the error of assumed co-latitude, and I thought, therefore, pending further enquiry, it would be better to use Bessel's Refraction Tables unaltered than to apply an uncertain correction.

The recent Cape observations do appear to offer grounds for serious doubts about the necessity of decreasing Bessel's Refractions, for small zenith distances, by 0·0053; but the evidence thus collected is not certainly such as should outweigh the accumulated evidence of the Greenwich circumpolar observations

unless it can be shown that the residual errors presented by the Greenwich results can be otherwise accounted for.

And it appears to me that, if any serious doubts have ever been entertained on the substantial accuracy of my work of 1867, great advantages would have resulted had the results been independently recomputed, and, whether they confirmed or disproved the accuracy of my work, printed in the Addendum of the Greenwich Nine Year Catalogue. If my work could have been shown to have been seriously inaccurate, there would have been an end of all difficulty, so far as the evidence there collected was concerned. On the other hand, if my work was confirmed and the residual errors, which I have given, found to be substantially correct, then, most certainly, in the best interests of the Greenwich Observatory, these residuals should be kept distinctly in view and not virtually shelved from false impressions of the inaccuracy of my work. For, so far as I can see, there are only four possible causes to which these residual errors can be referred:—

(1.) My work is wrong—*i.e.* the residual errors which I have given are not correctly deduced from the observations, and do not, therefore, represent their results as given in the annual volumes of the Greenwich Observatory.

(2.) The Greenwich observations, as reduced and published in the annual volumes of the Greenwich Observatory, are affected with systematic errors sufficiently large to give rise to the residual errors contained in the five groups A.

(3.) Bessel's Refractions do require to be diminished by about $0{\cdot}005 \times$ Bessel's Refraction; and the co-latitude of Greenwich does not differ much from $38^\circ\ 31'\ 21''{\cdot}60$.

(4.) Bessel's Mean Refractions have been deduced so imperfectly from theory as to give rise to these residual errors.

The residual errors A may, of course, arise, not from one, but from the conjoint action of two or more of these causes of error.

First, with respect to the accuracy of my work. The only evidence, so far as I am aware, which has been brought forward to show that my results are not reliable are the facts that the thermometer used during the year 1857 and part of the year 1858 read too high by about six tenths of a degree Fahrenheit, and that the barometer used read too low by about a hundredth of an inch. It has also been assumed that the necessary corrections had not been applied by me. I need hardly point out that these instrumental errors, if unallowed for, would not have affected my determination of y by more than $0{\cdot}0016$ had the thermometer with a large index error been used during the nine years over which my discussion extends, instead of only during the year 1857 and a part of the year 1858. Besides this, the thermometer read too high, and the Barometer too low, and the Refractions, therefore, used in the Greenwich reductions were already smaller than they would have been if correctly deduced from Bessel's

Tables. I ought, therefore, so far as these causes were effective, to have obtained an increase and not, as I did, a decrease of the Refractions used. At the January meeting of the Society, Mr. Christie did bring forward some results derived from the observations 1857 to 1865 which I, at first, hoped might have settled the simple point whether the residual errors A had been correctly deduced from the results of the Greenwich volumes 1857 to 1865. The observations were divided by Mr. Christie into five groups like mine, and were given on two assumptions: first, that the law of discordance R—D was $= a \sin$ Z.D.; and secondly, that it was $= a \sin$ Z.D. $\cos^2$ Z.D. The mean errors for the five groups were as follows:—

(1)	(2)	(3)	(4)	(5)	R–D
+0·19	−0·09	+0·22	+0·79	+1·20	$a \sin$ Z.D.
+0·21	+0·09	+0·49	+1·11	+1·56	$a \sin$ Z.D. $\cos^2$ Z.D.

whilst my results obtained in 1867 are

+0·24	+0·20	+0·57	+1·05	+2·19.

There is a rough agreement amongst all these values, and they all show that Bessel's Refractions unaltered do not represent the Greenwich observations; but the differences between the results given by Mr. Christie and myself differ with regard to the most important residual (5) by more than four times the theoretical probable error of my result, and either Mr. Christie's results or mine are in error, for the differences of the (R—D) corrections will not account for the discordances.

In my changes of residence between England and the Cape, I have, unfortunately, lost or mislaid the calculations upon which the results given in the paper of December 1867 were based. It is impossible for me to assert positively, from memory, what small corrections I applied to the results more than thirteen years ago; but from a general knowledge of the interest I took at the time in the investigation, and from my general rule of work, I have felt throughout that I must have applied all the corrections which could be applied with certainty to the results; but as these questions of inaccuracy have been raised, I have now recomputed the principal residual for group (5).

It may probably avoid useless discussion if I state clearly all the corrections applied to the Greenwich results.

First:—The observations have been extracted from the annual Catalogues of the Greenwich Observatory 1857 to 1865 inclusive.

Secondly:—I have not used the Reflexion observations. These observations have been already combined with the direct observations in the determination of the Nadir Points and in the deduction of the R—D corrections, and the corrections thus deduced have been applied in the Greenwich volumes to the

direct results. With this exception, all the results given in the Greenwich volumes have been used.

Thirdly:—I have assumed that the thermometer used in 1857 and part of 1858 read too high by six tenths of a degree Fahrenheit.

Fourthly:—I have assumed that the barometer used in the reductions read too low by a hundredth of an inch.

Fifthly:—As some of the observations contained in the group (5) have a greater zenith distance than 82°, and these observations have had their Refractions already decreased by 0·0023 × Tabular Refraction, I have therefore increased their Refractions by the same amount to bring them back to the results which would have been obtained had Bessel's Refractions been used unaltered.

Sixthly:—The observations have all been reduced to an assumed co-latitude 38° 31′ 21″·80.

Seventhly:—I have weighted the observations from the formula

$$W = \frac{1}{2\epsilon^2 + \frac{e^2}{n} + \frac{e_1^2}{n_1}}$$

assuming $2\epsilon^2 = 0{\cdot}04$, and extracting e and e_1 from my paper on probable errors in the *Monthly Notices* for 1869. The result obtained in 1867 was

2″·19 with a weight 28 (or probable error ± 0″·2).

The result which I now obtain is

2″·10 with a weight 24 (or probable error ± 0″·2).

The two results are essentially identical, and it is clear that all the corrections now applied were applied in 1867, but (as my paper on probable errors was not written before 1869) I must have worked out, with sufficient approximation, probable errors at different zenith distances and used them in my work: I believe, therefore, that I may safely assert that my work of 1867 correctly represents the results of the Greenwich observations as printed, and that we must look to one of the following causes for the solution of any difficulties presented by the appearance of the residuals A.

I now come to the consideration of the assumption (2).

There are undoubtedly systematic errors in the Greenwich results; but not such as can account for the residuals A.

During the years 1857 to 1865 there was no well-marked constant difference between the Nadir Point determinations with the reflecting eyepiece and from stars, and the differences between the Nadir Points from Northern and Southern stars were confined within small limits.

And not only is this the case but, so far as I know, no good reasons can be assigned why the small uncorrected systematic

instrumental errors should tend to increase rather than decrease the residual A (5). The determination of the horizontal flexure and its correction under the form a sin Z.D. should prevent any serious instrumental errors affecting the circle readings near the horizon; and although the good effects of this were virtually destroyed by the system formerly adopted at Greenwich of introducing another correction of the form b sin Z.D. to destroy the the mean difference between the Nadir Points determined from Northern and Southern stars, yet the corrections thus introduced, during the period 1857 to 1865, were not large, and have not affected the residual error (5) by 0″·3. The law assumed, since 1862, to destroy the difference between the results for Northern and Southern stars has been a sin Z.D. cos² Z.D., and a term under this form must exist in the flexure unless the yielding of the instrument in the two horizontal positions object-glass North and South remains unchanged, a fact which can only be known from experiment, and appears to be disproved when systematic differences are found to exist between the Nadir determinations from the Northern and Southern stars. So far as the observations 1857 to 1865 are concerned I can see no reason whatever for assuming it possible that the residual error A (5) has been increased 0″·5 from uncorrected systematic errors.

I now pass to the consideration of the assumption (3).

Upon this point I appeal to a comparison of the residual errors A and B; and, as a confirmation of this result, I give the corresponding residual errors from the Greenwich observations 1868–1876.

The residuals for the first three of the following groups are extracted from the Introduction to the Nine Year Catalogue, 1872. The two last are not given in that Introduction. But, as these are all important in questions of Refraction, I have extracted the necessary observations from the Introductions to the annual volumes of Greenwich Observations 1868 to 1876, and formed the mean errors with the proper theoretical weights.

1868–1876.

Excess of N.P.D. below over N.P.D. above pole.

(1)	(2)	(3)	(4)	(5)
″	″	″	″	″
−0·26	−0·63	−0·36	−0·72	−0·19

The first three groups give a correction of −0″·41 to the assumed co-latitudes, and, after applying this, the residuals become

	(1)	(2)	(3)	(4)	(5)
	″	″	″	″	″
C	+0·15	−0·22	+0·05	−0·31	+0·22

These residual errors are small, and certainly do not afford the slightest indication of any serious errors in the adopted refractions, which are those of Bessel diminished by 0·0053 × Tabular Refraction. The evidence afforded by a comparison of the

residual errors A and B and the smallness of the residual errors C, where the groups (4) and (5) contain observations extending to 85° and below 85° Z.D. respectively, appear to afford evidence in favour of the reduced refractions which it must be difficult to resist, unless it can be shown that these results can otherwise be accounted for. I believe, however, it will be proved, in the present paper, that it is possible to account for these results, to a great extent, at all events, under the Section (4).

This point appears important, and will probably meet with the more ready acceptance, as it does not necessitate any considerable changes in Bessel's refraction at the smaller zenith distances, and accounts for the different values of y which have been obtained from discussions extending over small and large ranges of zenith distance.

The mean refractions of the *Tabulæ Regiomontanæ* are merely those of the *Fundamenta* multiplied by 1·003282. But at the zenith distance 85° these refractions were found by Bessel to be too large, and he recommended that they should be replaced by quantities which he had derived more directly from observation. Attempts have been made from time to time to deduce corrections to Bessel's refractions by assuming, as Bessel had done, that the tabular refractions required some constant multiplier or divisor. These attempts have led to anomalous results. If we start with Bessel's Refractions—then, as might be expected, no corrections are found necessary for small zenith distances; at considerable zenith distances the tabular refractions are found to be too large, and the factor by which they require diminution increases with the zenith distance. The whole theory of Bessel's mean refractions is included in the following relations:—

$$(1)\quad \frac{dR}{d\mu}=\frac{1}{\mu}\,\frac{\mu_0 \sin z}{\sqrt{(1+x)^2\mu^2-\mu_0^2 \sin^2 z}}=\phi'(\mu) \text{ suppose.}$$

$$(2)\quad \mu^2 = 1+4k\rho,$$

$$(3)\quad \rho = \rho_0\epsilon^{-\beta x},$$

where R is the refraction, μ, ρ the index of refraction and density of the air at a height $x \times$ Earth's radius; μ_0, ρ_0 the corresponding quantities at the Earth's surface; κ a constant; β a quantity introduced to force an agreement between the calculated and observed refractions at considerable zenith distances; ostensibly β is introduced to give the law of diminution of ρ with increasing altitudes, but its real object is that which I have indicated. It is clear that the relation $\rho=\rho_0\epsilon^{-\beta x}$ is not the true relation which exists between the quantities ρ, ρ_0 and x. The absolute truth of this formula would require that the pressure and temperature of the air should be uniform over the Earth's surface. It is, I believe, equally clear that ρ cannot be expressed as a simple function of ρ_0 and x. All such functional relations must tacitly assume that the atmosphere is in equilibrium, thermal and otherwise, whereas we know that the changes are largely, perhaps principally, due to diffusion.

But whatever may be thought of the theory from which Bessel's Refractions have been deduced, they certainly represent the observations with considerable accuracy. If β be determined with reference to the horizontal refraction the errors of the computed refractions are still not very great. And if we determine β so that the calculated refraction agrees with observations at Z.D., 89°, 88°, 87°, 86°, and 85° respectively, we are certain to obtain theoretical curves, in succession, which lie closer and closer to the true curve of refraction. If, therefore, we could insure the near coincidence of the curves for the smaller zenith distances by the proper determination of ρ_0 or its equivalent μ_0, we should, with such a theory as Bessel's, be able to obtain any necessary degree of accuracy over a considerable range of zenith distance. It will be seen that I regard the theory chiefly as a useful guide to a convenient formula of interpolation. Since the mean refractions of the *Tabulæ Regiomontanæ* have not been directly computed from the formulæ, but derived from those of the *Fundamenta* by the mere multiplication of these refractions by a constant quantity, we can only indirectly infer the values of of μ_0 and β which have been really assumed by Bessel.

It appears that Bessel's refractions at Barometer 30 inches and Thermometer 50° can be sensibly reproduced from the theory when

$$\mu_0 = 1\cdot00028272, \ \log. \beta = 2\cdot8699628.$$

The refractions thus computed were found by Bessel too large at 85° Z.D., and I believe that all subsequent investigations have led to a similar result. There may be, and probably will be, some doubt about the exact excess of Bessel's Tables at 85° Z.D., but I shall assume that the true refraction for the Barometer 30 inches and Thermometer 50° is 589″·13.

Any change in the adopted value of the refraction at 85° will, of course, introduce corresponding changes into the results for the other zenith distances, but the necessary changes can be inferred from the differences between my results and those given by Bessel.

When Bessel found the refractions of the *Fundamenta* multiplied by 1·003282, although agreeing well with observations at the smaller zenith distances, too large by about 2″ at the zenith distance 85°, he should, in my opinion, have recomputed his refractions and determined the quantity β so that the necessary agreement near 85° Z.D. was obtained. This is what I have done. And, if the value of the refraction at 85° which I have assumed be nearer the truth than that deduced from Bessel's Tables, then the intermediate refractions given in my table must also be closer to the true results than Bessel's are.

The following table exhibits the values of the refraction for Barometer 30 inches and Thermometer 50°, which I have calculated, and the corresponding quantities computed from Bessel's *Tabulæ Regiomontanæ.*

I adopt

$$\mu_0 = 1{\cdot}0028272 \text{ and } \log. \beta = 2{\cdot}8505582.$$

Refractions for Barometer 30 *in. and Thermometer* 50° *F.*

Z.D.	Stone's Refraction.	Bessel's Refraction.	Excess of Bessel.
° ′	″	″	″
45	58·24	58·22	−0·02
55	83·08	83·06	−0·02
65	124·33	124·35	0·02
70	158·72	158·76	0·04
75	213·96	214·10	0·14
76	229·39	229·55	0·16
77	246·97	247·18	0·21
78	267·24	267·52	0·28
79	290·83	291·22	0·38
80	318·70	319·16	0·46
81	351·93	352·58	0·65
82	392·39	393·32	0·93
83	442·57	443·85	1·28
83 30	472·35	473·95	1·60
84	506·15	508·06	1·91
84 30	544·74	546·98	2·24
85	589·13	592·00	2·87

There appears a slight, but not important, difference between my results and Bessel's at 84° 30′. I believe my results are correct, but the calculations are intricate, and I may have made some slight error.

The importance of the changes thus introduced into Bessel's Refractions will, perhaps, be more apparent if we consider their effects when we attempt to compare Bessel's theory with observations by discussing circumpolar observations under the assumption that the true refraction is equal to Bessel's Refraction × $(1-y)$, and consider y to be constant over any wide range of zenith distances.

If $38° + \frac{1}{2}x$ be the assumed co-latitude and we have observations above and below pole at zenith distances 38°, 38°; 75°, 1°; 82°, −6°, 84°, −8°, our equation of condition will be of the form

$$(1) \quad x + 90y = a_1,$$
$$(2) \quad x + 215y = a_2,$$
$$(3) \quad x + 387y = a_3,$$
$$(4) \quad x + 500y = a_4,$$

when a_1, a_2, a_3, a_4, are the excess of observed N.P.D. below pole over that observed above pole reduced with Bessel's refractions.

If, therefore, Bessel's Refractions are in error by the differences between my computed quantities and those given in Bessel's Tables—and this must be the case if Bessel's ρ_0 be adopted and the assumed error of his tables at $85°$ Z.D. be $2''.87$—then, when the observations are sufficiently numerous and good to destroy any mere errors of observation, our equations of condition would assume the form

$$
\begin{aligned}
&(1) \quad x+90y = 0''.00,\\
&(2) \quad x+215y = +0.14,\\
&(3) \quad x+387y = +0.93,\\
&(4) \quad x+500y = +1.91,
\end{aligned}
$$

from which, for the determination of x and y, we have

$$
\begin{aligned}
4x + 1192y &= +\ 2.98,\\
1192x + 454094y &= 134.501,
\end{aligned}
$$

which give

$$x = -0.626, \quad y = +0.0046.$$

The residual errors would be the following:—

$$
\begin{aligned}
&(1) \quad -0.212,\\
&(2) \quad +0.223,\\
&(3) \quad +0.224,\\
&(4) \quad -0.236.
\end{aligned}
$$

A comparison should be made between these results and those obtained in 1867.

The residual errors are not large; and for the groups (3) and (4) are certainly not larger than the residual errors would be had they been derived from a large number of good observations.

It appears, therefore, that when the two quantities ρ_0 and β are both determined indirectly from a discussion of observations, we have in Bessel's Theory an *embarras de richesses*. If we assume that Bessel's Refractions in the *Tabulæ Regiomontanæ* are too great at $85°$ by $2''.87$, then we can force a somewhat close agreement between our circumpolar observations and Bessel's Refractions, either by adjusting the refractions by a redetermination of β, or by reducing all the refractions in the proportion of about 0.996 to 1, which agrees nearly with the result obtained in my paper of 1867, December. In the first case the Refractions at small zenith distances will remain unaltered, and, if our co-latitude has been originally deduced by the use of Bessel's Refractions, our co-latitude will also remain unaltered. In the second case all the Refractions will be proportionately altered and the assumed co-latitudes require correction of about $0''.3$. If we restrict the arc over which our discussion extends, we obtain smaller and smaller corrections to the assumed values of our Refractions and co-latitude, and by sufficiently contracting the range of zenith distances we shall merely reproduce our original assumptions.

Our real difficulty in these discussions appears to be to determine, amongst the small quantities involved in the equations of condition as residual errors, what proportion of the adjustment should be made with μ_0 or ρ_0 and what with β.

It will be seen that the results which I have obtained explain those which Mr. Main deduced from a discussion of the observations below Z.D. 70° made during the years 1836 to 1854.

At 82° my Refraction is less than Bessel's in the proportion of 0·9976 to 1, and at 85° in the proportion of 0·9949 to 1. These results agree almost identically at 82° and 85° with those obtained by Mr. Main, but it was formerly assumed that no sensible corrections were needed at smaller zenith distances than 82°, and that the correction 0·9976 : 1 continued sensibly unchanged from 82° to 85°. In my calculations the changes are, as they must be, progressive; but of course there may be some doubt whether my corrections at 85° may not be slightly too large.

On account of the rapidity with which the differences between the refractions given in the present paper and Bessel's Refractions increase at considerable zenith distances it is very difficult to correct the mean results when observations are combined with zenith distances differing by several degrees. I have, therefore, given the results for the different stars from 82° to 85° Z.D., 1857 to 1865, and also for stars 80° to 85° during the years 1868 to 1876. It may be noticed that the adopted co-latitudes for the same Refractions are different in the two sets of observations. The co-latitudes adopted are those given by the groups least affected by refraction. The co-latitude during the second period appears to have been affected by the constant difference which existed during the period 1870–1874 between the Nadir Point determinations, and the virtual rejection of the observations with the reflecting eyepiece, by the application of constant corrections to make their results agree, in mean results, with the zenith points found from the star observations. To some extent, however, any error which might arise from the course adopted will be compensated for when the co-latitude obtained from the observations themselves is adopted, as in the present comparison.

Observations 1857–1868.

Star's Name.	App. Z.D. at S.P.	Weight.	Excess of N.P.D. below, over N.P.D. above, Pole. Bessel's Ref. Co-latitude 38°31′21″·80	R = Bess. (1 − 0·0053) Co-latitude 38°31′21″·60	Ref. of present paper. Co-latitude 38°31′21″·92
	° ′				
Capella	82 40	5·5	+1·34	−0·30	+0·07
δ Cygni	83 41	4·7	+2·81	−0·80	+1·04
ω Ursæ Maj.	84 37	0·9	+3·50	+1·24	+1·08
13 Lyræ	84 45	2·3	+3·86	+1·46	+1·11
β Aurigæ	83 36	1.2	+3·07	+1·07	+1·18
α Cygni	83 44	0·6	+4·49	+2·49	+2·60
11 Lacertæ	84 57	0·8	+1·34	−1·17	+1·65

The co-latitude $38°31'21''.92$ has been obtained from group (1) A, and the residual errors for Bessel's Tables should be referred to the same co-latitude as the refractions are equal for the smaller zenith distances.

The mean results will then be :—

With Bessel's Refractions.	Tab. Ref. (1 − 0·0053).	Refrs. of the present Paper.
″	″	″
+ 2·26	+ 0·39	+ 0·65

Observations 1868–1876. Co-latitude, $38°31'21''.60$.

	° ′				
χ Ursæ Maj.	80 2	1.81	+ 1·51	− 0·14	+ 1·06
ψ Ursæ Maj.	83 19	1·62	+ 1·51	− 0·14	+ 1·06
α Cygni	83 44	3·02	+ 2·33	− 0·19	+ 0·73
δ Persei	81 8	0·88	− 0·98	− 2·81	− 1·58
θ Persei	79 50	0·58	+ 1·41	− 0·25	+ 0·95
13 Lyræ	84 45	0·28	− 3·52	− 6·49	− 6·03
δ Cygni	83 41	2·74	+ 2·29	+ 0·26	+ 0·76
ω Cygni	79 33	1·67	− 0·47	− 2·09	− 0·89
τ Herculis	81 52	0·99	+ 0·57	− 1·42	− 0·22
ξ Cygni	85 10	0·48	+ 0·76	− 2·34	− 2·08
ι Ursæ Maj.	79 57	6·18	+ 1·70	+ 0·04	+ 1·24
f Cygni	81 32	4·43	+ 1·37	− 0·60	+ 0·58
Capella	82 40	9·00	+ 0·87	− 1·29	− 0·16
β Aurigæ	83 36	4·00	+ 2·01	− 0·47	+ 0·46

The mean values are

″	″	″
+ 1·33	− 0·71	+ 0·35,

but from the co-latitude investigation the second group requires a correction $+0''.40$, and the first and third, on account of differences of adopted refraction, a correction of $+0''.40-50$ or $-0''.10$.

The mean errors then become

″	″	″
+ 1·23	− 0·31	+ 0·25.

Both comparisons show that Bessel's uncorrected results give much larger residual errors than those obtained either by diminishing these refractions by 0·0053 or using the refractions of the present paper.

These results have had several corrections applied to them, and do not profess a greater accuracy than $0''.1$. The errors may in particular cases amount to $0''.2$; but I think this will be the extreme range of error.

The systematic errors of the Greenwich Results during the second period are larger than during the period 1857–1868, but are still confined within limits of less than a second of arc.

So far as these observations can afford us information it appears that I have not over-corrected Bessel's Refractions at 85°.

The chief defect in Bessel's Mean Refractions, which appears to have been clearly proved, is the running away of his Refractions at considerable zenith distances. It appears from the results of the present paper that this can be corrected to 85° Z.D. by a proper determination of β without the necessity of any sensible alteration of the Refractions at small zenith distances. It would appear, therefore, that if we can feel confidence in Bessel's value of ρ_0 or μ_0, or in his Refractions at small zenith distances, his Tables must be considered a satisfactory practical solution of the problem of astronomical refraction for the present epoch. But I cannot feel that perfect confidence in Bessel's results which astronomers in general appear to feel. The tables in use have been obtained by a comparison between Observations and the Tables of the *Fundamenta*, on the supposition that Refraction = Tab. R. $(1-y)$; and the constant thus found, $(1-y) = 1{\cdot}003282$, must depend upon the range of Zenith distances over which the discussion extended. But the result appears to disprove the assumed constancy of y, for the Refractions at 85° are too large, and can only be adjusted to the Refractions at the smaller zenith distances by applying corrections which no longer render y a constant even over an arc of 75° Z.D.

And when we are restricted to a range of zenith distances of 75° the difficulties of determining y within such limits as 0·003 appear to me very great indeed. How great these difficulties are may best be seen by a numerical illustration. If we attempt the solution of this problem by a discussion of circumpolar observations in latitude 52° our limiting equations of condition will be about

$$x + 90y = a_1, \qquad x + 215y = a_2,$$

and an error of 0·003 in y cannot introduce into our residuals a_1 and a_2 greater differences than 0″·38, and it is from these differences alone that the value of y has to be deduced. Nor are we in a better position if we attempt to determine y from a discussion of the results at a Northern and Southern Observatory—Greenwich and the Cape, for instance; our limiting equations will then be confined within some such limits as

$$x + 116y = a_1, \qquad x + 230y = a_2,$$

and the extreme differences introduced in the residuals a_1 and a_2 by changes of 0·003 in y will not amount to more than 0″·34. When, therefore, it is considered that at 75° Z.D. the Refractions are already in doubt by 0″·14, on account of the corrections required by β, and that the results of our best Circles are not free from small systematic errors, it will, I think, be admitted that it is very

difficult, if not hopeless, to expect to determine y from such equations within such errors as 0·003. Whatever may be the difficulties connected with the making of good observations at considerable zenith distances, and in obtaining a theory of Refraction sufficiently strong to brace together the results with those made at smaller zenith distances, I believe it is absolutely necessary that we should extend our enquiry to sufficient zenith distances to separate the errors in the adopted refraction tables from the systematic instrumental errors to which our Circle observations are liable, and I cannot regard this as done when discussions are limited to zenith distances less than 75°. But these general considerations are not the only grounds upon which I doubt the extreme accuracy of Bessel's determination of μ_0.

It will be found a fundamental assumption of his work that at the limits of the atmosphere the density is so small that with his assumed law $\rho = \rho_0 \epsilon^{-\beta s}$ the integration may be extended to infinity without sensibly affecting the resulting refractions. But in this case we have sensibly, at the limits of the atmosphere, $\mu = 1$; and returning to our equations,

$$\frac{dR}{d\mu} = \frac{\mu_0}{\mu} \frac{\sin z}{\sqrt{(1+s)^2\mu^2 - \mu_0^2 \sin z}} = \phi'(\mu),$$

we have

$$R = \int_1^{\mu_0} \phi'(\mu) d\mu = \phi(\mu_0) - \phi(1)$$

$$= \phi'(\mu_0) \,.\, (\mu_0 - 1) - \phi''(\mu_0) \,.\, \frac{(\mu_0 - 1)^2}{1 \,.\, 2} + \ldots.$$

The first term, $\frac{1}{\mu^0} (\mu_0 - 1) \tan Z$, is independent of the law of variation between s and μ, and is, for small zenith distances, the only important term. In fact, with Bessel's Theory the remaining terms do not amount at 45° Z.D. to 0″·1.

If we accept Bessel's value of μ_0, we find for the first term, in seconds of arc, 58″·29. But μ_0 is supposed to be the refractive index of air at barometer 30 inches and thermometer 50°F. The value of μ_0 for dry air has been given by Biot $= \sqrt{1{\cdot}000588768}$ at barometer $0^{m}{\cdot}76$ and thermometer 0° C.; and this would correspond to the value $\mu_0 = 1{\cdot}00028462$ at 30 in. and 50°F.; but this value of μ_0 gives 58″·69 for the coefficient of tan Z. The difficulty presented by these differences has been, I believe, generally explained by the fact that Biot's value is for dry air, whilst Bessel's is for the moist air of the atmosphere. It appears to me that this explanation cannot be the true one. The coefficient of refraction should be larger for the moist air instead of smaller.

I have taken the trouble to compute the refractions which would result if we adopted Biot's value of μ_0 and Bessel's Theory.

The refractions thus deduced do not appear to me admissible. If Biot's determination of μ_0 for dry air is too large, this should be proved by an independent determination of that quantity; but

accepting its value we must, I fear, abandon the assumption that $\mu = 1$ at the limit of the Earth's atmosphere, and assume that its value is about 1·0000019.

In this way we can make Biot's value of μ_0 give us Bessel's Refraction at Z.D. 45°, for

$$R = \int_{\mu_0'}^{\mu_0} \phi'(\mu) \,.\, d\mu = \phi'(\mu_0) \,.\, \frac{\mu_0 - \mu_0'}{\mu_0} - \ldots .$$

$$\text{and the first term} = \tan z \,.\, \frac{\mu_0 - \mu_0'}{\mu_0} = 58''\!\cdot\!29 \text{ as before.}$$

This supposition gives with Bessel's law of density a height of about 27 miles to the atmosphere.

But, if this explanation be accepted, the agreement which is found between Bessel's Refractions and the observations at the smaller zenith distances is a forced agreement, arising from the errors of the theory being counteracted by an erroneous determination of μ_0, and although the errors thus introduced at considerable zenith distances may be destroyed by the aid of the second constant β, we must be liable to a separation between the true refractions and those obtained from such a strained theory at intermediate and perhaps unexpected points; such errors, if they were small, as they must be, would scarcely be found without a systematic examination, with a considerable number of observations, of the results at many of the intermediate zenith distances.

Upon this work, and an attempted revision of Bessel's Theory, on the lines here indicated, I am at present engaged.

A List of 410 *Radiation-points of Shooting Stars, deduced from Observations in Hungary in the Years* 1871 *to* 1878, *and of* 80 *probable Radiants deduced from the* 410 *Radiation-points.* By Prof. N. de Konkoly, Member of the Hungarian Academy at Budapest, Director of the Observatory at O'Gyalla.

The present paper is based upon observations in Hungary, in the years 1871 to 1878, of 2999 shooting stars; but of these only 1641 are used for the determination of Radiation-points, the others being rejected as relating to sporadic stars, or as being bad observations.

I have deduced from this large number of observations 410 Radiation-points.

The shooting stars were observed in Hungary at 5 stations:—

	Long. h m s		Lat. ° ′ ″
1 Astrophysical Observatory, O'Gyalla	λ = 0 7 13·9	E.*	D = 47 52 43·4
2. Real School at Agram	λ = 0 1 30·7	W.	D = 45 50
3. Forest Academy, Schemnitz ...	λ = 0 10 5	E.	D = 48 27
4. Economical School at Hodmezövasárhely	λ = 0 15 1	E.	D = 46 26
5. Gymnasium at Szathmárnémethi	λ = 0 26 1	E.	D = 47 46

The observers were: at O'Gyalla, the staff of the Observatory; at Agram, Professor Stozir; at Schemnitz, Professor Dr. Schwarz; at Hodmezövásárhely, Professor Nagy, and at Szathmárnémethi, Professor Toth.

I arrange as follows the whole series of observations, to show how they are distributed in the 8 years.

The first column gives the year; the second, the number of days on which observations were employed for the determination of Radiation-points; the third shows the number of Radiation-points; and the fourth shows how many shooting stars were employed for the calculation of the Radiation-points. I give separately the results for each station, and finally the sum from all the stations.

Table of Distribution of Meteors in different Years.

O'Gyalla—Observatory.

Year.	Observing Days.	Number of Radiation-points.	Number of Meteors employed.
1871	3	13	44
1872	12	71	278
1873	10	33	105
1874	7	68	271
1875	10	84	361
1876	6	22	89
1877	3	7	28
1878	8	18	66
Sum	59	316	1242

Schemnitz—Academy.

Year.	Observing Days.	Number of Radiation-points.	Number of Meteors employed.
1875	4	8	36
1876	2	6	20
1878	2	3	10
Sum	8	17	66

* E. or W. from Vienna Observatory.

Hodmezövásárhely—Economical school.

Year.	Observing Days.	Number of Radiation-points.	Number of Meteors employed.
1875	Only two sporadic fireballs.		
1876	5	29	120
1877	1	2	6
1878	3	8	31
Sum	9	39	157

Agram—R. Real School.

1875	2	11	37
1876	2	4	12
1877	3	10	52
Sum	7	25	101

Szathmárnémethi—Gymnasium.

1875	3	13	75
Total Sum	86	410	1641

The Radiation-points are deduced as follows: the right ascension and declination for the beginning and the end of each shooting star were laid down in a chart, constructed for this purpose. The charts are on the gnomonic projection, and large enough for estimated $0^{\circ}\cdot5$ at 45° declination, and 1° at 85° declination. The path of each shooting star was drawn with a sharp pencil with as much precision as possible, and after this operation the paths were prolonged backwards, until several paths met in or near a point, and the 410 Radiation-points were thus determined upon the chart. The shooting stars used for the determination were within at most four days of each other; but it was only in the case of very poor showers that so long an interval was taken.

The Radiation-points were laid down on a new chart, and from those near each other the 80 probable Radiants were calculated by the method of least squares.

In the following three tables, Table I. contains the Radiation-points deduced from the O'Gyalla observations; Table II., those from the other stations. In each of these tables the first column shows from how many meteors the Radiation-point was deduced: the second and third give the R.A. and Declination of the Radiation-point. The left-hand three columns are to be read separately from, and as preceding, the right-hand three columns.

The two tables show Radiation-points for—

1. *February*, 20 and 24, observed at O'Gyalla.
2. *April*: April 19, 20 and 21, 1874; April 19, 20 and 21, 1878, from O'Gyalla; 1876, April 20, 22, from Agram; and 1878, April 20, 21, from Schemnitz.
3. *July*: 1873, July 25, 26, 27, 28 and 29; 1875,

July 26, 27, 28, 29 and 31; 1876, July 26, 27 and 28; 1877, July 27; and 1878, July 28 and 29, from O'Gyalla. For the same shower: 1875, July 25, 26, 27 and 28, from Schemnitz; 1876, July 26 and 27; 1878, July 25 and 26 from Schemnitz; 1875, July 25 and 27, from Agram; and 1878, July 28, from Hodmezövásárhely.

4. *August*: 1871, August 9; 1872, August, 6, 7, 9, 10, 11, 12 and 13, from O'Gyalla; 1873, August 8, 9, 10; 1874, August 8, 10, 11 and 12; 1875, August 8, 9, 10, 11 and 12; 1876, August 12, 13, and 14; 1877, August 12 and 13; and 1878, August 9, 10 and 12, also from O'Gyalla; 1875, August 10, 11 and 12, from Szathmár; 1876, August 12 and 13; 1877, Aug. 10, from Hodmezövásárhely; 1877, August 11, 12 and 13, from Agram.
5. *October*: 1872, October 20, 22, 25 and 27; 1873, October 22, from O'Gyalla.
6. *November*: 1872, November 28; and finally, 1873, November 12, from O'Gyalla.

Table III. contains the deduction of the 80 probable Radiants: the first column gives the current Nos., 1 to 80; the second column shows the Radiation-points used in the determination: viz. these are referred to by means of the current Nos. of the observations for the year and day or days, as appearing by the first and second tables; the third and fourth columns give the R.A. and Declination of the probable Radiants.

TABLE I. (O'Gyalla.)

No. of Meteors.	Radiation-points. R.A.	Decl.	No. of Meteors.	Radiation-points. R.A.	Decl.
	°	°		°	°
	1871.	*February* 20, 24.	Nos. 1 to 3.		
3	50·0	56·0	3	205·0	31·0
3	90·0	50·0			
	1874.	*April* 19, 20, 21.	Nos. 1 to 5.		
3	223·5	59·5	4	267·0	37·0
3	229·0	67·5	3	269·5	28·0
3	259·0	19·0			
	1878.	*April* 19, 20, 21.	Nos. 1 to 4.		
3	216·0	60·0	4	257·0	43·0
3	217·5	55·5	4	264·0	23·5
	1873.	*July* 25, 26.	Nos. 1 to 9.		
4	3·0	67·5	6	312·0	−8·0
7	212·5	57·5	6	323·5	3·5
6	279·0	81·5	6	327·0	25·0
6	302·0	41·0	4	351·0	64·0
18	304·0	60·0			

1873. *July* 27. Nos. 10 to 14.

No. of Meteors.	Radiation-points. R.A.	Decl.	No. of Meteors.	Radiation-points. R.A.	Decl.
6	274°·0	74°·0	6	331°·5	5°·5
7	299·0	69·5	4	351·0	61·5
4	320·0	0·0			

1873. *July* 28, 28. Nos. 15 to 18.

5	0·0	90·0	4	325·0	65·0
5	316·0	−10·0	5	328·0	22·0

1875. *July* 26, 27, 28, 29. Nos. 1 to 14.

4	1·0	35·5	6	310·0	43·0
5	2·5	63·5	3	310·0	71·0
4	282·0	54·5	4	313·0	45·0
5	288·0	50·0	4	315·0	45·0
4	297·5	3·5	4	315·0	72·0
3	308·0	40·0	5	321·0	36·5
6	308·0	51·0	4	335·0	3·0

1875. *July* 31. Nos. 15 to 24.

5	276·0	39·0	6	316·5	17·0
3	293·0	−3·0	3	318·0	19·0
4	298·0	−8·0	5	332·0	24·5
3	213·0	30·0	4	339·0	27·0
3	315·0	31·5	4	353·0	47·5

1876. *July* 26, 27, 28. Nos. 1 to 15.

3	4·0	65·5	5	325·0	74·5
3	6·5	63·5	5	328·0	44·5
3	9·5	67·0	4	338·5	41·0
4	195·0	75·0	4	339·0	41·0
4	287·5	44·0	4	342·5	75·5
5	288·0	46·0	5	352·0	69·5
4	308·0	57·0	3	359·0	67·0
5	312·0	41·5			

1877. *July* 27. Nos. 1 and 2.

4	51·5	59·0	4	68·0	85·0

1878. *July* 28, 29. Nos. 1 to 11.

No. of Meteors.	Radiation-points. R.A.	Decl.	No. of Meteors.	Radiation-points. R.A.	Decl.
3	100°·0	84°·0	6	288°·0	45°·5
3	132·0	83·0	3	304·0	−4·5
3	284·0	30·0	3	306·0	−2·5
3	287·0	30·0	4	349·0	18·0
5	287·5	3·0	5	352·0	13·0
3	288·0	33·0			

1871. *August* 9. Nos. 1 to 10.

4	22·0	71·5	3	242·0	48·0
4	96·0	71·0	4	247·0	50·5
3	140·0	83·0	3	266·0	44·5
3	161·0	83·5	4	268·0	60·0
3	179·9	79·5	4	271·0	44·0

1872. *August* 6, 7. Nos. 1 to 5.

3	300·0	39·0	4	337·0	43·0
3	306·0	45·5	3	342·0	52·0
3	309·5	42·5			

1872. *August* 9. Nos. 6 to 19.

4	6·0	4·0	5	324·5	59·0
5	249·0	61·0	4	330·0	76·0
4	265·5	28·5	3	343·0	60·0
4	270·0	57·0	3	346·5	48·0
7	276·5	14·5	4	346·5	59·0
4	298·5	56·0	3	347·5	47·0
4	312·0	46·0	3	351·0	58·5

1872. *August* 10. Nos. 20 to 38.

4	8·0	67·5	7	326·0	73·0
5	41·0	58·0	5	340·0	68·0
3	43·5	58·0	4	341·0	38·0
4	248·5	25·0	4	342·5	56·0
4	294·0	28·0	4	343·0	52·5
4	300·0	26·0	4	343·0	66·5
5	309·0	56·0	6	343·0	66·5
4	311·0	48·5	4	353·5	57·5
6	322·0	69·5	7	355·0	68·0
4	323·5	38·0			

1872. *August* 11. Nos. 39 to 46.

No. of Meteors.	Radiation-points. R.A.	Decl.	No. of Meteors.	Radiation-points. R.A.	Decl.
	°	°		°	°
7	295·0	60·0	5	334·5	44·5
5	297·0	20·5	5	336·0	42·0
7	313·5	8·5	5	338·0	45·0
4	330·0	60·0	3	340·0	45·0

1872. *August* 12, 13. Nos. 47 to 59.

3	4·0	45·0	3	331·0	63·5
3	8·5	47·5	3	340·5	44·0
3	11·5	46·5	3	352·0	52·0
4	28·5	48·5	3	355·0	46·0
4	32·0	63·0	3	358·0	40·0
3	304·0	50·0	4	95·0	87·0
3	328·0	59·0			

1873. *August* 8, 9, 10. Nos. 1 and 2.

3	19·0	65·0	3	290·0	49·0

1874. *August* 8, 10. Nos. 1 to 40, and (next page) Nos. 41 to 56.

3	0·0	55·0	6	38·5	58·5
3	1·0	53·0	5	39·0	48·5
3	1·0	52·0	3	39·0	66·0
3	3·0	53·0	4	40·0	68·5
4	9·5	40·5	3	42·0	66·0
5	11·0	46·0	4	43·0	68·5
5	18·5	47·5	4	46·5	58·5
3	24·0	66·5	4	48·0	63·0
3	25·0	65·5	6	50·0	84·0
3	27·0	67·0	5	54·0	69·5
3	28·0	65·0	4	63·0	67·0
3	29·5	67·5	4	63·0	68·5
4	31·0	49·5	4	64·5	59·7
5	32·0	48·0	5	72·0	64·0
4	32·0	59·7	6	78·0	66·5
3	33·0	67·0	4	86·0	65·0
4	36·0	77·5	4	170·0	86·0
6	37·0	58·0	4	273·0	47·5
4	38·0	63·0	3	321·5	49·5
4	38·5	67·5	4	323·0	46·0

No. of Meteors.	Radiation-points. R.A.	Decl.	No. of Meteors.	Radiation-points. R.A.	Decl.
	°	°		°	°
3	325·0	49·5	3	355·0	52·0
7	325·5	18·0	3	357·0	51·5
4	326·0	16·0	4	357·0	39·5
4	326·0	−13·0	4	358·0	52·0
5	343·0	50·5	3	358·0	50·5
5	345·0	41·0	3	358·0	53·5
7	346·0	46·5	5	359·5	74·0
4	350·2	27·0	5	359·5	42·0

1874. *August* 11, 12. Nos. 57 to 63.

4	217·0	73·0	3	290·0	52·0
3	250·0	73·0	4	292·0	48·0
3	263·0	67·0	5	337·0	70·0
3	270·0	72·0			

1875. *August* 8. Nos. 1 to 4.

3	18·0	73·5	3	29·0	71·5
3	21·0	77·5	3	323·0	72·0

1875. *August* 9. Nos. 5 to 21.

5	8·0	48·5	3	309·0	31·5
6	28·0	51·0	3	326·0	56·0
4	58·0	63·5	5	328·0	55·0
4	62·0	68·5	3	330·0	56·0
4	260·0	66·0	3	330·0	54·0
5	279·0	17·5	3	332·0	52·5
4	298·5	55·5	3	335·0	56·0
3	307·0	33·0	5	355·0	52·5
4	308·0	43·5			

1875. *August* 10. Nos. 22 to 37, and (next page) Nos. 38 to 55.

4	6·0	77·5	5	40·5	63·0
4	22·5	37·0	6	46·5	52·0
5	22·5	52·5	5	47·0	50·5
7	28·0	49·5	3	48·5	51·0
5	28·5	55·0	5	52·0	67·0
5	32·0	57·5	4	63·5	66·0
5	34·5	55·0	6	65·0	83·0
6	38·0	67·0	5	85·0	72·5

No. of Meteors.	Radiation-points. R.A.	Decl.	No. of Meteors.	Radiation-points. R.A.	Decl.
4	88°·5	67°·0	4	317°·0	73°·0
4	248·0	55·5	6	322·0	47·0
4	250·0	52·5	4	322·0	82·0
5	268·0	43·5	7	326·5	58·0
4	270·0	49·5	4	327·0	48·5
5	274·0	47·5	6	337·0	77·0
5	280·0	83·5	5	339·0	70·5
4	282·0	45·0	6	343·0	76·5
6	296·0	43·0	4	348·0	78·0

1875. *August* 11, 12. Nos. 56 to 60.

3	39·5	43·0	3	257·0	47·0
5	40·0	79·5	3	258·5	50·0
4	57·0	79·5			

1876. *August* 12, 13, 14. Nos. 1 to 7.

5	102·0	61·7	5	271·5	56·5
5	216·5	55·5	4	303·0	48·0
5	220·0	56·0	4	310·0	75·0
5	254·0	36·5			

1877. *August* 12, 13. Nos. 1 to 5.

4	0·0	84·5	4	9·0	45·0
4	3·0	50·0	4	36·5	61·5
4	7·0	55·5			

1878. *August* 9, 10, 12. Nos. 1 to 3.

3	2·5	41·0	4	358·0	39·0
4	38·0	67·0			

1872. *October* 20, 22, 25, 27. Nos. 1 to 9.

3	5·0	38·5	3	325·0	67·5
3	243·0	42·0	3	330·0	53·0
3	276·0	50·0	3	337·5	64·0
3	302·5	35·0	3	341·0	20·0
4	325·0	62·5			

1873. *October* 22. No. 1.

No. of Meteors.	Radiation-points. R.A.	Decl.	No. of Meteors.	Radiation-points. R.A.	Decl.
3	47°·0	53°·5			

1872. *November* 21. Nos. 1 to 3.

3	77·5	50·0	4	333°·5	59°·0
3	319·0	68·0			

1873. *November* 12. Nos. 1 and 2.

4	335·5	28·5	3	351·0	45·0

Table II. (Other Stations.)

Agram.

1876. *April* 20, 22. Nos. 1 to 4.

3	181·0	55·0	3	225·0	46·0
3	224·0	−3·0	3	272·0	43·5

1875. *July* 25, 27. Nos. 1 to 11.

4	3·5	61·0	3	314·0	− 8·5
4	16·0	81·0	3	320·0	66·5
3	300·0	59·0	3	322·0	− 6·0
3	305·0	−11·5	3	326·5	8·0
3	309·5	− 7·5	5	350·5	56·5
3	313·5	4·5			

1877. *August* 11, 12, 13. Nos. 1 to 10.

6	249·0	9·0	5	276·0	− 7·5
4	251·8	−24·0	4	278·0	−15·2
5	260·5	5·0	6	288·0	4·0
4	270·0	1·5	5	296·0	19·0
5	272·5	−20·2	8	309·0	13·5

Schemnitz.

1875. *July* 25, 26, 27, 28. Nos. 1 to 8.

4	276·0	15·5	4	343·0	57·5
4	282·5	−15·0	4	344·0	20·0
6	308·0	− 3·5	4	345·5	56·5
4	334·0	57·5	6	352·5	19·0

1876. *July* 26, 27. Nos. 1 to 6.

No. of Meteors.	Radiation-points. R.A.	Decl.	No. of Meteors.	Radiation-points. R.A.	Decl.
	°	°		°	°
4	105·0	71·0	4	245·0	11·5
3	232·0	17·0	3	273·5	41·0
3	237·5	30·0	3	274·5	16·0

1878. *July* 25, 26. Nos. 1 to 3.

4	263·0	52·5	3	279·0	46·0
3	271·0	36·0			

SZATHMÁR-NÉMETHI.

1875. *August* 10, 11, 12. Nos. 1 to 13.

4	5·0	21·0	7	46·5	50·0
5	8·5	16·5	12	49·5	58·5
5	17·0	28·5	8	63·0	46·0
8	18·0	8·0	4	59·0	25·0
4	23·0	58·5	4	71·0	53·0
8	38·0	53·5	4	338·0	71·5
4	44·0	41·5			

HODMEZÖ-VÁSÁRHELY.

1878. *April* 20, 21. Nos. 1 to 5.

3	249·0	23·0	7	275·0	40·5
3	249·0	27·5	4	303·0	47·5
4	256·0	23·5			

1878. *July* 25. Nos. 1 to 3.

3	90·0	90·0	4	329·0	45·5
3	327·0	26·0			

1876. *August* 9, 10, 11. Nos. 1 to 20.

5	4·5	44·0	5	60·0	78·5
4	5·0	2·5	4	72·0	72·0
4	9·0	2·0	4	90·0	90·0
4	7·0	40·0	3	139·0	59·0
4	12·5	25·0	5	320·0	70·0
4	20·0	67·0	4	330·0	58·0
5	22·5	59·0	4	340·0	30·0
6	43·0	63·0	5	346·0	41·0
5	51·5	56·5	4	347·0	46·0
6	60·0	77·5	3	348·0	43·0

1876. *August* 12, 13. Nos. 1 to 9.

No. of Meteors.	Radiation-points. R.A.	Decl.	No. of Meteors.	Radiation-points. R.A.	Decl.
	°	°		°	°
4	31·0	58·5	3	330·0	88·0
4	43·0	59·0	3	337·0	49·0
3	269·0	21·5	4	347·0	61·0
4	283·5	18·0	4	358·0	51·5
3	326·0	46·0			

1877. *August* 10. Nos. 1 and 2.

3	173·0	76·0	3	212·5	70·5

Table III. (The 80 probable Radiants.)

No. of Radiant.	Nos. of the Radiation-points.	Position of Radiant. R.A.	Decl.
	1874. *April* 19, 20, 21.		
		°	°
1	1, 2	226·3	63·5
2	4, 5	268·1	33·1
	1878. *April* 19, 20, 21.		
3	1, 2	216·8	57·3
	1873. *July* 25, 26.		
4	1, 9	357·0	65·7
	1873. *July* 27.		
5	10, 11	287·5	71·6
	12, 13	326·9	3·3
	1875. *July* 26, 27, 28, 29.		
7	3, 4	285·3	52·0
8	6, 7, 8, 10, 11	310·3	45·4
9	9, 12	312·8	71·6
	1875. *July* 31.		
10	16, 17	295·9	− 5·8
11	18, 19	314·0	30·7
12	20, 21	317·0	17·7
13	22, 23	335·1	25·6
	1876. *July* 26, 27, 28.		
14	1, 2, 3, 14, 15	1·0	66·8
15	5, 6	287·8	45·1
16	10, 11, 12	334·6	42·3
17	9, 13, 14	339·6	73·0

No. of Radiant.	Nos. of the Radiation-points.	Position of Radiant. R.A.	Decl.
	1871. *July* 28, 29.		
18	3, 4, 6	286°·3	31°·0
19	8, 9	305·0	− 3·5
20	10, 11	350·5	15·5
	1871. *August* 9.		
21	3, 4, 5	160·0	82·0
22	8, 10	268·8	44·2
	1872. *August* 6, 7.		
23	1, 2, 3	305·2	42·3
24	4, 5	339·1	46·9
	1872. *August* 9.		
25	15, 16, 17, 18, 19	346·9	54·8
	1872. *August* 10.		
26	21, 22	41·9	58·0
27	24, 25	297·0	27·0
28	26, 27	309·9	52·7
29	28, 30, 31, 35, 36	333·8	69·0
30	33, 34, 37	346·3	55·3
	1872. *August* 11.		
31	39, 42	307·7	60·0
32	42, 43, 44, 45, 46	336·9	44·0
	1872. *August* 12, 13.		
33	47, 48, 49	8·0	46·3
34	53, 54	329·5	61·3
35	56, 57, 58	355·0	46·0
	1874. *August* 8, 10.		
36	5, 6, 7	13·3	44·9
37	13, 14	31·6	48·7
38	{7, 8, 9, 10, 11, 16, 19, 20, 23, 24, 25, 26, 27, 28}	36·6	65·6
39	15, 18, 21, 22, 27	38·5	56·6
40	31, 32	63·0	67·8
41	34, 35	75·3	65·4
42	39, 40, 41	323·2	48·1
43	1, 2, 3, 4, 49, 50, 52, 53, 54	359·0	52·5

1874. *August* 11, 12.

No. of Radiant.	Nos. of the Radiation-points.	Position of Radiant. R.A.	Decl.
		°	°
44	58, 59, 60	261·0	70·7
45	61, 62	291·1	49·7

1875. *August* 8.

46	1, 2, 3	22·7	74·2

1875. *August* 9.

47	12, 14	308·0	32·2
48	15, 16, 17, 18, 19, 20	330·0	54·9

1875. *August* 10.

49	26, 27, 28	31·7	55·8
50	29, 30	39·1	65·2
51	31, 32, 33	47·1	51·3
52	34, 35	57·1	66·5
53	39, 40	249·0	54·0
54	41, 42, 43	270·7	46·6
55	48, 51	324·0	47·6
56	53, 54, 55	341·3	75·5

1875. *August* 11, 12.

57	59, 60	257·8	48·5

1876. *August* 12, 13, 14.

58	2, 3	218·3	55·8

1877. *August* 12, 13.

59	2, 3, 4	6·3	50·2

1872. *October* 20, 22, 25, 27.

60	5, 6, 7, 8	329·0	61·8

1872. *November* 28.

61	1, 2, 3	327·3	62·9

ZÁGRÁB (AGRAM).

1875. *July* 25, 27.

62	4, 5, 6, 7, 9, 10	315·1	− 3·5

1877. *August* 11, 12. 13.

No. of Radiant.	Nos. of the Radiation-points.	Position of Radiant. R.A.	Decl.
63	3, 4, 6, 7	270°8	− 3°7
64	8, 9, 10	298·5	11·9

SCHEMNITZ.

1875. *July* 25, 26, 27, 28.

65	4, 5, 7	340·8	57·2
66	6, 8	349·1	19·4

1878. *July* 25, 26.

67	1, 2, 3	270·2	45·6

SZATHMÁR-NÉMETHI.

1875. *August* 10, 11, 12.

68	1, 2, 3, 4	15·1	16·1
69	5, 6	40·0	49·5
70	7, 8, 9	52·7	52·6

HÓDMEZÖ-VÁSÁRHELY.

1876. *August* 9, 10, 11.

71	1, 4	5·6	42·2
72	2, 3	7·0	2·3
73	6, 7	21·4	62·6
74	8, 9	46·9	60·0
75	10, 11, 12	63·2	76·4
76	17, 18, 19	345·1	39·9

1876. *August* 12, 13.

77	1, 2	7·0	58·8
78	5, 7	31·5	47·5
79	8, 9	352·5	56·3

1878. *April* 20, 21.

80	1, 2, 3	251·8	24·0

Meteors observed during Voyage from London to Melbourne and home.

By Mr. D. W. Barker, 2nd. Officer on board ship 'Superb.'

Explanation of Symbols used in "Mag." Column.

... denotes very brilliant.
× " brightness of 1st mag. stars.
\+ " 2nd mag. stars. ⌃ added shows it left a stream behind.
− " all below 2nd mag.

Date		Time.	Lat.	Long.	From ✱	To ✱	No.	Mag.
1879, July	28	...	7 S	25 W	δ Cephei	β Cephei	1	—
	28	...	"	"	Between β & γ Trianguli	16 Persei	1	—
	28	...	"	"	ι Ceti	Mira	2	—
	28	...	"	"	κ Fornacis	δ Horologii	2	—
	28	...	"	"	κ Fornacis	η Horologii	1	—
	28	...	"	"	α Doradûs	γ Pictoris	1	—
	28	...	"	"	52 Arietis	through Pleiades	1	—
	28	...	"	"	α Reticuli	α Pictoris	1	—
	28	...	"	"	between ε & τ Eridani	between ν & 54 Eridani	1	—
	28	...	"	"	ν Ceti	τ Eridani	1	—
	28	...	"	"	1 Eridani	ζ Phœnicis	1	×
	28	...	"	"	ι Eridani	between ν^1 & ν^2 Eridani	1	×
	28	...	"	"	α Eridani	α Hydri		— ⌃
	28	...	"	"	41 Arietis	South of Pleiades	1	

Meteors numerous, average from 0 to 4 A.M. 8 in a minute.

Date.		Time.	Lat.	Long.	From ✱	To ✱	No.	Mag.
1879, July	29	...	10 S	27 W	ε Sculptoris	α Eridani	1	—
	29	...	„	„	δ Phœnicis	φ Eridani	1	—
	29	...	„	„	γ Phœnicis	κ Eridani	1	—
			Meteors numerous, average from 0 to 4 A.M. 2 in a minute.					
Sept.	13	...	41 S	118 E	between α & μ Canis maj.	5 Monocerotis	1	+
	15	...			Very few meteors.			
1880, Feb.	5	...	35 S	30 W	λ Leonis	North of γ Cancri	1	+
	5	...			Very few meteors.			
	10	1.30 A.M.	26 S	34 W	ρ Coronæ	12 Boötis	1	+‹
	10	2.30 A.M.	„	„	μ Centauri	γ Muscæ	1	—‹
	10	3.35 A.M.	„	,	Spica	φ Ophiuchi	1	×‹
	10	...	„	„	12 Boötis	θ Coronæ	1	—
	11	3.20 A.M.	25 S	33 W	69 Boötis	9 Boötis	1	+‹
	12	3.0 A.M.	„	„	23 Comæ	between 2 & 6 Canum Venaticorum	1	—‹
	12	4.0 A.M.	„	„	Scorpionis	θ Ophiuchi	1	—‹
	14	1.30 A.M.	23 S	„	α Canum Venat.	83 Ursæ maj.	1	—
	14	2.15 A.M.	„	„	X Argûs	π^1 Columbæ	1	—
	16	2.0 A.M.	19 S	32 W	δ Crucis	η Chamæleontis	1	+‹
	16	7.0 P.M.	„	„	ζ Persei	ν Persei	1	×‹
	16	7.30 P.M.	„	,	ε Volantis	κ Argûs (Carina)	1	—‹

Date.		Time.	Lat.	Long.	From ✻	To ✻	No.	Mag.
1880, Feb.	18	0.30 A.M.	16 S	32 W	ϵ Centauri	225 (neb) Lupi	1	–
	19	1.40 A.M.	14 S	31 W	ρ Serpentis	β Herculis	1	–
	19	3.15 A.M.	,,	,,	N. of β Scorpionis	N. of Antares	1	–
	19	3.30 A.M.	,,	,,	χ Virginis	16 Leonis min.	1	+
	20	2.0 A.M.	11 S	30 W	μ Centauri	3496 Centauri	1	–
	21	...	10 S	30 W	51 Herculis	68 Herculis	1	–
March	5	2.0 A.M.	19 N	34 W	ζ Herculis	θ Draconis	1	–
					Very few meteors.			
	7	1.0 A.M.	23 N	37 W	ι Centauri	η Centauri	1	–
	9	0.45 A.M.	23 N	40 W	δ Urs. maj.	towards 1625 Σ Urs. min.	1	–
	9	...		Through	5 Urs. min.	5° each way	1	+
	9	1.0 A.M.	,,	40 W	ν^1 Draconis	30 Draconis	1	–
	9	1.30 A.M.	,,	,,	ζ Herculis	68 Herculis	1	–
	9	3.30 A.M.	,,	,,	37 Urs. maj.	38 Urs. maj.	1	–
					Numerous meteors.			
	10	1.0 A.M.	24 N	39 W	δ Urs. min.	76 Draconis	1	–
					Very few meteors.			
	11	1.0 A.M.	,,	,,	between β & γ Urs. min.	Towards Polaris	1	–
	11	2.30 A.M.	,,	,,	δ Scorpionis	3 Centauri	1	–
	11	2.40 A.M.	,,	,,	δ Draconis	α Cephei	1	–

Date.		Time.	Lat.	Long.	From ∗	To ∗	No.	Mag.
1880, March	11	2.50 A.M.	24 N	39 W	ξ Draconis	Σ 2403 Draconis	1	—
	11	3.0 A.M.	"	"	ε Ophiuchi	η Scorpionis	1	$\underset{\wedge}{\times}$
	11	3.0 A.M.	"	"	45 Draconis	15 Cygni	1	—
	11	6.40 P.M.	"	"	α Piscium	? double 33 H Piscium	1	$\underset{\wedge}{\times}$
	12	2.45 A.M.	24 N	39 W	γ Urs. min.	4 Urs. min.	1	—
	12	3.0 A.M.	"	"	ξ Urs. maj.	μ Urs. maj.	1	$\underset{\wedge}{-}$
	12	7.15 A.M.	"	"	γ Arietis	64 Piscium	1	$\underset{\wedge}{\times}$
	13	3.15 A.M.	24½ N	"	δ Urs. min.	Polaris	1	+
					Very few meteors.			
	14	1.0 A.M.	25 N	"	δ Urs. min.	28 Cephei	1	—
	18	1.15 A.M.	28 N	37 W	φ Urs. maj.	52 Cameli	1	—
	18	3.15 P.M.	"	"	60 Leonis	between ρ & α Leonis	1	—
					Few meteors.			
	21	2.0 A.M.	31 N	31 W	σ Scorpionis	χ Lupi	1	×
	21	3.0 A.M.	"	"	φ Urs. maj.	θ Urs. maj.	1	+

Meteorological Office,
1880, *April* 5.

On the Theoretical Value of the Acceleration of the Moon's Mean Motion in Longitude, produced by the Change of Excentricity of the Earth's Orbit. By Sir G. B. Airy, K.C.B. Astronomer Royal.

1. In the *Monthly Notice* of the Royal Astronomical Society for 1874, January, I explained the principles of a novel method of treating the Lunar Theory. My object was to dispense with long algebraic expansions, far as as possible. For this purpose, I proposed to use the numerical coefficients found by a preceding investigator (Delaunay), not as accurate, but as very approximate, requiring numerical corrections (treated, in the first instance, in a symbolical form) so small that multiples of those symbols (when reduced to numbers) to the first power only would suffice for the correction of the numerical value of every function depending on the coefficients. The immediate object was to find the numerical relation between the theoretical movements of the Moon and the forces (themselves depending on those movements) which ought to account for the movements; and to find whether, by varying the movements and varying in correspondence therewith the forces which depend on those movements, the relation between the movements and the forces can be made perfect.

2. It is evident that this form of investigation is not limited to the correction of assumed coefficients, but can also be applied to the examination of the effects of introducing small forces not contemplated in the original theory. These small forces may in some instances depend on geometrical considerations (as in the effect of the Earth's oblateness), in other cases they may depend on time (as in the slow change of solar elements producing certain disturbances in the motions of the Moon). In such cases, the changes from the original suppositions are in fact the introduction of new forces; and here we have to consider how, by varying the movements and varying in correspondence therewith the forces which depend on those movements (as stated in the first paragraph), and also introducing the new forces and their variations depending on the movements, the relation between the movements and the forces can be made perfect. The original fundamental supposition must, however, be maintained, that the new variations of forces and movements are to be so small that the first power of the algebraic terms expressing them will suffice.

3. The units employed in the further investigation which I am now to explain (the same which are used in my general Lunar Theory) are the following:—

The unit of longitudinal measure is the mean distance of the Moon from the Earth at the epoch (say A.D. 1900). In respect of absolute measure, that distance is called 1, but in respect of variability, it is called a. In the present investigation, we shall not need to consider the variability.

The unit of angular measure is the angle corresponding to an arc of a circle whose length is equal to radius 1. It is equal, in degrees, to $\frac{360°}{2\pi}$ or, in seconds, to $\frac{1296000''}{2\pi}$.

The unit of time is the time occupied by the Moon, with her mean angular motion at the epoch, in describing the unit of angle. We shall have to compare it with the length of a Julian year: it is easily seen that, as referred to our unit, the Julian year =

$$\frac{\text{Moon's mean angular motion}}{\text{Sun's mean angular motion}} \times 2\pi = 13{\cdot}368753 \times 2\pi = 83{\cdot}998352.$$

The unit of the Moon's angular motion is therefore necessarily the angular velocity with which the Moon in mean angular motion describes the angle 1 in the time 1. In respect of absolute measure of angular motion, this is called 1; in respect of variability (not, however, considered here), it is called n.

4. The Moon's radius vector, the projection of radius vector on the plane of the ecliptic, her longitude, her latitude, and all combinations of these, are (in the most general theory) expressed in indefinite series of periodical terms, whose arguments consist of sums of various multiples (sometimes positive, sometimes negative) of the following quantities:—

D = excess of Moon's mean longitude above Sun's mean longitude.

f = excess of Moon's mean longitude above mean longitude of node.

l = excess of Moon's mean longitude above mean longitude of perigee.

S = mean anomaly of Sun.

These are all treated as increasing uniformly with the time.

In the present investigation, however, it is proposed to neglect the variable terms related to the excentricity of the Moon's orbit and its inclination to the ecliptic, not as non-existent, but as producing nothing of sensible amount additive to the terms employed in the present inquiry; and thus the symbols f and l will not appear. The modifications which they produce in the constant term will, in the first instance, be retained. We shall not treat of the disturbance of the Moon's latitude. As regards the excentricity of the Sun's orbit, we shall use a mean of the Sun's effect through each year; and thus S will not be required. Thus the only argument remaining is D, and the only multiple of that argument which we shall use is 2D, of which the effects far exceed those of D or 3D. And for 2D we shall write F.

5. The equations which we use in the general theory to determine the Moon's motion apply to T (a force in the plane of the Sun's orbit, normal to the projection of the Moon's radius

vector, and accelerating the Moon), P (a force in the same plane, in the projection of the radius vector, and directed from the Earth), and Z (a force normal to the plane of the Sun's orbit, in the direction in which z is considered positive). Z, however, has here no existence, as the inclination of the two orbits is neglected.

6. By investigations of which the introductory parts are printed, a "Factorial Table" has been prepared, exhibiting in a symbolical form, as factors of the lunar geometrical disturbances, the forces which must be introduced as acting on the Moon in order to explain the disturbed movements of the Moon. Any possible disturbances, subject to the limitation above mentioned in regard to inclination of orbit, may be expressed by disturbances of $\frac{a}{r}$ and v, where r is the radius vector of the Moon, and v the Moon's true longitude. For such disturbances we put the symbols $\delta\frac{a}{r}$ and δv. Then the Factorial Table, limited as is above mentioned, gives the following equations, in which the value of $\frac{r}{a}$ is $1 - \cdot 0077 \cos F$:

$$T\frac{r}{a} = \left\{\begin{array}{lll} +\delta\left(\frac{a}{r}\right) & \times(\quad \cdot 0000 & -\cdot 0281 \sin F) \\ +\frac{d}{dt}\left(\delta\frac{a}{r}\right) & \times(-1\cdot 9919 & +\cdot 0061 \cos F) \\ +\delta v & \times(\quad \cdot 0000 & +\cdot 0168 \cos F) \\ +\frac{d}{dt}(\delta v) & \times(\quad \cdot 0000 & +\cdot 0268 \sin F) \\ +\frac{d^2}{dt^2}(\delta v) & \times(+1\cdot 0006 & -\cdot 0144 \cos F) \end{array}\right\}$$

$$P\frac{r}{a} = \left\{\begin{array}{lll} +\delta\left(\frac{a}{r}\right) & \times(+2\cdot 9964 & -\cdot 0236 \cos F) \\ +\frac{d}{dt}\left(\delta\frac{a}{r}\right) & \times(\quad \cdot 0000 & -\cdot 0528 \sin F) \\ +\frac{d^2}{dt^2}\left(\delta\frac{a}{r}\right) & \times(-1\cdot 0053 & +\cdot 0212 \cos F) \\ +dv & \times(\quad \cdot 0000 & +\cdot 0168 \sin F) \\ +\frac{d}{dt}(\delta v) & \times(-1\cdot 9888 & -\cdot 0090 \cos F) \end{array}\right\}$$

These equations give, in the first instance, the values of forces which can produce an assigned geometrical disturbance; our object, however, will be, by a reversal-process, to find the geometrical disturbances which will be required to correspond to an assigned external force.

7. It is important to observe that, in forming these equations, the variations of terrestrial attraction and of solar attraction,

depending on the change of the Moon's place produced by the disturbance of which we treat, are accurately taken into account, whatever the change of place may be.

We now proceed to compute the Disturbing Forces.

8. Taking the mean parallaxes of the Sun and Moon as $8''\cdot91$ and $3422''\cdot3$, the following values are obtained by known developments:—

Mean tangential disturbing force at epoch		$-\cdot008284 \,.\, \sin F.$
Mean radial disturbing force at epoch	$+\cdot002683$	$+\cdot008245 \,.\, \cos F.$

(These numbers have been computed in a general process, embracing all the inequalities of the lunar motions.)

Now suppose that, in the progress of time, the solar energy which produces these forces varies from the force at tabular epoch in the proportion of $1 : 1+bt$. This is the same thing as introducing new forces, whose values are equal to the values above-given, multiplied by bt. Thus we obtain—

$$T = \qquad\qquad -\cdot008284 \times bt \,.\, \sin F;$$
$$P = +\cdot002683 \times bt + \cdot008245 \times bt \,.\, \cos F;$$

and these quantities, multiplied by $1 - \cdot0077 \cos F$, are to be substituted on the left side of the equation in Article 6. But the multiplication of the small term by T, and by the second part of P, will produce terms depending on 2F, which are everywhere neglected. The multiplication of the first part of P, however, introduces $-\cdot002683 \times \cdot0027 \times bt. \cos F$. This is similar to the second part, and will be associated with it.

9. In future we shall adopt the following notation:—

For $\frac{dF}{dt}$ we shall write m: the numerical value of m is $+1\cdot8504$. For

	$\cdot008284$,	$\cdot002683$, and	$\cdot008245$ (numbers without signs),
we shall use	A,	B,	C.

A and C are sensibly equal.

The equations of Article 6 now become—

Equation for the Moon's ecliptic longitude:—

$$-A.bt. \sin F = \left\{\begin{array}{ll} +\delta\frac{a}{r} & \times(\quad\cdot0000-\cdot0281 \sin F) \\ +\frac{d}{dt}\left(\delta\frac{a}{r}\right) & \times(-1\cdot9919+\cdot0061 \cos F) \\ +\delta v & \times(\quad\cdot0000+\cdot0168 \cos F) \\ +\frac{d}{dt}(\delta v) & \times(\quad\cdot0000+\cdot0268 \sin F) \\ +\frac{d^2}{dt^2}(\delta v) & \times(+1\cdot0006-\cdot0144 \cos F) \end{array}\right\}$$

Equation for the projection of the Moon's radius vector:—

$$\left.\begin{array}{r} +\mathrm{B}.\delta t. \\ +(\mathrm{C}-\cdot 0077.\mathrm{B})\delta t.\cos\mathrm{F} \end{array}\right\} = \left\{\begin{array}{ll} +\delta\dfrac{a}{r} & \times(+2\cdot 9964-\cdot 0236\cos\mathrm{F}) \\ +\dfrac{d}{dt}\left(\delta\dfrac{a}{r}\right) & \times(\quad\cdot 0000-\cdot 0528\sin\mathrm{F}) \\ +\dfrac{d^2}{dt^2}\left(\delta\dfrac{a}{r}\right) & \times(-1\cdot 0053+\cdot 0212\cos\mathrm{F}) \\ +\delta v & \times(\quad\cdot 0000+\cdot 0168\sin\mathrm{F}) \\ +\dfrac{d}{dt}(\delta v) & \times(-1\cdot 9888-\cdot 0090\cos\mathrm{F}) \end{array}\right\}$$

9. In the following Articles we shall use C′ for (C − ·0077. B), and in future calculations we shall use the integral numbers − 2, + 1, + 3, − 1, − 2, for the broken numbers in the first column, 2nd, 5th, 6th, 8th and 10th lines.

10. It is perhaps vain to expect that equations so complex as those exhibited above can be solved directly. The only hope to solve them rapidly and successfully must be in carefully selecting a form for the solution, and in substituting its terms with indeterminate coefficients. The form adopted has been established from the following considerations:—

(1.) Whatever takes place in the Moon's mean parallax will be repeated, lunation after lunation, in the same way. Thus $\delta\frac{a}{r}$ may have a term $+ct$.

(2.) A corresponding change will take place in the mean arc of longitude described in every portion of time, so that the alteration of mean arc described in one portion, which is the sum of all alterations in the portions up to that time, will be proportional to t; and the sum of these alterations of mean arc up to that time, which is the alteration of longitude, will be proportional to t^2. Thus δv may have a term ht^2.

(3.) and (4.) Since the Sun's motion is not altered, F, which = 2 Moon's longitude − 2 Sun's longitude, will receive the addition $2ht^2$; and this term will geometrically (not by multiplication of any other perturbation) modify the original quadrantal terms, so that $\delta\frac{a}{r}$ will have a term such as $ft^2.\sin\mathrm{F}$, and δv will have such a term as $kt^2.\cos\mathrm{F}$.

(5.) and (6.) During the progress of these changes, it is not unreasonable to expect that the quadrantal modification of the orbit may undergo changes proportional to the time; and it seems likely that $\delta\frac{a}{r}$ may require such a term as $gt.\cos\mathrm{F}$. and that δv may require $lt.\sin\mathrm{F}$.

(7.) and (8.) Something will depend on the time when we suppose the intrusion of the new force and the disturbance of movements to begin. It may happen that the new force will

not begin at Lunar Conjunction or Opposition. Our assumption $T = -Abt \sin F$ implies that there will never be a disturbing force in longitude when F has one special value + integral multiples of π. It appears that this can be well met by the term $e. \sin F$ in $\delta\frac{a}{r}$ and a constant term i in δv.

11. The following assumptions, therefore, have been made :—

$$\delta\frac{a}{r} = +c\,.\,t + e\,.\sin F + ft^2\,.\sin F + gt\,.\cos F;$$
$$\delta v = +ht^2 + i \qquad + kt^2\,.\cos F + lt\,.\sin F,$$

containing eight indeterminate quantities; and these expressions and their differentials have been substituted in the equations of Article 9. It is proper to remark that in the multiplication of series forming every step of this operation, sin F and cos F are most carefully preserved; but $\sin^2 F$, $\cos^2 F$, and sin F. cos F, are neglected; they all relate to the argument 2F or 4D, which has been omitted from the adopted portion of the Factorial Table; and their factors are all very small.

The results of substitution admit of being arranged in eight groups, as follows :—

(1) $-2c + 2h = 0.$

(2) $(+3c - 4h - Bb)t = 0.$

(3) $(-2mf + \cdot 0168.h - m^2k)t^2\,.\cos F = 0.$

(4) $(+3f + m^2f + \cdot 0168.h + 2mk)t^2\,.\sin F = 0.$

(5) $(-\cdot 0281.c - 4f + 2mg + \cdot 0536.h - 4mk - m^2l + Ab)t\,.\sin F = 0.$

(6) $(-\cdot 0234.c - 4mf + (m^2+3)g - \cdot 180.h - 4k - 2ml - C'b)t\,.\cos F = 0.$

(7) $(+\cdot 0061.c - 2me - 2g - \cdot 0288.h + \cdot 0168.i + 2k + 2ml)\,.\cos F = 0.$

(8) $(-\cdot 0528.c + (m^2+3)e - 2f + 2mg + \cdot 0168.i - 2l)\,.\sin F = 0.$

12. The solution of these equations, though troublesome, is not difficult. The following appear to be the results. In the last column, the expressions are simplified by considering A=3B, C= 3B :—

$c =$				$-Bb.$
$e =$				$+ \cdot 6195\,.\,Bb.$
$f =$				$+ \cdot 0144\,.\,Bb.$
$g =$	$+ \cdot 4485\ Ab$	$+ \cdot 0238\,.\,Bb$	$+ \cdot 4141\,.\,C'b.$	$= + 2\cdot 6116\,.\,Bb.$
$h =$				$-Bb.$
$i =$				$- 370\cdot 83\ .\,Bb.$
$k =$				$- \cdot 0205\,.\,Bb.$
$l =$	$+ \cdot 7769\,.\,Ab$	$+ \cdot 0457\,.\,Bb$	$+ \cdot 4476\,.\,C'b$	$= + 3\cdot 7192\,.\,Bb.$

It will be remembered that the signs of c, e, f, and g, are those which apply to $\delta\left(\frac{a}{r}\right)$. The signs are to be changed for $\delta\left(\frac{r}{a}\right)$.

The term which expresses the secular acceleration in longitude is $ht^2 = -Bb.t^2$. To the numerical value of this term we will now give attention.

13. First, we have to ascertain the value, in ordinary language, of B=·002683. The value is here expressed in decimal parts of the radius; we require it in seconds of arc. Now

$$2\pi = 360 \times 60 \times 60 \times 1'' = 1296000 \times 1'';$$

$$\text{therefore } \cdot 002683 = \frac{1296000''}{2\pi} \times \cdot 002683.$$

14. Secondly, to find the value of b in terms of ΔE. Let σ, A, E, R, V, T, be the Sun's mass, the orbital semiaxis major, the excentricity of the solar (or terrestrial) orbit, the orbital radius vector, the Sun's true longitude, and the Sun's periodic time (one year.) The mean effect (as regards special points of the Moon's orbit) of the Sun, in a given state of his orbit and a given position in his orbit, to disturb the Moon's motion, is represented by $\frac{\sigma a}{R^3} \times \Delta t$, Δt being an element of time. Let ΔV be the corresponding element of longitude. The area described by the radius vector is $\frac{1}{2}R^2\Delta V$. The whole area of the ellipse is $\pi A^2 \sqrt{(1-E^2)}$. Then this proportion holds—

$$\Delta t : T :: \tfrac{1}{2}R^2\Delta V : \pi A^2\sqrt{(1-E^2)};$$

$$\text{and therefore } \Delta t = \frac{TR^2 . \Delta V}{2\pi A^2\sqrt{(1-E^2)}};$$

and the disturbing effect of the Sun in the element of time is

$$\frac{\sigma a}{R^3} . \frac{TR^2 . \Delta V}{2\pi A^2\sqrt{(1-E^2)}} = \frac{\sigma a T}{2\pi . A^2\sqrt{(1-E^2)}} . \frac{\Delta V}{R}.$$

Now

$$\Delta v = \Delta \text{ (true anomaly), and } R = \frac{A(1-E^2)}{1+E . \cos \text{(true anomaly)}};$$

therefore the disturbing effect

$$= \frac{\sigma a}{2\pi . A^3} \times \frac{1+E . \cos \text{(true anomaly)}}{1-E^2} \times \frac{T . \Delta \text{(true anomaly)}}{\sqrt{(1-E^2)}}.$$

Integrating this through an entire orbital revolution of the Sun, the value is

$$\sigma a \times \frac{T}{A^3} \times \frac{1}{(1-E^2)^{\frac{3}{2}}} = \sigma a . \frac{T}{A^3}\left\{1+\frac{3}{2}E^2+\frac{3.5}{2.4}E^4+\&c.\right\}.$$

The variation of this quantity depending on the variation δE is

$$+\sigma a\frac{T}{A^3} \times \left\{3E+\frac{15}{2}E^3\right\}\delta E$$

and the proportion of the variation to the unvaried quantity is

$$+\frac{3E+\frac{15}{2}E^2}{1+\frac{3}{2}E^2} = +3E\times\frac{1+\frac{5}{2}E^2}{1+\frac{3}{2}E^2} = +3E(1+E^2)\delta E, \text{ nearly.}$$

15. Thirdly, to find the values of E and δE, and the proportion of the variation to the unvaried quantity just mentioned:

By Le Verrier's *Annales de l'Observatoire de Paris*, tome iv., page 103, E='01676927 − '0000004338 × number of years; and the proportion of the variation above mentioned, for one year, to the unvaried quantity, is

$$-3\times{}^{\cdot}01676927\times\{1+({}^{\cdot}01677)^2\}\times{}^{\cdot}0000004338.$$

To infer from this the proportion for one orbital revolution of the Moon, we must multiply by

$$\frac{\text{time of Moon's orbital revolution}}{\text{time of Sun's orbital revolution}}, \text{ or } \frac{\text{Sun's motion in longitude for 30 days}}{\text{Moon's motion in longitude for 30 days}}$$

$$\text{or } \frac{1064457}{14230466}.$$

And to infer the proportion for our unit of time, we must further multiply by

$$\frac{\text{unit of time}}{\text{units of time in Moon's orbital revolution}} \text{ or } \frac{1}{2\pi}.$$

Thus we obtain for the proportion of the variation to the unvaried quantity during one unit of time, or for b,

$$-3\times{}^{\cdot}01676927\times1{}^{\cdot}0002812\times{}^{\cdot}0000004338\times\frac{1064457}{2\pi\times14230466}.$$

16. And to obtain the value of δv for one year, we must multiply this by the square of $\frac{\text{time of Sun's orbital motion}}{\text{time of Moon's orbital motion}}\times$ units of time in Moon's orbital motion, or $\left(\frac{2\pi\times14230466}{1064457}\right)^2$.

Thus we obtain finally for the value of δv or $-Bb.t^2$,

For one year, $0''{}^{\cdot}00101477$.

For a century, $10''{}^{\cdot}1477$.

I present this result to the Society with much confidence.

It is to be remarked that this numerical value is founded upon the numerical value attached to the solar parallax in the first step of the investigations—namely $8''{}^{\cdot}91$. And the magnitudes of the quantities A,B,C, depend upon the inverse cube of the Sun's distance, or upon the cube of parallax. If, for instance,

the parallax be diminished by $\frac{1}{80}$ part, the magnitudes of A,B,C, and of the acceleration, will be diminished by $\frac{1}{20}$ part.

17. The following deductions are unimportant, but they may be interesting. In the Moon's orbit, $1'' = 6000$ feet, very nearly. In a century, therefore, the Moon is accelerated 60000 feet. In the first year, the acceleration is 6 feet; in the second year, 18 feet additional, &c. This is additive to the computed longitude, whether before or after the epoch.

In a century, the Moon's distance is changed by $\frac{60000 \text{ feet}}{t}$. Now t for 100 years is 8400, and the Moon's distance is changed in a century by $\frac{60000}{8400}$ feet or 7·14 feet; in one year it is changed by 0·0714 feet, or less than an inch. This change proceeds uniformly; for every year before the epoch, the distance is additionally greater than the computed distance by the multiple of 0·0714 feet, and for every year after the epoch, it is additionally less.

Wherever Bb occurs, we may use the value found thus. In a century, δv, or kt^2, or $-\text{B}b\,.\,t^2$, or $-\text{B}b \times (8400)^2, = 60000$ feet. Therefore Bb (without respect of sign) $= \frac{60000 \text{ feet}}{(8400)^2} = \frac{6 \text{ feet}}{(84)^2}$ $= 0·0101$ inch.

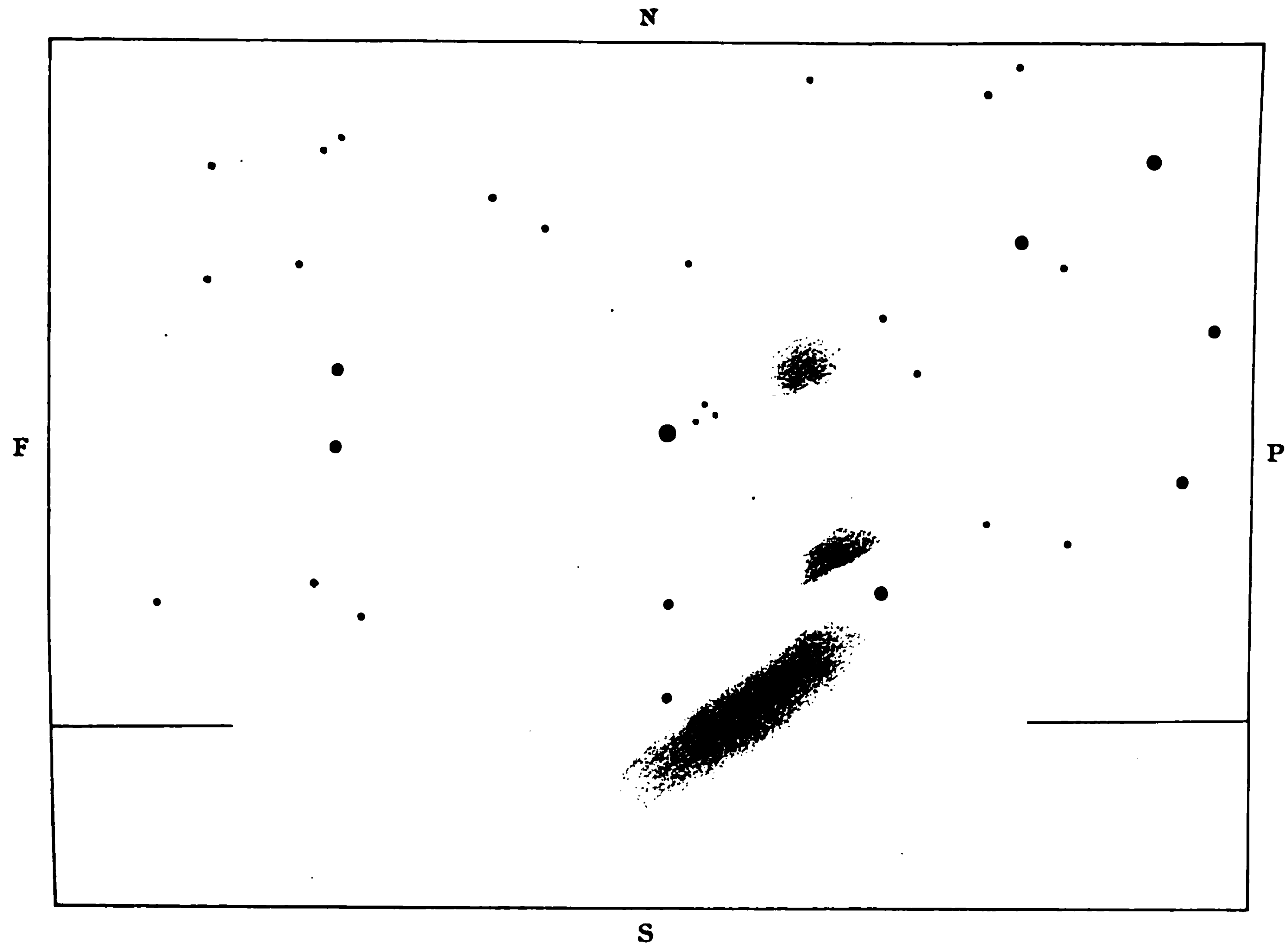
Royal Observatory, Greenwich,
1880, April 24.

The Nebula in the Pleiades. By A. A. Common, Esq.

An observation of this Nebula was made on February 8th, 1880, with my three-feet telescope, and the sketch made at this time, a copy of which is attached, differs so from that of Mr. Maxwell Hall in the January number of the *Monthly Notices* (which came to hand soon after) that it may be worth recording. The stars on this sketch are traced from two photographs taken that night with one and a half minute's exposure, and are given as a guide to the positions of the nebulæ seen.

The oval patch of light near *Alcyone* was only seen on this night; later observations made with a view to correct the place of the nebula proper did not show it, but the nights subsequent to the first were not so fine. The smaller patch of light north of *Merope* was always seen; the edge near that star is pretty sharply defined, with dark sky between. The general shape and direction of the nebula proper was as shown. A fine night was waited for with the hope of seeing better the extent of this nebula, but without success. There were pretty certain indications of an extension beyond *Merope* in the direction of *Electra*. In apparent brightness the sharp edge of the smaller nebula was equal to the brighter part of the large one; but this may have been due

NEBULÆ IN THE PLEIADES.



February 8th 1880.
A. A. Common,
Ealing.

... & Co Lith. London.

to the contrast the edge made with the sky in the one case, and the gradually brightening towards the middle in the other.

There is a great deal yet to be settled as to the extent and number of the nebulæ in this cluster, and the remarkable difference in the direction given of the principal line of the large nebula is quite incomprehensible.

Rotation Period of Jupiter. By T. D. Brewin, Esq.

In observing Jupiter for the first time last Opposition, August 7th, 1879, $11^h\ 30^m$ G.M.T., I saw a very conspicuous red spot in the southern hemisphere; the preceding end of red spot at the time was as near central as I could judge by eye-estimation. I made a drawing of the planet, thinking it would be a favourable opportunity to obtain the rotation period from sketches taken when the preceding end was central, and at as great an interval of time between successive sketches as possible.

The last sketch of the spot when the preceding end was central I obtained February 4th, 1880, $5^h\ 30^m$ G.M.T., a period of 180 days 18 hours from the first drawing, equal to 437 rotations, the result obtained being $9^h\ 55^m\ 34^s{\cdot}1$, a result so near to Mr. Pratt's described in the January *Notices* of the R.A.S. that I thought it would be of interest to send an account as confirmatory of Mr. H. Pratt's.

14, *St. Nicholas, Leicester*,
1880, *April* 9.

Observations of the Great Southern Comet, 1880, *made at the Melbourne Observatory.* By R. L. J. Ellery, Esq., Director of the Observatory.

The Comet whose appearance I announced by the last mail was observed here from the 9th till the 17th instant. After its first apparition it became rapidly fainter, while the tail increased in length up to 45°; by the 12th, however, the tail had almost disappeared, and on the 17th—the date of our last observations—the nucleus could only be observed with considerable difficulty in a dark field. The moonlight and the Comet's rapid diminution in brightness have now put it out of reach of further measures. A table of apparent places deduced from our observations is annexed.

Observatory, Melbourne,
1880, *February* 20.

Apparent Places of Comet, 1880, from Observations at the Melbourne Observatory.

Melbourne Mean Time.	Comet α	Comet NPD	Observer	Instrument.	No. of Measures.	Comet—Star α	Comet—Star NPD	Name of Star.	Remarks.
d h m s	h m s	° ′ ″				m s	′ ″		
Feb. 9 9 9 30·5	23 41 6·73	123 43 43	W	N	3	+4 1·90	− 1 8·4	Wash. 10462	
9 9 31 44·6	23 41 22·22	123 44 0	W & E	N	3	−4 18·57	− 3 24·0	Wash. 10520	
10 8 40 28·7	23 57 55·76	123 45 8	W	N	3	−2 11·30	+15 59·3	Wash. 10648	
10 9 18 42·6	23 58 22·96	123 44 59	E	S	3	−1 44·10	+15 50·4	Wash. 10648	
14 9 44 46·0			W	N	3	+1 33·67	− 8 2·0	7½ Mag.	{about 1° 0′ 23″ RA / 122 29 20 NPD} comp.*
14 9 59 26·7	1 2 5·62	122 21 8	W	N	1	+5 22·55	+ 9 10·5	σ Sculptoris	
15 8 46 4·2	1 15 22·38	121 46 26	W	N	6	−2 34·51	+11 59·9	BAC 421	
16 8 27 17·3	1 28 23·92	121 5 36	E	S	3	+2 13·06	+11 34·4	Wash. 719	Comet excessively faint
16 9 43 23·7	1 29 5·35	121 3 24	W	N	7	+2 54·49	+ 9 22·0	Wash. 719	"
17 8 57 29·8	1 41 9·51	120 21 43	W	N	6	−1 7·74	+ 1 16·9	Lacaille 522	{Comet barely visible; measures little better than guesswork.
17 9 42 2·7	1 41 31·32	120 20 36	E	S	7	−0 45·93	− 0 0·4	Lacaille 522	

In the observer's column E refers to Mr. Ellery, the director, and W to Mr. White, the chief assistant.

In the instrument column S refers to the South Equatoreal of 8 inches' aperture, and N to the North Equatoreal of 4½ inches' aperture.

The observations were made with a rhombus micrometer.

Observations of the Great Southern Comet, 1880, *made at the Observatory, Sydney.* By H. C. Russell, Esq., Director of the Observatory.

I send the following account of a Comet seen here, for insertion in the R. A. S. *Notices* if it is deemed of sufficient importance.

The earliest notice of the Comet which I can obtain is from a gentleman living a '*Moree*' in the northern part of this colony. On Sunday, February 1, he went out shooting quail just after sunset, and on looking to S.W. was surprised by a bright streak of light, stretching from the horizon towards the South Pole. It was a striking object from its brightness, and about three degrees wide (he does not give the length). On Monday he saw it again but it was much fainter; this evening it was seen by many persons in different parts of this and the neighbouring colonies. The nights of February 1 and 2 were cloudy in Sydney, but on the 3rd I saw the comet's tail; between the clouds I could trace it for 30° to a bank of clouds 5° above the horizon. At its widest part, near β *Gruis*, the tail was $1\frac{1}{4}$ degree wide (I enclose sketch taken on 5th). On 4th and 5th the tail was seen; but owing to clouds I could not detect the nucleus. On the 6th it became cloudy and wet until February 13th when I saw the nucleus in (approximately) R. A. $0^h\ 41^m$, Decl. 32° 57′ south. On the 14th another glimpse was obtained, giving approximate position R.A. $1^h\ 1^m\ 9^s$, Decl. 32° 23′ south. In both cases only small stars of comparison could be used, and they will have to be observed with the transit before exact position can be obtained. On the 13th comet had faded so much that it was difficult to see any part of it with the naked eye. The nucleus was very faint, about 5 seconds (time) long, and about 1 minute (arc) wide; the tail at the head was about 30″ wide, about 15′ at its widest part, and about 15° long. On the 14th it turned cloudy again after a few minutes clear sky, and it does not seem probable that the comet will be again observed, as it is fading so rapidly. In the southern colonies the weather has been more favourable.

Sydney Observatory,
1880, *February* 16.

(*From a Circular of the Vienna Academy.*)

COMET 1880 *b* (SCHAEBERLEON).

Elements :—

Per. Pass. 1880, June 11·7666 Berlin M.T.

	°	′	″	
Long. Per.	25	51	19	Mean Eq. 1880·0
Long. Node	254	17	15	
Inclination	123	40	13	

Log. Per. Dist. 0·28667.

Computed by Drs. Holetschek and Zelbr, from observations on April 10, 11, and 13.

On the relative Star Magnitude of Mars in February and March, 1880. By Lord Lindsay.

At the request of Mr. Marth, the planet Mars was observed at Dunecht with a Zöllner Astro-photometer on the most favourable nights at the end of February and during March, using *α Tauri* and *α Orionis* as comparison stars. The sky was generally far from favourable, but by watching faint stars in the immediate neighbourhood of the object under observation it is hoped that the final results at least are not seriously influenced by cloud. No readings have been rejected, except two sets specially noted at the time as being affected by haze.

The single results, derived from two readings for each object, are very discordant. This is doubtless owing in part to the very red colour of the planet, which is far more intense than that of either of the stars. The observer, too, found a singular difficulty in avoiding blunders in reading the four quadrants into which the intensity-circle is divided, counting, as they do, in various directions. The very slight arithmetical advantage got by this arrangement is probably far more than negatived by this additional source of error.

The final results for about $8\frac{1}{4}^h$. G. M. T., corrected for the varying extinction at different altitudes, are as follows:—

		Observations.	α Tauri ×	α Orionis ×
1880, Feb.	24	♂ = (3)	1·30	...
	26	(2)	1·12	1·23
	29	(1)	1·39	...
March	3	(2)	1·19	1·01
	7	(5)	1·03	1·04
	8	(5)	1·26	1·07
	9	(3)	1·09	0·98
	21	(4)	0·85	0·83

From these it seems that Mars = *α Tauri* March 13·8; and Mars = *α Orionis* March 10·3; or Mars half way between magnitude of *α Tauri* and *α Orionis* at Greenwich noon, March 12, 1880. On the average *α Orionis* = 1·067 × *α Tauri*. All the observations were taken by Ralph Copeland.

Dunecht Observatory,
1880, April 6.

MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. XL. MAY 14, 1880. No. 7.

J. R. HIND, Esq., President, in the Chair.

Thomas Buckney Esq., 61 Strand, W.C.;

Lindsay Atkins Eddie, Esq., Oatlands, Grahamstown, Cape of Good Hope;

Thomas Gullon, Esq., Northenden, Cheshire; and

Signor N. Perini, 261, Hampstead Road, N.W.;

were balloted for and duly elected Fellows of the Society.

Notice.—The Council have made arrangements for distributing by post to Fellows of the Society Ephemerides of Comets and other information of immediate interest. Any Fellow who may wish to receive such information is requested to forward his name to the Assistant Secretary, Royal Astronomical Society, Burlington House, London, W.

It is desired that early information respecting Comets may be sent direct to W. H. M. Christie, Royal Observatory, Greenwich, London, S.E., in order that it may be distributed without delay.

On the Preparations to be made for observation of the Transit of Venus 1882, December 6. By Sir G. B. Airy, K.C.B., Astronomer Royal.

Whatever we may consider as concluded from the observations of the Transit of *Venus*, 1874, I think we may well believe that the scientific world will not be satisfied unless we take the opportunity of securing all that can be obtained from the Transit

of 1882: such an opportunity as will recur only after the extinction of three generations of mankind. It is not now too early to examine into the circumstances of that Transit, and to decide provisionally on the sites to be adopted, the observations that may most advantageously be taken, and the means of carrying them out.

We approach this Transit under conditions far more favourable than those under which we undertook the observations of 1874. We have the personal experience of observers who are still living among us, and our minds are still occupied with the discussions to which their observations gave rise. In external circumstances a great step has been gained by the extension of electric telegraphs, upon which we may rely for delivering us from that which was the most laborious and most expensive part of the Transit of 1874—namely, the determination of longitudes of stations. And, finally, I believe I can show that, by departing in some respects from the plan which I proposed many years ago, and laying greater stress upon considerations which I then treated more lightly, we may adopt a course which seems to possess all the elements of success.

I wish first to call attention to this point: that a most important condition for trustworthy observation is a sufficient elevation of the Sun above the horizon. For this advantage it will be well sometimes to sacrifice something in the magnitude of the parallax-factor; especially where observations can be obtained at several neighbouring stations: I am inclined to fix upon the elevation 20° as one that ought to be secured if possible: though an elevation of 15° may sometimes be accepted as sufficient.

This being premised, I proceed to indicate the stations which I consider most advantageous.

Transit of Venus 1882.

Ingress Accelerated.

For this phenomenon, I propose to rely entirely on the Cape Colony, employing as many stations as can be manned from the Cape Observatory to D'urban.

As regards absolute longitude: relying upon that of Aden (determined in 1874), and remarking that telegraph is complete thence to D'urban, and that all authorities—and especially the Telegraph Company—are anxious to promote this determination, I cannot doubt that, under Mr. Gill, the longitude of the Cape Observatory will be quickly ascertained. Then as regards the actual observation of the Transit: telegraph exists along the whole coast, and all observations will be at once referred to Cape Observatory time, and thus to Greenwich time.

The Sun's elevation ranges from about 33° to about 46°. (At

Kerguelen and Mauritius the elevation is only 10° : at the south point of Madagascar it is 21°.)

The factor of parallax is about 0·65. This is the smallest that I propose to use; but its smallness is, in my opinion, abundantly compensated by the other very favourable circumstances.

The Ingress will also be visible in Britain, Belgium, France, Spain, and part of Africa. At Land's End and Valentia the Sun's elevation will be about 14°, at Greenwich about 10°, with factor 0·65. For the other countries, the circumstances are less favourable.

Ingress Retarded.

I proposed formerly to refer principally to the coasts of the Canadian Dominion and the United States of North America. But (without wishing to discourage observations there) I now think that the elevation of the Sun, 15° to 18°, is too small. I propose to substitute the whole chain of the West India Islands, from the eastern extremity of Cuba to Barbadoes: or stations on the neighbouring continent of Central America. The Hydrographic Office of the United States has determined with great accuracy the longitude of Havana, Santiago de Cuba, Kingston (Jamaica), Aspinwall, Panama, San Juan de Puerto Rico, St. Thomas, St. Croix, St. John of Antigua, St. Pierre of Martinique, Bridgetown of Barbadoes, Port Spain of Trinidad. At any of these it will be only necessary to determine local time at the observation.

The Sun's elevation ranges from about 23° to 43°.

The factor of parallax from about 0·75 to 0·85.

Bermuda also is very favourably situated, with Sun's elevation about 25° and factor of parallax about 0·90. But I believe that its longitude is not so well determined.

Egress Accelerated.

All the stations suggested for Ingress retarded may be adopted with advantage for Egress accelerated. The Sun's elevation at Bermuda will be about 13°, that for Charleston 22°, that for the chain of islands about 25°, and that for the continental stations about 35°. The parallax-factor ranges from about 0·7 to 0·9.

Egress Retarded.

Although the Egress can be observed at Melbourne with elevation 10°, and at Sydney and the whole eastern coast of New South Wales with elevation 14°, and the French will probably adopt New Caledonia with 22° elevation, yet I should be inclined to rely more completely upon New Zealand. There is nothing to prevent determination of the longitude of points of Australia, if a proper department of State would take it up, the telegraphic

communication being complete. For that of New Zealand, though considerable progress was made by Major Palmer in a series of Lunar Transits at Burnham, yet I think them scarcely sufficient for our purpose; and, if no telegraph be completed, I would propose some runs of chronometers, on which I have learned to place much reliance.

The Sun's elevation on the east coast of Australia (as stated above) would be perhaps 14°; that at Hobart Town 14°; that in New Zealand 30° to 35°.

The factor of parallax in Australia about 0·96; that in New Zealand about 0·83.

I will now advert to the instruments to be carried out.

For Local Time, at a station where the residence is to be one of considerable length, no instrument is comparable to the ordinary transit-instrument. But for short residences, intended to be no longer than is necessary for determining the state of the clock, and (if required) the local latitude, the most convenient instrument is a vertical circle, or altazimuth with no azimuth-circle or no accurately-divided azimuth-circle. (If, however, a survey for defining the place for observation should be contemplated, the azimuth-circle would be useful.) By using the vertical circle on two known stars in azimuth distant nearly 90° (it matters not in what portions of the azimuthal circumference nor at what elevation), latitude and clock-error are determined without difficulty. I should recommend the adoption of this instrument, where it can be obtained, in preference to any other. As everything depends upon its fitness for use, it ought to be furnished with duplicates of the parts most likely to be injured or lost—as levels and eye-pieces.

A good sidereal clock is indispensable, and a solar chronometer almost equally necessary.

For observation of the Transit, there ought to be a 6-inch telescope equatoreally mounted, with clock-work movement, and with careful provision in the eye-piece against the sun's radiant heat. Experiments on the efficiency of this protection ought previously to be made in the heat of the summer sun. And, if possible, another 4 or 5-inch telescope, similarly protected, ought to be furnished. A portion of these instruments can at once be supplied from the Royal Observatory.

It is decided, by almost unanimous consent, that no real assistance can be obtained by application of photography.

I should endeavour to dispense with framed huts and revolving roofs, thinking it possible that we may sufficiently trust to waterproof cloth coverings, or to such buildings (greenhouse, for instance, or ordinary shed) as may be found on the spot.

If at any station another competent observer can be found, and if the climate is uncertain, and if communications of time can be kept up (but not otherwise), a separation of the telescopes might

be advantageous. And it may be an important element of success that there be easy communication with the telegraph-office.

I have only further to remark that it is desirable that the history of the phenomena noted at each of the stations of past Transits should be carefully perused in making arrangements for this which is approaching. And I avow that some injury resulted to the observations in 1874 for want of this preparation. Attention had been so completely concentrated on the Transit of 1769, as affording a determination of Solar Parallax free in great measure from the effect of doubts on longitude, that the observations made in the Transit of 1761 were almost entirely neglected. But the Transit of 1761, partly visible in Europe, was, in fact, more extensively observed, and perhaps by abler professional observers, than that of 1769. There are numerous records of the observations of 1761 in the following volumes of the *Philosophical Transactions*. Among the results there is one—namely, the ring of light round the planet *Venus*—which, had it been known to our observers, would have saved them from some trouble, and possibly some inaccuracy, in the observations of 1874.

Note in Correction of a Passage in his Paper "On the Theoretical Value of the Acceleration of the Moon's Mean Motion in Longitude" (Monthly Notice 1880, April). By Sir G. B. Airy, K.C.B, Astronomer Royal.

It has been pointed out to me by Professor Adams that the second part of Paragraph 16 in the paper on Acceleration has no foundation. This is perfectly clear, inasmuch as the factor $\frac{\sigma}{A^3}$, which is proportional to $\frac{1}{T^2}$ (T being put for the length of the year), is invariable so long as the year is invariable. With change of A, σ must be changed. I cannot explain the occurrence of this error, except by the remark that the paragraph in question was written last of all—a circumstance which, as I well know, tends powerfully to increase probability of error.

1880, *May* 15.

On the Determination of the Solar Parallax by Means of the Parallactic Inequality in the Motion of the Moon. By James Campbell, Esq., and E. Neison, Esq.

As the accurate determination of the true value of the distance of the Sun is one of the most pressing requirements of modern astronomy, and as the various attempts to determine its value which have been made of late years have only resulted in increasing the uncertainty as to its real value, it seems to be most important to carefully discuss each method which can be used for determining the true value of the solar parallax, with the view of ascertaining the most advantageous manner of employing the method, and of determining, in the instances where it has already been employed, whether any systematic error has vitiated the results which have been obtained. In the present paper is presented such a discussion of the method of determining the solar parallax by means of the parallactic inequality in the motion of the Moon.

Viewed from its theoretical aspect, the most feasible method which can be employed at the present time for accurately determining the distance of the Sun is undoubtedly afforded by the parallactic inequality in the motion of the Moon. For in this inequality the solar parallax is multiplied by a coefficient which increases the quantity to be determined by nearly fifteen, so that by deducing from observation the value of the parallactic inequality to only one-seventh of a second of arc, it would enable the solar parallax to be ascertained to within less than one-hundredth of a second of arc. If, therefore, it were possible to deduce from observation the true value of the parallactic inequality, it would furnish the most advantageous method of ascertaining the true distance of the Sun that is likely to be at our disposal for many years. It is true that in time it will have to yield this theoretical advantage to the methods founded on the determination of the amount of the secular variations in the elements of the orbits of the planets, but it will be probably a century before this is the case.

When the first of these methods comes to be applied in practice, a very grave difficulty is encountered—so grave a difficulty that it has been regarded as insurmountable by modern investigators, and held to be fatal to this method of determining the solar parallax. This difficulty is due to the fact that when we discuss a series of observations of transit of the Moon's limb, what is obtained is the correction, not to the true value of the parallactic inequality, but the difference between the correction to the true value of the parallactic inequality and a small periodical variation in the apparent semi-diameter of the Moon due to the varying contrast between the brightness of the lunar limb and that of the sky which it is seen against. When the parallactic inequality tends to increase the Moon's longitude, then

this variation in the apparent semi-diameter tends to decrease the longitude; and when this variation in semi-diameter tends to increase the longitude, then the parallactic inequality tends to decrease it. Accordingly, the *apparent* value of the parallactic inequality as immediately derived from the observations is systematically less than the *true* value, which is what is required for determining the distance of the Sun.

To obtain the *true* value of the parallactic inequality from the *apparent* value furnished by the discussion of transits of the Moon's limb, it is necessary to apply a correction for the effect of this variation in the apparent semi-diameter of the Moon. The whole difficulty lies in deducing the proper value of this correction, and it has hitherto been found to be impracticable to do this. It may amount to anything between two tenths and twenty tenths of a second of arc, and to the moiety of this extent the parallactic inequality must be regarded as uncertain.

A number of attempts have been made to utilise this method of determining the solar parallax, by endeavouring to deduce the true value of the parallactic inequality from a discussion of the observations of the Moon, and then by comparing this with the theoretical value of the inequality to ascertain by how much the theoretical value of the solar parallax requires to be altered in order to give the real value. Rejecting some of the very early attempts, the principal determinations which have been made of the value of the parallactic inequality appear to be the following:—

1812 BURCKHARDT. From a discussion of over 4000 early Greenwich observations, the value 123″·5 was obtained. But the value of the lunar semi-diameter employed by Burckhardt was fully 2″·0 too small, and the effect of this error would be to decrease the apparent value of the parallactic inequality by nearly 1″·0, so that its apparent value from Burckhardt's discussion would seem to be really 124″·5.

1848 AIRY. From a discussion of the Greenwich Lunar Observations made between 1750 and 1830, the value 122″·37 was deduced for the value of the coefficient of this inequality. This was regarded as very uncertain.

1854 HANSEN. From a discussion of a number of observations made at Greenwich and at Dorpat, the value 126″·46 was obtained for this coefficient.

1859 AIRY. From a discussion of the entire series of meridian observations of the Moon made at Greenwich between 1750 and 1851 there was deduced a value of 124″·79. From four years' observations with the Greenwich altazimuth instrument, the value 125″·50 was deduced. As considerable uncertainty existed with regard to the proper value of the Moon's semi-diameter to be used in the early observations, the Astronomer Royal considered it better to use only the meridian observations made since the year 1811. These gave the value 124″·37. Finally, Sir G. Airy expressed his conviction that the value of the parallactic inequality could not differ much from 124″·70.

1867 STONE. From the discussion of over 2000 observations made at Greenwich between 1847 and 1866, the value 125″·36 was obtained.

1868 NEWCOMB. From a discussion of the Washington observations made during the four years 1862–1865, the value 125″·46 was obtained.

When these are examined it will be found that the apparent discordance between these values is to a certain extent fictitious, and arises from some being the values of the apparent coefficient of the parallactic inequality, and others the values of the real coefficient of the inequality. Properly separating these, they are

	Apparent Coefficients.	Real Coefficients.
	″	″
Burckhardt	124·5	
Airy	122·37	
Hansen		126·46
Airy	124·70	
Stone		125·36 ?
Newcomb	124·36	125·46

The four last values only need retain our attention, and these we proceed to examine. For convenience, let the following notation be employed :—

l = mean longitude of the Moon,
l' = mean longitude of the Sun,
α = mean anomaly of the Moon,
μ = mean anomaly of the Sun,
β = mean departure of the Moon from the ascending node of its orbit,
θ = the difference between the mean longitude of the Moon and Sun.

Further for brevity put

$$P \sin p = P \sin \theta = \text{Parallactic Inequality},$$
$$V \sin v = V \sin 2\theta = \text{Variation},$$
$$M \sin \mu = \text{Annual Equation}.$$

Next let P_0, V_0, M_0, denote the values of the coefficients P, V, M which are employed in the tables of the Moon with which the observations are compared, and let δP, δV, δM, denote the corrections which these values require, these corrections being defined by the equation

$$P + \delta P = P_0,$$

so that they indicate the augmentation of P, &c., necessary to bring it up to the value used in the tables.

Finally, suppose that (P), (V), (M) denote the apparent values of these coefficients—that is to say, the values which would be obtained from a discussion of the observations of the Moon without the elimination of the systematic errors due to variation in the apparent diameter of the Moon, &c., &c.

The first step to be taken is to ascertain what precautions ought to be used to ensure the accuracy of the value found for the coefficient of the parallactic inequality by a discussion of the observations. It will then be easy to see whether any of these precautions have been neglected in the determinations which have already been made of the value of the coefficient of this term.

It will be well to ascertain first of all the conditions which ought to be fulfilled by the lunar tables with which the observations are compared, as it is from the discordances between the tables and the observations that the corrections are to be deduced. The imperfections in the tables may be divided into the following:—

I. Imperfections in the theory of the terms of very long period and in the values of the secular mean motion and secular acceleration. In the discussion of the corrections to the terms of short period like the parallactic inequality, the imperfections of this class are of no importance, as they can be completely eliminated.

II. Imperfections in the theory of the terms of moderately long period. In general, errors in the values of these terms will not be of importance, the only exception being in the case of the annual equation, as this may have considerable indirect influence. For the annual equation has the effect of decreasing the longitude of the Moon in the spring of the year and of increasing its value in the autumn; whereas most observations of the Moon when the parallactic inequality is positive are obtained in the spring, and most when the parallactic inequality is negative are obtained in the autumn. The effect of this is to throw a portion of the correction to the annual equation on to the correction to the parallactic inequality. For suppose in the spring, when the argument of the annual equation is positive, that out of 50 observations 30 are made before Full moon, when the argument of the parallactic inequality is positive, and only 20 are made after Full moon, when the argument of the parallactic inequality is negative; then, the errors of the tables for these observations being a' and b', equations would result of the form

$$30\{\delta P\} + 30\{\delta M\} = a', \qquad 20\{-\delta P\} + 20\{\delta M\} = b',$$

$$\therefore \qquad 50\{\delta P\} + 10\{+\delta M\} = a' - b'.$$

Similarly, if in the autumn, when the argument of the annual equation is negative, the numbers be supposed to be reversed, then the equations will be

$$20\{\delta P\} + 20\{-\delta M\} = a'', \qquad 30\{-\delta P\} + 30\{-\delta M\} = b'',$$

$$\therefore \qquad 50\{\delta P\} + 10\{\delta M\} = a'' - b''.$$

Consequently, to determine δP, there exists the equation

$$100\{\delta P\} + 20\{\delta M\} = (a' - b') + (a'' - b'').$$

It follows, therefore, that in the above case one-fifth of the amount of any error in the annual equation would be thrown on the value found for the coefficient of the parallactic inequality. For this reason it is necessary that the tabular value of the apparent annual equation should be accurate, or its error be determined. In forming the above equation it has been assumed that the coefficients of the factors δP, δM are unity for each observation: this is not the case, so that the values 30 and 20 ought to be replaced by the sum of this number of coefficients of these quantities.

III. Imperfections in the theory of the terms of short period which are not nearly coincident in period with the parallactic inequality, nor with a multiple of it. These terms constitute nearly ninety-nine hundredths of the terms in the lunar theory, and unless they are affected by errors of large magnitude, their imperfections are without influence on the accurate determination of the parallactic inequality. This is in consequence of their mutually destroying each other when a number of observations are united. In determining, therefore, the value of the parallactic inequality by comparison with any modern tables, it may be assumed that the tabular values of these inequalities are correct. In fact, in the case of Hansen's tables it is probable that the error in these coefficients is rarely sensible.

IV. Imperfections in the theory of the terms of short period which have periods nearly coincident with that of the parallactic inequality, or of a multiple of it. These terms require careful consideration.

Suppose that the theory of the motion of the Moon in longitude which forms the basis of the tables is imperfect owing to its not containing a small variable term with a period differing from that of the parallactic inequality by the small quantity b, this quantity b being an angle increasing by a few degrees per annum. The tabular longitude of the Moon would thus be systematically incomplete by this term, which may be denoted by

$$B \sin (\theta + b).$$

As δP denotes the correction required by the tabular coefficient of the parallactic inequality, when the tables are compared with the observations, they will be found to be discordant by the quantity

$$\delta P \sin \theta + B \sin (\theta + b) = \{\delta P + B \cos b\} \sin \theta + B \sin b \cos \theta, = E.$$

Suppose at the epoch of any observation the value of $\sin \theta$ be $= a$, then the value of the coefficient of $\sin \theta$, or of the supposed correction to the parallactic inequality, will be given by the equation

$$\delta[P] \times a = E.$$

In reality, what would be equated to E would be

$$\{\delta P + B \cos b\} \times a = E.$$

When the sum of a number of observations is taken, the term depending on $\cos\theta$ will be entirely destroyed by the cases where $\cos\theta$ is negative destroying those where it is positive, so that it may be neglected. But the quantity δ [P] which would be obtained as the correction to the parallactic inequality from a year's observations would not be the true correction to this term, but would be the sum of the true correction and the mean value for the year of the quantity B cos b. Suppose the existence of this term B sin $(\theta+b)$ to be unsuspected. Then each year would yield a different value to the correction to the parallactic inequality owing to the variation in the value of the factor B cos b as the argument b increases from year to year; and, on comparing these yearly values of the correction to the parallactic inequality, they would show a periodical variation with a period equal to that of b. Therefore, if b had a moderately long period, so that the observations from which the correction to the coefficient of the parallactic inequality was to be deduced extended only over a fractional portion of this period, the resulting value would be erroneous by the mean value of the factor varying with b. If b had merely a comparatively short period, so that the observations extended over several periods of b, then its effects might be considered as eliminated from the mean of all, as the positive values at one part of the period would be destroyed by the negative values at the other part. But even then the existence of this term would give rise to discordances in the results for separate years, which would have the effect of throwing doubt on the accuracy of the resulting mean value. It is essential, therefore, that the lunar tables should contain no such imperfection, or that means should be taken to discover the existence of any such error and to determine its effect on the results.

When the lunar theory is examined in order to see if there be any terms of this nature, it will be found that the perturbations due to the direct action of the Sun contain only three sensible inequalities which have periods nearly coincident with that of the parallactic inequality. Any error in the value assigned to the coefficients of these terms will produce, obviously, the same effect as the existence of a new term with the same argument and with a coefficient equal to the quantity which must be added to the tabular value in order to render it accurate. These three terms are those with the argument

$$\alpha-\mu, \qquad 2\beta-\alpha-\mu, \qquad 2\theta-\alpha+\mu.$$

The periods of the first and third of these differ from that of the parallactic inequality by the difference between the mean motion of the perigees of the Moon and Sun, a quantity which increases by over 40°·6 per annum. This quantity is so large that the observations of a few years would neutralise the effect of any small errors in these coefficients. As the theoretical value of these terms is easily computed, it is highly improbable that there is any sensible error in the modern values employed in

the lunar tables. It may be safely assumed, therefore, that the accuracy of the value of the parallactic inequality will not be affected by errors in the coefficients of these two terms. The second term has a period differing from that of the parallactic inequality by less than 1° per annum. It would affect the value found for the parallactic inequality by a most dangerous systematic error. It would be an error very difficult to detect, and it would have the effect of making the coefficient of the parallactic inequality appear too large for nearly a century, and too small for the next century. Fortunately, it is a term which has so small a coefficient—less than one-fifth of a second of arc—that it can be regarded as nearly insensible, whilst it is not difficult to determine its true value with great accuracy; so that it is improbable that its theoretical value is inaccurate by as much as one twenty-fifth of a second of arc. It may therefore be safely neglected.

The only important terms of the kind having arguments close accordant in period with that of some multiple of that of the parallactic inequality are those with the arguments

$$2\theta \qquad 3\theta \qquad 4\theta \qquad 5\theta \qquad 6\theta.$$

All these are exact multiples of the argument θ of the parallactic inequality. It may be stated that the terms with arguments differing by a small quantity from that of a multiple of the argument θ are far less important than those differing from θ itself by a small quantity. For they introduce a sensible error during a part only of the period, and that error more quickly vanishes. But a still more important reason is the fact that they are nearly all extremely small; and, as their values are very easily calculated, it is most improbable that the modern theoretical values are erroneous to any important amount.

Of the terms whose arguments are given above, those with the arguments 4θ and 6θ may be neglected, as not only is it unlikely that they involve any sensible error, as they are small, but the effect of any error is almost completely eliminated when a number of observations are combined.

The terms depending on the arguments 3θ and 5θ might exert very considerable effect on the value found for the parallactic inequality if their coefficient were sensibly erroneous. The term depending on 5θ may be, however, disregarded at once, for its theoretical value is unquestionably less than two hundredths of a second of arc, and in all the tables it is regarded as insensible. The term with the argument 3θ is more important, for it has a most systematic effect on the value of the parallactic inequality. Suppose the parallactic inequality is to be determined from observations made within two days on either side of the first or third Quarters of the Moon's age. When the Moon is within two days of the first Quarter, the value of θ, the argument of the parallactic inequality, will range between 65°

and 115°, and $\sin\theta$ will range between the values +0·90 and +1·00, its mean value being about +0·95. During the same period the value of 3θ must be between 195° and 345°, so that $\sin 3\theta$ will range between −0·26 and −1·00, and its mean value may be taken at −0·71. Consequently, if δP and δQ denote the errors in the coefficients of these terms, they will produce an average error in the tabular place of the Moon at about the first Quarter amounting to

$$0{\cdot}95\,\delta P - 0{\cdot}71\,\delta Q.$$

Similarly, when the Moon is within a like period on either side of the third Quarter, the values of θ will range between 245° and 295°, and the mean value of $\sin\theta$ will be about −0·95, whilst the values of 3θ will range between 15° and 165°, and the mean value of $\sin\theta$ be about +0·71. Therefore, the error produced will average at the third Quarter

$$-0{\cdot}95\,\delta P + 0{\cdot}71\,\delta Q.$$

The difference D between these two values will be, therefore,

$$D = +1{\cdot}90\,\delta P - 1{\cdot}42\,\delta Q;$$

whence

$$\delta P = +0{\cdot}53\,D + 0{\cdot}75\,\delta Q.$$

If $\delta(P)$, therefore, be the value of the correction found by neglecting the effect of any error in the coefficient of the term with the argument 3θ, the true value of the correction will be

$$\delta P = \delta(P) + \frac{3}{4}\delta Q.$$

When the parallactic inequality is determined from the entire mass of observations, but double weight is given to all observations in which the value of $\sin\theta$ is large, it can be shown in exactly the same manner that a greater part of this error is eliminated and that the true correction is

$$\delta P = \delta(P) + \frac{1}{3}\delta Q.$$

As it is possible to determine the value of the coefficient Q of this term with great accuracy, the error in the tabular value of the coefficient can be easily discovered and the proper correction applied.

The only other term requiring consideration is the *Variation*, the argument of which is denoted by 2θ. If the observations from which is to be determined the value of the parallactic inequality were equally distributed on either side of the first and third Quarters, the effect of any error in the tabular value of this term would be zero; but unless a great number of observa-

tions are rejected this will not be the case, as far more observations are made after the first Quarter and before the third Quarter than before the first and after the third Quarter, so that in the sum of the observations made before and after each Quarter the effect of an error in the variation will not be completely eliminated. This outstanding error will be negative at the first Quarter and positive at the third Quarter, so that when the difference between the tabular errors at these times is taken, these two portions will coalesce, and a portion of the error in the coefficient of the variation be thrown on the correction to the coefficient of the parallactic inequality. The method of approximately determining the amount of the error thus introduced can be determined exactly as in the case of the term with the argument 3θ. If δV be the error in the tabular value of the coefficient of the variation, the correct value of δP will be given by one of the following two expressions.

If the correction $\delta(P)$ be deduced from the difference between the sum of the tabular errors within two days on either side of the Quarters,

$$\delta P = \delta(P) + \frac{1}{9}\delta V.$$

If the correction $\delta(P)$ be deduced from all the observations, weighted as stated before,

$$\delta P = \delta(P) + \frac{2}{3}\delta V.$$

It may here be stated that these values are perfectly in accordance with the actual coefficients found from a discussion of a large number of observations, as will be shown subsequently. For this reason, unless only observations are employed which are equally distributed on both sides of the Quarters it will be essential to correct the resulting value of the correction to the parallactic inequality for any error in the coefficient of the variation.

V. Imperfections in the tabular constants. For the present purpose the only constant in which an error would be important is the semi-diameter. If the tabular semi-diameter be too great by the quantity δS, it will have the effect of increasing the longitude before Full Moon, and decreasing the longitude after Full Moon. As this is exactly contrary to the effect of the parallactic inequality, it will tend to make the observed value of the parallactic inequality too small. For this reason the observed value (P) should receive a correction and the true value will be

$$P = (P) - \delta S.$$

The tabular value of the semi-diameter must represent the apparent value of the semi-diameter of the Full Moon, and should afford the exact interval between the transit of the two limbs when the Moon is Full. The variation in the semi-diameter due

to the varying contrast in the brightness of the Moon's limb and the sky is the most important factor in the accurate determination of the value of the coefficient of the parallactic inequality, and has considerable influence on the apparent values of other of the inequalities in the motion of the Moon. It is a correction, however, rather to be applied to the observations than to the tabular place of the Moon.

The practical difficulties which occur in the treatment of the investigation will be examined as they occur.

The second step to be taken is to examine the methods which have been used by previous investigators in order to see how far they have taken into account and eliminated any errors of the kind which have been discussed in the preceding.

In his investigations Sir G. Airy has employed Tables of the Moon partially constructed by himself and founded on Plana's development of the theory of the lunar perturbations. The tables are affected, therefore, by nearly all the errors contained in Plana's lunar theory. On comparing the theory of the Moon embodied in these tables used by Sir G. Airy with the more correct theory as developed in the investigations of Hansen and Delaunay, it will be found that the tables are affected by no less than three errors of nearly 4″, by two errors of nearly $2\frac{1}{2}''$, and by two errors of $1\frac{1}{2}''$ (see Delaunay, *App. Conn. des Temps*, 1869), besides great numbers of smaller errors. In consequence, when these tables are compared with observations, they must exhibit large and rapidly varying discordances. Fortunately, none of these larger errors are in the coefficients of the terms which can systematically affect the value of the parallactic inequality.

In their several investigations Professors Hansen and Newcomb have both taken Hansen's *Tables de la Lune* as the basis of their work. These tables have shown a peculiar error in the value assigned to the mean longitude of the Moon, but in other respects accord far closer with observation than any other tables have done. The values found by Hansen for the coefficients of the terms of short period are with rare exceptions in close accord with the values found by Delaunay and by Pontecoulant when the theory of the latter has been extended far enough. In fact, there is reason to believe that these values represent with great accuracy the true perturbations of the Moon produced by the direct action of the Sun. A few of the terms may require very small corrections, and so may some of the coefficients which, like those of the parallactic inequality and equation of the centre, have been deduced from observation and not from pure theory. There may be also more than one term omitted from the tables.

Mr. Stone in his investigation has also nominally employed Hansen's tables as the basis of his labours, but for the greater part of the observations what has been employed is really a kind of Burckhardt-Hansen table. As the details of Mr. Stone's

investigation have never been published, there must remain some uncertainty as to the exact nature of the means employed. From 1862–1866 the Greenwich observations have been compared with Hansen's tables, and the observations employed by Mr. Stone are quite satisfactory. But it does not appear that the Greenwich Observations for the years 1859, 1860, and 1861, have ever been compared with Hansen's tables, and if any of these observations were used by Mr. Stone, it must have been as compared with Burckhardt's tables. For the period 1848–1858 Mr. Stone has probably used the comparisons between Hansen's tables and observations, given by the Astronomer Royal as an Appendix to the Greenwich Observations for 1859. But though the Greenwich Observations for these years are nominally compared with Hansen's tables, this is not strictly the case. The differences between observation and theory really consist of the difference between Burckhardt's tables and observation added to the difference between Burckhardt's tables and Hansen's tables *at midnight*. If, then, between midnight and the epoch of observation the difference between the tabular longitudes of Burckhardt and Hansen would have changed, the supposed comparison between Hansen's tables and observation would be as much in error. If, therefore, there are errors in Burckhardt's tables which change at the rate of 0''·08 per hour, as there well may be, then at the Quarters when the Moon is observed six hours away from midnight these errors would produce discordances amounting to nearly half a second of arc. As these errors would be systematic, it cannot be assumed that they are unimportant, and in deducing the value of the parallactic inequality it must not be assumed without careful examination that this indirect comparison may be regarded as equivalent to a direct comparison between Hansen's tables and observation.

It remains now to compare the differences between these tables in the case of the six terms which have been distinguished from the rest by their systematic effect on the parallactic inequality. These six terms are the following:—

Argument	Values of the Coefficients according to Burckhardt	Airy	Hansen	Delaunay
μ	$-\ 673''\cdot 3$	$-\ 669''\cdot 0$*	$-\ 669''\cdot 83$	$-\ 668''\cdot 91$
$a-\mu$	$+\ 147\cdot 5$	$+\ 148\cdot 1$	$+\ 148\cdot 02$	$+\ 148\cdot 24$
$2\beta-a-\mu$	$+\ 0\cdot 2$	$+\ 0\cdot 0$	$+\ 0\cdot 08$	$+\ 0\cdot 12$
2θ	$+2373\cdot 4$	$+2370\cdot 7$*	$+2369\cdot 86$	$+2369\cdot 74$
$2\theta-a+\mu$	$-\ 27\cdot 6$	$-\ 28\cdot 8$	$-\ 28\cdot 53$	$-\ 28\cdot 28$
3θ	$+\ 2\cdot 7$	$+\ 0\cdot 9$	$+\ 0\cdot 41$	$+\ 0\cdot 53$

The values marked with an asterisk under the heading Airy are not the tabular values, but these values *plus* the corrections deduced by Sir G. Airy from his discussion of the observations,

and are, therefore, tantamount to the values actually employed by him in determining the value of the parallactic inequality.

In comparing the values used by Burckhardt with those employed by Hansen, it must be remembered that, though there may be considerable discordances between the values employed in the two tables, we have to deal only with such discordances as will in the space of a few hours near the time of the Moon's Quarters cause sensible systematic differences in the tabular places of the Moon—that is to say, differences which will vitiate the assumption made by Sir G. Airy that there will be no sensible difference between the tables of Burckhardt and Hansen at midnight and an epoch six hours before or after midnight.

The values assigned to the coefficient of the three arguments $(\alpha-\mu)$, $(2\beta-\alpha-\mu)$ and $(2\theta-\alpha+\mu)$ are so nearly alike that the effects of the differences may be considered insensible. In the case of the coefficient of the term with the argument μ (the annual equation) there is a difference of nearly $1''$ between the values employed by Airy and Hansen. The difference may exert a sensible influence on the values found for the parallactic inequality. Airy's value is really that of the apparent value of the annual equation as affected by any systematic error due to variation in semi-diameter, &c., and, being derived from the observations, was the proper value to be employed by him. The difference between Burckhardt and Hansen will not give rise to sensible variations in the space of a few hours, so that it is immaterial.

The value assigned by Airy to the coefficient of 2θ differs by nearly $1''$ from the values found by Hansen and Delaunay; but here again Airy's value is that of the apparent variation, and Hansen's and Delaunay's that of the true variation. This difference must again give rise to systematic differences in the values found for the coefficient of the parallactic inequality. Airy's value being that given by the observations is the correct one to have been used by him. If the apparent variation has really a coefficient $0''.9$ greater than the theoretical variation, then both Stone and Newcomb's values for the parallactic inequality require the correction

$$\frac{1}{9}\delta V = \frac{1}{9} \times \{-0''.9\} = -0'.10.$$

But in this case, as in that of the annual equation, it is the apparent coefficients which must be used in deducing the true values of the other coefficients—that is to say, the values as affected by changes in the apparent semi-diameter, &c., and as these variations will usually differ with different instruments, it is essential to determine the correction for the effect of any error in the variation and annual equation from the observations themselves. If there be introduced into the equations from which the value of the inequality is to be determined a correction for the effect of change in semi-diameter, then, of course, the true values of

the inequalities must be employed, and the results obtained will be the more accurate of the two.

Burckhardt's value of the variation differs by over $3''\cdot5$ from the value employed in Hansen's tables, and as this difference will change quickly, it is of importance. Its effects may be ascertained as follows:—

When the Moon is in its first Quarter it will transit about six hours before midnight, and as the argument of the variation increases at a rate of rather more than 1° per hour—so that on the average the argument of the variation will be 6° less at transit than at midnight—therefore, if δV denote the excess of Burckhardt's value over that of Hansen, and as at first Quarter the argument of the variation has a mean value of 180°, the difference between the two tables at midnight will be greater than at six hours earlier by the quantity

$$\delta V \sin (180° + 6°) = +0{\cdot}105\,\delta V.$$

That is to say, that about the time of the first Quarter of the Moon's age, if B denotes Burckhardt's value of the longitude of the Moon at midnight, and H denotes Hansen's value at the same epoch, at the average time of the transit of the Moon, about six hours before midnight, the difference between the two tables would be

$$(H-B)-0{\cdot}105\,\delta V.$$

On the other hand, when the Moon is about the time of third Quarter, so that it would transit on the average about six hours after midnight, the argument of the variation at the time of transit would be on the average 6° greater than at midnight, and as the mean value of the argument of the variation at third Quarter is 180°, there would be at transit a similar difference of

$$\delta V \sin (180° + 6°) = -0{\cdot}105\,\delta V;$$

whence if $(H'-B')$ were to denote the difference between the two tables at midnight, at the time of transit, six hours later, the difference would be

$$(H'-B')+0{\cdot}105\,\delta V.$$

Now, the supposed difference between Hansen's tables and observation was found by adding the difference $(H-B)$ to the difference $(B-O)$, where O denotes the observed longitude, whereas the values given above ought to have been used. Consequently, instead of the residuals being really $(H-O)$ for the time of observation, they are really at the time of observation

$$(H-O)\ +0{\cdot}105\,\delta V \quad \text{at first Quarter, and}$$

$$(H'-O')-0{\cdot}105\,\delta V \quad \text{at third Quarter.}$$

If, then, D be the difference between the errors of Hansen's

tables at the first and third Quarters, instead of obtaining this quantity by subtracting the two expressions given above, what will be obtained will be

$$D + 0{\cdot}21\,\delta V.$$

If, therefore, this result be equated to the assumed correction required by Hansen's value of the coefficient of the parallactic inequality, instead of obtaining the true equation

$$1{\cdot}90\,\delta P = D,$$

we shall obtain the expression

$$1{\cdot}90\,\delta P = D + 0{\cdot}21\,\delta V.$$

The resulting value of δP will be, therefore,

$$\delta P = \frac{10}{19}D + \frac{1}{9}\delta V,$$

or will be too large by one ninth of the correction required by the coefficient of the variation. If, therefore, $\delta[P]$ be the value found for the correction to the coefficient of the parallactic inequality where the term in δV is neglected, the true value will be

$$\delta P = \delta[P] - \frac{1}{9}\delta V.$$

Now, Burckhardt's value of the variation is greater than Hansen's by

$$\delta V = 3''{\cdot}85,$$

this being the sum of the correction to the variation and to some analogous terms which will act on the whole like an apparent increase to the value of the variation. Consequently, the value found from these observations for the correction to the parallactic inequality must be decreased to the amount

$$\delta P = \delta[P] - \frac{3''{\cdot}85}{9} = \delta[P] - 0''{\cdot}43.$$

The importance of this result is evident, for it shows that the results deduced for the correction to the parallactic inequality from the comparison of Hansen's tables with the Greenwich Observations for the years 1847–1858 are all affected by a systematic error which will render the apparent correction erroneous by nearly half a second of arc. But these observations form three fifths of the data employed by Mr. Stone, so that his result will be affected by a proportionately large error.

The only remaining term to be considered is that with the argument 3θ. The important effects of an error in this term

have been already shown, and its amount approximately determined. From a comparison of Hansen's and Delaunay's values for Q, the coefficient of this term, with that made use of by Airy, it appears that Airy has adopted too large a value. Delaunay's value is $+0''\cdot53$ but an examination of the algebraical form of his coefficient shows that he has not pushed his approximation far enough, and, as every additional term tends to reduce its value, it is probable that his value is too great by $0''\cdot07$ at least. This would reduce it to $+0''\cdot46$. Neison's value of the coefficient is $+0''\cdot44$, but this extends an order further than Delaunay's. Hansen's value is $+0''\cdot41$, and is probably very nearly correct. Comparing these values with Airy's, it is evident that Airy's coefficient is too large by

$$\delta Q = +0''\cdot49.$$

Consequently, the value found by the Astronomer Royal for the value of the parallactic inequality ought to be increased by

$$\frac{1}{3}\delta Q = +0''\cdot16.$$

On comparing the theoretical value of this coefficient with that employed by Burckhardt, which was $2''\cdot7$, it appears that Burckhardt's value is too great by

$$\delta Q = +2''\cdot29.$$

Consequently, if Mr. Stone had determined the parallactic inequality from the observations made at Greenwich in the years 1859, 1860, 1861, as these are compared with Burckhardt's tables, the resulting value of the parallactic inequality would require to be increased by

$$\frac{3}{4}\delta Q = +1'\cdot71.$$

When the term with the argument 3θ is examined, it will be found to exert a perfectly similar effect on the comparison between Hansen's and Burckhardt's tables, to that already shown to occur with the variation. There is this distinction, however, that whereas in the case of the variation the effect before and after the Quarter had the same sign, in the case of this term with the argument 3θ they will differ in sign and so tend to neutralise each other. If, therefore, the number of observations before the day of first Quarter or after that of third Quarter were nearly equal to those made after the first Quarter and before the third Quarter, the errors from this source would counterbalance one another. This, however, is not the case, as the latter class of observations far exceed in number the former, consequently some considerable portion of the correction will remain outstanding. By the same method which was employed in the case of the variation, the effect may be shown to be represented by the expression

$$\delta P = \delta[P] - \frac{1}{25}\delta Q.$$

Substituting the value found for δQ,

$$\delta P = \delta[P] - 0''\cdot 09.$$

The result, therefore, of this examination may be summed up as follows:—

Airy's value requires a correction of

$$+0''\cdot 16.$$

The portion of Stone's results which are based on the comparison between Hansen's tables and the Greenwich Observations for the years 1848 to 1858 require a correction of

$$-0''\cdot 52.$$

The portion, if any, of Stone's results founded on the Greenwich Observations of 1859, 1860 and 1861, requires the correction

$$+1''\cdot 71.$$

Both Stone's and Newcomb's results will require correction, if there be any error in the tabular value of the coefficient of the variation employed in Hansen's tables.

The results obtained by Sir G. Airy (*Memoirs Roy. Ast. Soc.* vol. xxix. 1–24) are founded on an elaborate discussion of the entire series of Greenwich Observations made between 1750 and 1851. The observations are divided into nine-year groups, and from each of these groups there are simultaneously deduced corrections to the epoch, eccentricity, motion of the perigee, and coefficients of the evection, annual equation, variation, and parallactic inequality. Therefore, the influence of errors in the tabular values of all these terms is eliminated from the resulting value of the parallactic inequality. The early groups can be omitted, as they must be inferior in accuracy to the more modern observations. The remaining nine groups yield the following results.

Group.	Mean Year.	Correction to the Assumed Coefficient.	Resulting Coefficient of the Parallactic Inequality.	Difference from Mean.
1806–1815	1810·5	$-0'\cdot 88$	$-122'\cdot 98$	$+1'\cdot 24$
1811–1819	1815·0	−1·99	−124·09	+0·13
1816–1824	1820·0	−2·40	−124·50	−0·28
1820–1829	1824·5	−3·72	−125·82	−1·60
1825–1833	1829·0	−3·10	−125·20	−0·98
1830–1838	1834·0	−1·91	−124·01	+0·21
1834–1842	1838·0	−1·47	−123·57	+0·65
1839–1847	1843·0	−1·14	−123·24	+0·98
1843–1851	1847·0	−2·43	−124·53	−0·31

It will be seen from the last column that there is a considerable discordance between the separate results for each group of years, but, treating these discordances as due to accidental or unsystematic errors, the value found for the coefficient of the parallactic inequality is

$$(P) = -124''.22 \pm 0''.20.$$

The mean of the differences of the separate results from the mean of all the results is $0''.71$, a very large quantity. The Astronomer Royal, by omitting the first of these groups, has found the value

$$(P) = -124''.37.$$

The coefficient whose value has been determined by Sir G. Airy is not the real coefficient of the parallactic inequality, and which can be alone used for determining the distance of the Sun, but it is the apparent coefficient of the parallactic inequality, or the sum of the true coefficient *plus* the factor which expresses the influence of the variation in the semi-diameter. No attempt was made by the Astronomer Royal to determine the effect of this variation in semi-diameter, so that, unless this be subsequently done by a further discussion of the observations, it will be impracticable to determine the real coefficient of the parallactic inequality from these investigations, or to use them for ascertaining the solar parallax.

The periodical character of the differences between the mean value and the separate results is obvious, the values obtained for the parallactic inequality at different times seeming to be affected by a periodical inequality running through all its changes in about 30 years and reaching its maximum value between the years 1825 and 1826. It has already been shown that this would indicate the existence of some imperfection in the lunar tables due to the omission of a term with a period differing slightly from that of the parallactic inequality. Suppose, therefore, it be assumed that

$$-X = \delta(P) + B\cos b,$$

where $\delta(P)$ denotes the true correction to the apparent coefficient of the parallactic inequality, and $B\cos b$ denotes a periodical inequality in X, the values obtained from any group of observations for the value of the apparent coefficient of the parallactic inequality. As a first approximation it may be assumed that

$$b = 12^\circ \times (Y - 1825.5).$$

Then the nine values of X obtained by Sir G. Airy from the discussion of the Greenwich Observations will give nine equations from which to determine the values of $\delta(P)$ and B. Solving these equations by the method of least squares, they give the values

$$\delta(P) = +2''.37, \qquad B = +1'.18.$$

The comparison of the correction calculated from these values with the correction actually obtained from the observations is as follows:—

Epoch	Observed Correction.	Difference from Mean.	Calculated Correction.	Difference from Observed.
1810·5	$-0''{\cdot}88$	$+1''{\cdot}24$	$-1''{\cdot}18$	$+0''{\cdot}30$
1815·0	$-1{\cdot}99$	$+0{\cdot}13$	$-1{\cdot}67$	$+0{\cdot}32$
1820·0	$-2{\cdot}40$	$-0{\cdot}28$	$-2{\cdot}85$	$-0{\cdot}45$
1824·5	$-3{\cdot}72$	$-1{\cdot}60$	$-3{\cdot}49$	$-0{\cdot}23$
1829·0	$-3{\cdot}10$	$-0{\cdot}98$	$-3{\cdot}25$	$+0{\cdot}15$
1834·0	$-1{\cdot}91$	$+0{\cdot}21$	$-2{\cdot}11$	$+0{\cdot}20$
1838·0	$-1{\cdot}47$	$+0{\cdot}65$	$-1{\cdot}34$	$-0{\cdot}13$
1843·0	$-1{\cdot}14$	$+0{\cdot}98$	$-1{\cdot}34$	$+0{\cdot}20$
1847·0	$-2{\cdot}43$	$-0{\cdot}31$	$-2{\cdot}11$	$-0{\cdot}32$

The smallness of the residuals shows how closely the hypothesis of a missing term accounts for the variation in the values found by the Astronomer Royal for the apparent coefficient of the parallactic inequality. If the hypothesis which has been made is correct, then the term missing from the tables would have the form

$$+1''{\cdot}18 \sin \{\theta + 12^\circ \times [Y - 1825{\cdot}5]\},$$

or it may be

$$+1{\cdot}18 \sin \{(a-\mu) + 52^\circ{\cdot}7 \times [Y - 1819{\cdot}5]\}.$$

The weakest point in the form assigned to the preceding inequality is its period. It is obvious that a term of longer period would satisfy all the observations except the group for 1843—1851, or rather that portion of the group included between the years 1847—1851. When this group is reached the inequality seems to suddenly change sign, thus limiting its period to thirty years. It is thus solely this group which suggests the adoption of a thirty-year inequality.

When the results for the earlier groups of observations, those made prior to 1800, are compared with the values found by the preceding hypothesis, the comparison is far from being so satisfactory and reveals much larger discordances. It is true that these earlier observations are much less precise than the more modern ones, so that they might fairly be expected to show larger discordances. These earlier values were obviously affected by an analogous inequality to the more modern, but the variations indicated an inequality of longer period than thirty years. These earlier groups gave corrections to the assumed coefficient of the parallactic inequality which rendered it more than a second smaller than the value indicated by the more modern groups. This constant difference, however, is no more than might be expected, for the instruments employed in the two periods

were different, and the earlier and smaller instrument would certainly give a smaller value for the *apparent* coefficient of the parallactic inequality than would be the case with the finer modern instrument, as in the earlier instrument there would be a greater variation in the irradiation of the lunar limb, and consequently a greater decrease in the apparent value of the coefficient of the inequality.

It being only the final group for the period 1847–1851 which prevented a longer period being assigned to the inequality indicated by the more modern groups of observations, the group depending on these observations was omitted, it being the final one of the series, and the remaining twenty values of the correction to the parallactic inequality were considered. They gave the results for the ninety-seven years 1750–1846. All the earlier observations were reduced by Sir G. Airy with the same value for the mean semi-diameter of the Moon. It was probable, however, that more accurate results would be obtained by using for each group of observations the value of the apparent semi-diameter given by the observations made during the period covered by the group. Moreover, by adopting this system the greater part of the effects of any personality would be eliminated. The data requisite to effect this are given in the Introduction to the *Reduction of the Greenwich Lunar Observations* (vol. i. pages lxiv.–lxv.). From these data corrections were obtained to the value of the semi-diameter employed by Sir G. Airy in reducing the observations contained in each group between 1750 and 1816. From these corrections it appeared that the observations had been reduced with too large a semi-diameter from 1800 to 1775, and with too small a value from 1775 to 1750. The resulting corrections to the value found for the apparent coefficient of the parallactic inequality varied between $+ 0''{\cdot}70$ for the year 1755 to $- 1''{\cdot}20$ for the year 1780. The application of these corrections rendered still more apparent the regular periodical variation in the values found for the correction to the coefficient of the parallactic inequality.

As before, let $\delta(P)$ denote the correction to the parallactic inequality, B denote the coefficient of the forty-five-year inequality, and A denote the constant correction required by the earlier observations; then the twenty values for the apparent correction to the coefficient of the parallactic inequality furnish twenty equations from which to determine $\delta(P)$, B, and A. Solving these by the method of least squares, the following values are obtained:—

$$\delta(P) = +1''{\cdot}85, \qquad B = +1''{\cdot}20, \qquad A = +1''{\cdot}60.$$

From these values was calculated the theoretical apparent coefficient of the parallactic inequality for each of these groups of observations, and it was compared with the observed value.

Epoch.	Observed Coefficient.	Calculated Coefficient.	Difference.	Epoch.	Observed Coefficient.	Calculated Coefficient.	Difference.
	$''$	$''$	$''$		$''$	$''$	
1843·0	−123·24	−123·00	−·24	1796·5	−121·34	−121·56	+·22
1838·0	−123·57	−123·67	+·10	1792·0	−121·95	−122·15	+·20
1834·0	−124·01	−124·29	+·28	1787·0	−122·26	−122·92	+·66
1829·5	−125·20	−124·95	−·25	1783·0	−123·56	−123·41	−·15
1824·0	−125·82	−125·50	−·32	1778·0	−123·93	−123·42	−·51
1820·0	−124·50	−124·85	+·35	1773·5	−122·54	−122·99	+·45
1815·0	−124·09	−124·15	+·06	1769·0	−122·10	−122·32	+·22
1810·5	−122·63	−122·60	−·03	1764·0	−122·03	−121·56	−·47
1806·0	−121·46	−121·25	−·21	1759·5	−121·00	−121·14	+·14
1801·0	−120·82	−121·13	+·31	1754·5	−120·73	−121·20	+·47

The small residuals between the calculated and observed places, although the observations extend over more than a double revolution of the inequality, show how strongly the existence of this inequality is indicated by the observations. Small as the residuals are, they could be still further reduced by slightly altering the period and changing by a small interval the time when the inequality is supposed to reach its maximum value.

The computed value of the correction for the group 1843–1851 is (P) = $-122''·75$, whilst that actually found from the observations is (P) = $-124''·53$, there being a difference of $1''·78$. Almost the entire portion of this difference arises from the high values found for the parallactic inequality from the observations made during the years 1846–1851, as those made for the earlier years were much lower. It would, in fact, seem as if the apparent semi-diameter of the Moon had increased by $2''·0$ during these years, either from a change of observers or some other cause. This would have increased the apparent value of the parallactic inequality by $2''·6$, and as this increase affects two thirds of the observations, it would correspond to a fictitious increase of $-1''·73$, whilst that actually observed was $-1'·78$. The further consideration of this point must be deferred.

Sir G. Airy obtained the value for the apparent coefficient of the parallactic inequality of

$$(P) = -124''·37.$$

Neglecting the effects of the supposed inequality, but increasing the earlier observations by the constant factor A = $1''·60$ to allow for the difference in instruments, the value becomes

$$(P) = -123·78.$$

Upon the hypothesis of a thirty-year inequality, the value is

$$(P) = -124·47.$$

Upon the more probable hypothesis of a forty-five-year inequality, the value of (P) becomes

$$(P) = -123''.95.$$

This last value seems to be entitled to far greater weight than any of the others, and is probably close to the truth. It involves the assumption of the real existence of the forty-five-year inequality, a question whose further consideration must be deferred for the present.

Since the above results were completely finished, a reference to the earlier memoir by Sir G. Airy (*Memoirs Roy. Ast. Soc.*, vol. xvii. page 42) shows that the periodical nature of the variations in the values found for this inequality did not escape his attention. The Astronomer Royal remarks:—"We might almost imagine, from the succession of values of V, that it is subject to a periodical factor whose period is about 46 years. Such a factor would be represented by adding to the Moon's longitude two inequalities whose daily increases of argument are $13^{s}.5686$ and $13^{s}.5220$. I cannot conceive that there is any physical foundation for this supposition." Is the existence of such inequalities inconceivable?

From a discussion of the four years' observations made with the Altazimuth between 1847 and 1851, the Astronomer Royal derived the value

$$(P) = -125''.50.$$

This excessive value he seemed disposed to regard as accidental. This, however, is by no means the case. The Astronomer Royal has derived the supposed differences between his own tables and observation, through the medium of Burckhardt's tables, in exactly the same manner as he derived the supposed differences between Hansen's tables and observation for the years 1847–1858. It has been already shown that this will introduce grave systematic errors. In this case these errors will require the value obtained by Sir G. Airy to be reduced by $-0''.46$, as a greater portion of the error in the term 3θ will be eliminated. Further, in reducing the Altazimuth observations Sir G. Airy has employed the same semi-diameter as in reducing the Meridian observations, whereas the former seems to require a semi-diameter at least $0''.6$ greater, and for this reason the Astronomer Royal's value will be about $0''.75$ too large. Applying these corrections, its value becomes

$$(P) = -124''.29.$$

Much weight cannot, however, be given to this result, from the uncertainty as to the exact difference in semi-diameter between the Altazimuth and old Transit Instrument (the difference between the Altazimuth and Transit Circle is known to be fully

$1''\cdot0$), and partly from the small period over which the observations extend.

All the values in the preceding statement require the correction $-0''\cdot16$ for the errors in Airy's tables. The value found by the Astronomer Royal from the modern observations should be

$$(P) = -124''\cdot53,$$

and the value found from all the observations by the assumption of a forty-five-year inequality becomes

$$(P) = -124''\cdot11.$$

All these values are those of the apparent coefficient of the parallactic inequality, and are therefore in their present state useless for the purpose of yielding the solar parallax. Nor are they strictly comparable in this form with the results obtained from other instruments, for, being affected by the error due to the variation in the irradiation at the limb, they are affected by an error which varies with the aperture and excellence of the telescope.

Prof. Hansen does not seem to have published any details of the method by which he determined his value of the parallactic inequality, so that there is no means of ascertaining in what manner he eliminated the effect of the variation in the semidiameter of the Moon. Practically the only existing information seems to be contained in the Note in the *Monthly Notices* for November 1854 (vol. xv. page 9). He states there that from a comparison of his own theory with some of the Greenwich observations, and some of the Dorpat observations of the Moon, he deduced as the value of the coefficient of the parallactic inequality

$$P = -126\cdot46.$$

It appears to be probable that the observations employed by Prof. Hansen were made between the years 1820 and 1835, for it is certain that he did compare many of the Greenwich observations made at this period with his tables. It is noteworthy that at this period the Greenwich observations give the largest value for the apparent coefficient of the parallactic inequality, a value more than $1''$ greater than what is probably the true value, and Hansen's value is in turn apparently $1''$ too large. Unless, however, the papers of Prof. Hansen which are now at Leipzic are found to afford some further information on the subject, nothing can be done with this result obtained by Prof. Hansen.

The only published account of Mr. Stone's investigation is to be found in a short abstract of his memoir published in the *Monthly Notices* for May 1867 (vol. xxvii. page 271). The memoir

itself seems never to have been published, and is not now to be found. In consequence of this misfortune, the details of the method which was followed have to be surmised from the very short abstract of the Introduction, which was all that was ever published. The main principle of the method adopted is stated to have been as follows. The longitude deduced from the observations made near the maximum of the parallactic inequality was compared with the tabular longitude; and (it being assumed that the error in the mean results arises entirely from error in the coefficient of the parallactic inequality) the comparison furnishes a correction to the coefficient in question, and consequently also to the value of the solar parallax used in the calculation of that coefficient. It is further stated that 2,075 observations were used, all made at Greenwich between 1848 and 1866. The value obtained for the coefficient was

$$(P) = -125''\!\cdot\!36 \pm 0''\!\cdot\!40.$$

This has been put down as the apparent coefficient because, if the preceding description fully describes what was done, it is obvious that no steps were taken to eliminate the effect of the variation in the semi-diameter due to differences in contrast between the sky and the limb of the Moon. If this be so, then it is obvious that Prof. Newcomb in using it as the value of the real coefficient of the parallactic inequality has been misled. Prof. Newcomb lays great stress on the importance of distinguishing between the two, and his own investigations show that the apparent value of the inequality is less than the real value by over 1″. Consequently, had he perceived that the value obtained by Mr. Stone is really that of the apparent coefficient and not that of the real coefficient, he would have seen that, so far from confirming his own results, there was a discordance of fully 1″.

From the number of observations used, which is stated to be 2,075, or at the rate of at least 109 a year, it is obvious that both the Meridian and Altazimuth observations were used. Although not explicitly stated, it is probable that the actual figures used were those for the years 1848–1858 given as the comparison between the Greenwich Observations and Hansen's tables in the Appendix to the Greenwich Observations in the volume for 1859 and those for the years 1862–1866 given in the annual volumes for these years. All these may be said to be founded on Hansen's tables. It is uncertain whether the observations made at Greenwich during the years 1859, 1860, 1861 were used, as they are compared with Burckhardt's tables. No information exists as to the value employed for the semi-diameter, but it is probable that Hansen's tabular value was used unchanged. This is, in fact, implied in the sentence in brackets, that the mean difference between observation and theory was supposed to arise entirely from error in the coefficient of the parallactic inequality. For

the same reason it is probable that the same semi-diameter was used for both Meridian and Altazimuth observations.

The apparent semi-diameter of the Moon with the Altazimuth is fully 1″ greater than with the Transit Circle so that the parallactic inequality derived from the Altazimuth observations will be over 1″ greater than that derived from the Transit Circle. Of late years the observations with the Altazimuth have been reduced with a large value of semi-diameter, thus reducing the error; but probably the principal observations made use of by Mr. Stone would not be corrected for the difference in semi-diameter. On this account, as one half of the observations employed by Mr. Stone are affected by this correction, his result would require to be reduced by 0″·50.

It has already been shown that the results derived from the observations made during the years 1848–1858 ought to be decreased by −0″·52. As these results affect three-fifths of the observations employed by Mr. Stone, his result will require a correction of −0″·31. It will be, therefore,

$$(P) = -125''\cdot05.$$

If this be further reduced by the correction for the difference in semi-diameter required by the Altazimuth and Transit Circle observations, it will become

$$(P) = -124''\cdot55.$$

The investigation by Prof. Newcomb is contained in Appendix II. to the Washington Observations for 1865. In this memoir a sufficiently full account is given of the method by which the value of the parallactic inequality was deduced, so that its real character can be properly made out. It is founded on the Meridian observations of the Moon made at Washington during the four years 1862, 1863, 1864 and 1865. The number of observations which were used probably did not exceed one hundred.

According to Prof. Newcomb, the method employed by him to determine the value of (P), the apparent coefficient of the parallactic inequality, was as follows: "The Washington Observations of the Moon, from 1862 to 1865 inclusive, are regularly compared with Hansen's tables. I have discussed those made within two days of the time of maximum and minimum parallactic inequality, on the supposition that the effect of errors in the other inequalities will destroy each other in the course of the four years. Thus the following corrections to Hansen's parallactic inequality are obtained for the several years:—

$$1862 = +2''\cdot2 \qquad 1863 = +2''\cdot2 \qquad 1864 = +2''\cdot0 \qquad 1865 = +2''\cdot0.$$

These results are still subject to correction for adopted semi-

diameter of the Moon. Seven transits of both limbs of the nearly Full Moon were observed during the above period. The mean correction to Hansen's semi-diameter was zero. If, then, we suppose this same semi-diameter applicable to the Moon at first and third Quarters, the coefficient of the parallactic inequality will be"

$$(P) = -126''\cdot46 + 2''\cdot10 = -124\cdot36.$$

Prof. Newcomb next considers how to convert this, the value for the apparent coefficient, into the true value of the coefficient. He points out that as many of the observations at the Quarters are made in bright twilight, or even daylight, from the diminished irradiation the apparent semi-diameter of the Moon must be considerably smaller than at Full, and that consequently the above value of the parallactic inequality must be increased to make it correspond to the real value of the parallactic inequality.

After a careful discussion of the probable amount of difference between the semi-diameter at Full Moon and at its quarters when observed at daylight, Prof. Newcomb arrives at the conclusion that from this variation in the semi-diameter the real value of the parallactic inequality will exceed the apparent value by $1''\cdot10$. In this manner he deduces for the real value

$$P = -124'\cdot36 - 1''\cdot10 = -125'\cdot46.$$

As Prof. Newcomb points out, it is the uncertainty as to the real value of this augmentation due to the variation of the lunar semi-diameter which forms the whole difficulty in the problem. The value obtained by him can be scarcely said to rest on observation.

Prof. Newcomb regards his value as in perfect accord with that obtained by Mr. Stone, for the result of the Greenwich observations. We have seen that this is a mistake, and that Mr. Stone's is really the value of the apparent coefficient and corresponds really to the value $124''\cdot36$ obtained by Prof. Newcomb.

It may here he remarked that both Mr. Stone and Prof. Newcomb expressly base their investigations on the assumption, that in the mean result the errors in any other of the inequalities will destroy each other. This assumption it has been shown is untrue for at least three inequalities in the motion of the Moon, and that any error in Hansen's values for the annual equation or variation will be in part thrown on the values found by them for the coefficient of the parallactic inequality. These errors may be very approximately taken as

$$= +\frac{1}{9}\,\delta V - \frac{1}{7}\,\delta M.$$

Moreover, if there really exists an inequality of forty-five-year period in the apparent value of the parallactic inequality,

and if, further, this inequality does not arise from imperfections in Airy's tables which do not exist in Hansen's tables, then all these values will be affected by this inequality. Introducing then all these elements of uncertainty, the results can be written in the form

$$\text{Airy} \quad (P) = -124{\cdot}53 + \frac{1}{3}B,$$

$$\text{Stone} \quad (P) = -124{\cdot}55 - \frac{1}{7}B - \frac{1}{9}\delta V - \frac{1}{7}\delta M,$$

$$\text{Newcomb} \quad (P) = -124{\cdot}36 + \frac{2}{5}B + \frac{1}{9}\delta V - \frac{1}{7}\delta M.$$

These results are those which, we believe, really represent the true values of the apparent coefficient of the parallactic inequality to be derived from the investigation of these astronomers when properly corrected. A comparison of these results with those originally given will show the importance of the correction which we have been obliged to introduce, and the necessity which existed for a rigid examination of the theoretical basis of their labours.

[The concluding portion of this Paper, containing a new discussion of the Greenwich Meridian observations of the Moon in the years 1862–1876, will be published in the June No.—ED.]

Note on the Astronomer Royal's Investigation of the Theoretical Value of the Acceleration of the Moon's Mean Motion. By Professor J. C. Adams, M.A., &c.

I lose no time in pointing out briefly the reason why the Astronomer Royal, in the investigation which he communicated to the Society at the last Meeting, has failed to find my value of the coefficient of the Lunar Acceleration.

It may be useful, in the first place, to recall to mind that, according to my theory, the secular changes of

n, the Moon's mean motion,

and e', the eccentricity of the Earth's orbit,

are connected by the following relation:—

$$\frac{dn}{ndt} = \frac{e'de'}{dt}\left\{-3m^2 + \frac{3771}{32}m^4 + \frac{34047}{42}m^5 + \ldots\right\},$$

where m denotes, as usual, the ratio of the Sun's mean motion to that of the Moon.

If we stop at the first term of the series within the brackets the result is identical with that found by Laplace.

We do not know why Laplace did not carry his investigations further than this first term; but he probably thought that the succeeding terms would prove to be inconsiderable.

It is seen, however, that these terms have very large numerical coefficients and that their sign is contrary to that of the first term, and on calculation it is found that the sum of the series is less than its first term nearly in the ratio of 3 to 5.

Hence the secular acceleration will be diminished in the same ratio, and its amount in a century, instead of being about 10″, will be reduced to nearly 6″.

No investigation of the Moon's secular acceleration can be satisfactory which does not take into account terms of the nature of those which give rise to the terms involving m^4, m^5, &c., above referred to.

There is nothing to object to in the general principles of the method adopted by the Astronomer Royal, but in the practical application of the method I notice very grave defects.

In the first place, the only periodic terms which are included in the Astronomer Royal's expressions for $\mathrm{T}\frac{r}{a}$ and $\mathrm{P}\frac{r}{a}$ and for the factors multiplying

$$\delta\frac{a}{r},\quad \frac{d}{dt}\left(\delta\frac{a}{r}\right),\quad \delta v,\quad \frac{d}{dt}(\delta v),\quad \&c.,$$

on the right-hand side of the equations, are those which involve the angle 2D or F; whereas it will be seen by a reference to my paper in the *Philosophical Transactions* for 1853, that a great part of the coefficient of m^4 in the value of $\frac{dn}{ndt}$ there obtained arises from the combination of terms involving the angles S, F−S and F+S in the expressions for the Moon's coordinates with similar terms in

$$\delta\left(\frac{a}{r}\right),\quad \delta v,\quad \&c.$$

In the present investigation terms of the forms last mentioned are simply ignored.

In the next place, it is to be noted that, although periodic terms depending on the angle F are introduced into the assumed values of $\delta\frac{a}{r}$ and δv, yet in art. 12, the value of h which is the coefficient of t^2 in the value of δv, is found equal to $-\mathrm{B}b$, quite independently of the values of the coefficients e, f, g, k and l, which occur in the terms thus introduced.

The result of this is to reduce the secular acceleration practically to its first term only; which accounts for the coincidence of the Astronomer Royal's value with that of Laplace.

It may also be remarked in reference to art. 11, that although

terms involving the argument $2F$ or $4D$ may be properly omitted, we must put

$$\sin^2 F = \frac{1}{2} - \frac{1}{2} \cos 2F,$$

$$\text{and } \cos^2 F = \frac{1}{2} + \frac{1}{2} \cos 2F,$$

and the constant terms in these latter quantities should be taken into account.

After these general remarks, we will enter a little more closely on the consideration of one or two points in the investigation which are important.

Adopting the Astronomer Royal's notation, let

σ denote the Sun's mass,

A the semiaxis major of the Sun's (or Earth's) orbit,

E the eccentricity of the orbit,

R the radius vector at any time.

Then it may be shown, as in the paper before us, that the mean value of

$$\frac{\sigma}{R^3} \text{ is } \frac{\sigma}{A^3} \frac{1}{(1-E^2)^{\frac{3}{2}}}, = \frac{\sigma}{A^3}\left(1 + \frac{3}{2}E^2\right) \text{ nearly.}$$

Hence if E receive the variation δE in the time t, this quantity will be increased in the ratio of $1 + 3E\delta E$ to 1 nearly, or in the ratio of $1 + bt$ to 1, calling

$$3\frac{E\delta E}{t} = b.$$

Having arrived at this point, the Astronomer Royal assumes that the variation of the disturbing forces due to the variation δE in the eccentricity of the Sun's orbit will be represented by supposing

T to be replaced by $T(1 + bt)$,

and similarly P to be replaced by $P(1 + bt)$,

and therefore that the new forces, the effects of which are to be found by the present method, are Tbt and Pbt respectively.

On consideration, however, it will appear that this is only true for the non-periodic term in P, and that the periodic terms, whether in P or T, will be changed by any given variation of E in very different ratios.

For instance, the periodic terms in both T and P which depend on the angle $2D$ or F will vary nearly in the same ratio as $1 - \frac{5}{2}E^2$ does, instead of in the ratio in which $1 + \frac{3}{2}E^2$ varies as in the above case.

Hence these terms will be changed by the above-mentioned variation of E in the ratio of $1 + b't$ to 1, where

$$b' = -5\frac{E\delta E}{t} \text{ nearly.}$$

Again, the periodic terms in T and P which depend on the angles S, F−S and F+S will vary nearly in the same ratio as E does, so that these terms will be changed in the ratio of $1 + b''t$ to 1, where

$$b'' = \frac{\delta E}{Et} \text{ nearly.}$$

Hence we see that the values of b' and b'' are quite different from that of b which belongs to the non-periodic term, and that b'' is much larger than the other two quantities.

The correct way of finding δT and δP, the changes of the disturbing forces T and P due to change in the eccentricity of the Sun's orbit, is to express T and P in terms of the Moon's coordinates v and r, the Sun's mean longitude L and its mean anomaly S, and the eccentricity E.

Hence δT and δP may be at once expressed in terms of δv, δr, and δE.

Thus calling V the Sun's longitude, and employing the other symbols in the sense before explained, we have

$$P = \frac{1}{2}\frac{\sigma r}{R^3} + \frac{3}{2}\frac{\sigma r}{R^3}\cos(2v-2V),$$

$$T = \quad -\frac{3}{2}\frac{\sigma r}{R^3}\sin(2v-2V).$$

Or,

$$Pr = \frac{1}{2}\frac{\sigma r^2}{R^3} + \frac{3}{2}\frac{\sigma r^2}{R^3}\cos(2v-2V),$$

$$Tr = \quad -\frac{3}{2}\frac{\sigma r^2}{R^3}\sin(2v-2V).$$

Now, by the formulæ of elliptic motion, we may find

$$\frac{1}{R^3} = \frac{1}{A^3}\left[1 + \frac{3}{2}E^2 + 3E\cos S\right],$$

$$\frac{1}{R^3}\cos(2v-2V) =$$

$$\frac{1}{A^3}\left\{\left(1-\frac{5}{2}E^2\right)\cos(2v-2L) + \frac{7}{2}E\cos(2v-2L-S) - \frac{1}{2}E\cos(2v-2L+S)\right\},$$

$$\frac{1}{R^3}\sin(2v-2V) =$$

$$\frac{1}{A^3}\left\{\left(1-\frac{5}{2}E^2\right)\sin(2v-2L) + \frac{7}{2}E\sin(2v-2L-S) - \frac{1}{2}E\sin(2v-2L+S)\right\},$$

neglecting terms involving 2S, and powers of E above the second.

Substituting, and then taking the variation, we have

$$\delta(\mathrm{P}r) = \frac{\sigma}{\mathrm{R}^3} r\delta r + 3\frac{\sigma}{\mathrm{R}^3} r\delta r \cos(2v-2\mathrm{V}) - 3\frac{\sigma}{\mathrm{R}^3} r^2\delta v \sin(2v-2\mathrm{V})$$

$$+\frac{1}{2}\frac{\sigma r^2}{\mathrm{A}^3}\Big[3\mathrm{E}\delta\mathrm{E} + 3\delta\mathrm{E}\cos\mathrm{S}\Big]$$

$$+\frac{3}{2}\frac{\sigma r^2}{\mathrm{A}^3}\Big[-5\mathrm{E}\delta\mathrm{E}\cos(2v-2\mathrm{L}) + \frac{7}{2}\delta\mathrm{E}\cos(2v-2\mathrm{L}-\mathrm{S}) - \frac{1}{2}\delta\mathrm{E}\cos(2v-2\mathrm{L}+\mathrm{S})\Big]$$

$$\delta(\mathrm{T}r) = -3\frac{\sigma}{\mathrm{R}^3} r\delta r\sin(2v-2\mathrm{V}) - 3\frac{\sigma}{\mathrm{R}^3} r^2\delta v\cos(2v-2\mathrm{V})$$

$$-\frac{3}{2}\frac{\sigma r^2}{\mathrm{A}^3}\Big[-5\mathrm{E}\,\delta\mathrm{E}\sin(2v-2\mathrm{L}) + \frac{7}{2}\delta\mathrm{E}\sin(2v-2\mathrm{L}-\mathrm{S}) - \frac{1}{2}\delta\mathrm{E}\sin(2v-2\mathrm{L}+\mathrm{S})\Big]$$

in which $-r^3\delta\left(\frac{1}{r}\right)$ may be written for $r\delta r$, and the expressions given by the ordinary lunar theory in the case of unvaried eccentricity are to be substituted for v and r.

Hence, the expressions for $\delta\left(\mathrm{T}\frac{r}{a}\right)$ and $\delta\left(\mathrm{P}\frac{r}{a}\right)$, which are employed in the paper, are wholly incorrect, except in the case of the non-periodic term, which gives rise to the principal term of the secular acceleration or that found by Laplace.

The remark made near the close of the paper, viz. that the magnitudes of the quantities A, B, C, and therefore also that of the secular acceleration are proportional to the inverse cube of the Sun's distance, or to the cube of the Sun's parallax, can only be the result of inadvertence, as the Astronomer Royal himself will be the first to acknowledge.

In fact, the quantities A, B, C involve the factor $\frac{\sigma}{\mathrm{A}^3}$ and this is equal to n'^2, where n' is the Sun's mean motion and is known. The Sun's mass σ is determined by means of the parallax from this equation; or conversely, if the Sun's mass be known the parallax is thereby determined.

The values of A, B, C are approximately as follows

$$\mathrm{A} = \frac{3}{2}m^2,\quad \mathrm{B} = \frac{1}{2}m^2,\quad \mathrm{C} = \frac{3}{2}m^2,$$

where m denotes, as before, the ratio of the Sun's mean motion to that of the Moon.

Errata in *Monthly Notices*, vol. xix. p. 207, l. 6, for $-3m^4$ read $-3m^2$.
p. 208, l. 15 for $\frac{dn}{n^2dt}$ read $\frac{dn}{ndt}$.

Ephemeris for Physical Observations of Jupiter, 1880–81. By A. Marth, Esq.

Greenw. Noon.	Angle of Position of ♃'s Axis.	Latitude of Earth above ♃'s Equator.	Latitude of Sun above ♃'s Equator.	Annual Parallax.	Equat. Diam.	Greatest Phase.	Longitude of ♃'s Meridian directed to the Earth.	Diff.	Corr. of Long.
1880.	°	°	°	°	"	"	°	°	°
June 18	335·698	+2·616	+2·322	−11·26	37·83	0·364	196·69	4352·68	+0·55
23	335·792	2·655	2·338	11·50	38·38	·385	229·37	2·74	·57
28	335·879	2·693	2·353	11·68	38·95	·403	262·11	2·80	·59
July 3	335·959	2·730	2·369	11·80	39·55	·418	294·91	2·87	·61
8	336·032	2·766	2·384	11·86	40·17	·429	327·78	2·94	·61
13	336·096	2·800	2·400	11·84	40·82	·434	0·72	3·01	·61
18	336·152	2·832	2·415	11·76	41·48	·435	33·73	3·08	·60
23	336·198	2·863	2·430	11·60	42·15	·430	66·81	3·16	·59
28	336·232	2·892	2·445	11·36	42·84	·420	99·97	3·22	·56
Aug. 2	336·255	2·919	2·460	11·04	43·53	·403	133·19	3·29	·53
7	336·265	2·944	2·474	10·64	44·22	·380	166·48	3·36	·49
12	336·262	2·966	2·489	10·16	44·90	·352	199·84	3·43	·45
17	336·247	2·985	2·503	9·59	45·57	·318	233·27	3·49	·40
22	336·219	3·001	2·517	8·94	46·22	·281	266·76	3·55	·35
27	336·179	3·014	2·531	8·21	46·83	·240	300·31	3·60	·29
Sept. 1	336·129	3·023	2·545	7·41	47·41	·198	333·91	3·64	·24
6	336·068	+3·028	+2·559	−6·53	47·93	·155	7·55	4353·68	+0·19

Greenw. Noon.	Angle of Position of ♃'s Axis.	Latitude of Earth above ♃'s Equator.	Latitude of Sun above ♃'s Equator.	Annual Parallax.	Equat. Diam.	Greatest Phase.	Longitude of ♃'s Meridian directed to the Earth.	Diff.	Corr. of Long.
1880.	°	°	°	°	″	″	°	°	°
Sept. 11	335.999	+3.030	+2.573	−5.59	48.39	0.115	41.23		+0.14
16	335.924	3.027	2.586	4.59	48.79	.078	74.94	4353.71	.09
21	335.845	3.020	2.599	3.54	49.10	.047	108.66	3.72	.05
26	335.762	3.008	2.612	2.45	49.34	.023	142.38	3.72	.03
Oct. 1	335.679	2.993	2.625	1.33	49.48	.007	176.09	3.71	+0.01
6	335.596	2.974	2.637	−0.21	49.52	.000	209.78	3.69	.00
11	335.516	2.951	2.650	+0.93	49.47	.003	243.44	3.66	.00
16	335.441	2.925	2.662	2.05	49.33	.016	277.04	3.60	−0.02
21	335.371	2.897	2.674	3.15	49.09	.037	310.58	3.54	.04
26	335.307	2.866	2.686	4.21	48.77	.066	344.05	3.47	.08
31	335.251	2.834	2.698	5.22	48.37	.100	17.44	3.39	.12
Nov. 5	335.203	2.802	2.710	6.18	47.89	.139	50.74	3.30	.17
10	335.162	2.769	2.721	7.07	47.35	.180	83.93	3.19	.22
15	335.129	2.737	2.732	7.88	46.76	.221	117.03	3.10	.27
20	335.104	2.705	2.743	8.62	46.12	.260	150.02	2.99	.32
25	335.087	2.675	2.754	9.28	45.45	.297	182.90	2.88	.37
30	335.077	2.647	2.765	9.85	44.76	.330	215.67	2.77	.42
Dec. 5	335.074	+2.621	+2.776	+10.33	44.05	.357	248.34	2.67	−0.46
								4352.57	

Greenw. Noon.		Angle of Position of ♃'s Axis.	Latitude of Earth above ♃'s Equator.	Latitude of Sun above ♃'s Equator.	Annual Parallax.	Equat. Diam.	Greatest Phase.	Longitude of ♃'s Meridian directed to the Earth.	Diff.	Corr. of Long.
1880.		°	°	°	°	"	"	°	°	°
Dec.	10	335·079	+ 2·598	+ 2·786	+ 10·72	43·33	·0·378	280·91	4352·47	−0·50
	15	335·092	2·577	2·796	11·02	42·61	·393	313·38	2·37	·53
	20	335·112	2·559	2·806	11·25	41·91	·403	345·75	2·29	·55
	25	335·139	2·544	2·816	11·39	41·21	·406	18·04	2·20	·56
	30	335·175	2·532	2·826	11·46	40·53	·404	50·24	2·13	·57
1881.										
Jan.	4	335·220	2·523	2·836	11·44	39·87	0·396	82·37	2·06	0·57
	9	335·273	2·516	2·845	11·36	39·23	·384	114·43	1·99	·56
	14	335·335	2·513	2·854	11·21	38·62	·368	146·42	1·94	·55
	19	335·406	2·512	2·863	10·99	38·04	·349	178·36	1·88	·53
	24	335·487	2·514	2·872	10·71	37·49	·327	210·24	1·84	·50
	29	335·579	2·518	2·880	10·37	36·97	·302	242·08	1·80	·47
Feb.	3	335·681	2·524	2·889	9·99	36·48	·277	272·88	1·76	·43
	8	335·795	2·533	2·897	9·56	36·02	·250	305·64	1·74	·40
	13	335·920	2·543	2·905	9·08	35·59	·223	337·38	1·71	·36
	18	336·056	2·556	2·913	8·56	35·19	·196	9·09	1·70	·32
	23	336·204	2·570	2·921	8·01	34·83	·170	40·79	4351·68	·28
	28	336·365	+ 2·584	+ 2·928	+ 7·42	34·50	0·144	72·47		−0·24

The "annual parallax" is the difference of the Jovicentric longitudes of the Sun and the Earth, reckoned in the plane of *Jupiter's* equator. If the correction given in the last column is applied to the "longitude of ♃'s meridian directed to the Earth," the longitude of the meridian is found which bisects the illuminated disk. The First (or Zero) Meridian, from which the longitudes are reckoned is that which at the Greenwich midnight preceding January 1, 1872, was apparently directed to the Earth, and which is assumed to rotate at the daily rate of $870^{\circ}\cdot60$, the corresponding period of rotation being $9^{h}\ 55^{m}\ 27^{s}\cdot08$. If suitable observations for ascertaining the true rate of rotation for different spots have been made during the apparitions of *Jupiter* from 1872 to 1878, for which Ephemerides were published, they have not yet come to my knowledge. The numerous observations of passages across the central meridian which have been made during the last season appear to have been confined to those of the red spot in the planet's southern hemisphere. Though their proper discussion must be deferred till the end of this exceptional appearance, and till all the accounts are available, a preliminary examination of the results will not be uninstructive, and I give, therefore, a list of the observed passages which have hitherto come to my knowledge, together with the corresponding longitudes of the meridian which bisected the illuminated disk, as derived from the Ephemeris, and also with the longitudes corrected by the quantity $-0^{\circ}\cdot17$ (t – Oct. 17·0 1879). The period of rotation corresponding to the daily rate $870^{\circ}\cdot43$ is $9^{h}\ 55^{m}\ 34^{s}\cdot1$. A small correction, depending on the Jovicentric latitude of the spot and on the correction in the last column of the Ephemeris, is still required in those cases in which the observed passage does not refer to the meridian which is equidistant from the two limbs (one of which is affected by the phase), but to the line perpendicular upon and bisecting the apparent equatoreal diameter. Except at opposition, this line does not pass through the poles of the planet, and is not a meridian line, and though the difference is small for spots not far from the equator, it may become very sensible for spots in higher latitudes.

The sources from which the list is derived are mentioned *post*, p. 429 : the times are Ath. Athens, Br. Brussels, Gr. Greenwich, Mo. Moscow mean times, as supplied by the observers; in the cases, however, where seconds are given, tenths of minutes have been substituted for the sake of uniformity.

1879.	Observed Passages. Prec. End. h m	Middle. h m	Foll. End. h m	Observer.		Corresponding Longitudes. Rot. 870°·60. Prec. End. °	Middle. °	Foll. End. °	Rot. 870°·43. Prec. End. °	Middle. °	Foll. End. °
July 10	—	19 11·3	—	Gr.	Pritchett	—	243·9	—	—	260·6	—
22	—	19 1·3	—	„	„	—	246·2	—	—	260·9	—
26	—	12 25·0	—	„	Pratt	—	249·4	—	—	263·4	—
28	14 18·4	14 38·4	14 48·5	Br.	Niesten	248·8	260·9	267·1	262·5	274·6	280·7
29	10 4·7	10 19·8	10 34·8	„	„	246·2	255·3	264·4	259·8	268·8	277·
31	11 30·6	11 54·6	12 15·6	„	„	239·5	254·0	266·7	252·7	267·2	279·
Aug. 2	12 50	13 27·8	—	„	„	229·0	249·4	—	241·8	262·3	—
4	—	15 8·2	15 29·8	„	„	—	254·0	267·1	—	266·5	279·6
6	—	16 25·3	—	Gr.	Pritchett	—	252·7	—	—	264·8	—
7	11 30	—	—	„	Brewin	224·9	—	—	236·8	—	—
7	—	22 12·3	—	„	Pritchett	—	253·2	—	—	265·2	—
8	—	18 2·3	—	„	„	—	252·8	—	—	264·6	—
9	13 38·5	14 0·5	—	Br.	Niesten	233·5	246·8	—	245·1	258·4	—
10	9 39·1	10 8·1	10 31·6	„	„	239·4	256·9	270·8	250·9	268·4	281·5
12	11 17	11 38	12 1·5	„	„	240·1	252·8	267·0	251·2	263·9	278·1
15	—	18 42·3	—	Gr.	Pritchett	—	252·1	—	—	263·7	—
16	—	14 34·1	—	„	„	—	252·8	—	—	263·2	—
17	10 28·6	10 43·9	11 7·2	Br.	Niesten	244·5	253·8	267·8	254·8	264·1	278·1
18	—	16 11·9	—	Gr.	Pritchett	—	253·4	—	—	263·5	—
20	—	17 48·3	—	„	„	—	253·2	—	—	262·9	—

1879.	Observed Passages. Prec. End. h m	Middle. h m	Foll. End. h m	Observer.		Corresponding Longitudes. Rot. 870°·60. Prec. End. °	Middle. °	Foll. End. °	Rot. 870°·43. Prec. End. °	Middle. °	Foll. End. °
Aug. 22	—	19 25·3	—	Gr.	Pritchett	—	253·3	—	—	262·7	—
24	—	21 0·6	—	„	„	—	252·4	—	—	261·4	—
25	—	16 51·8	—	„	„	—	252·7	—	—	261·6	—
27	—	18 29·3	—	„	„	—	253·1	—	—	261·7	—
28	—	14 19·0	—	„	„	—	252·5	—	—	260·9	—
29	—	10 14	—	„	Backhouse	—	255·1	—	—	263·3	—
29	—	20 7·3	—	„	Pritchett	—	253·8	—	—	262·0	—
30	—	15 58·1	—	„	„	—	253·9	—	—	261·9	—
31	11 33·6	12 4·4	—	Br.	Niesten	234·1	253·7	—	242·0	261·6	—
Sept. 1	—	17 33·9	—	Gr.	Pritchett	—	253·3	—	—	261·0	—
3	9 10	—	—	Br.	Niesten	239·5	—	—	246·9	—	—
3	—	9 19	—	Gr.	Pratt	—	255·5	—	—	262·9	—
3	—	19 9·8	—	„	Pritchett	—	252·7	—	—	260·1	—
4	—	15 2·3	—	„	„	—	253·8	—	—	261·0	—
6	—	16 40·7	—	„	„	—	254·7	—	—	261·6	—
8	—	8 25	8 55	„	MacCance	—	256·4	274·6	—	263·0	281·1
8	—	18 15·8	—	„	Pritchett	—	253·7	—	—	260·2	—
10	—	10 1·5	—	„	Pratt	—	256·2	—	—	262·4	—
10	—	19 51·7	—	„	Pritchett	—	253·1	—	—	259·2	—
11	—	8 25·9	—	Mo.	Bredichin	—	258·2	—	—	264·3	—

	Observed Passages.			Observer.		Corresponding Longitudes.					
						Rot. 870°·60.			Rot. 870°·43.		
1879.	Prec. End. h m	Middle. h m	Foll. End. h m			Prec. End. °	Middle. °	Foll. End. °	Prec. End. °	Middle. °	Foll. End °
Sept. 12	—	11 59·1	—	Br.	Niesten	—	258·1	—	—	264·0	—
13	—	17 22·3	—	Gr.	Pritchett	—	254·8	—	—	260·5	—
14	—	13 13·9	—	„	„	—	255·3	—	—	260·9	—
15	8 35	9 7	—	„	MacCance	237·4	256·7	—	242·8	262·1	—
15	—	11 43·7	—	Mo.	Bredichin	—	260·6	—	—	266·0	—
15	—	18 59·7	—	Gr.	Pritchett	—	255·1	—	—	260·4	—
16	—	14 50·9	—	„	„	—	255·4	—	—	260·5	—
17	—	13 18·6	—	Mo.	Bredichin	—	259·4	—	—	264·4	—
18	—	9 11·3	—	„	„	—	260·5	—	—	265·4	—
20	—	10 48·2	—	„	„	—	260·5	—	—	265·0	—
20	—	18 6·3	—	Gr.	Pritchett	—	256·2	—	—	260·7	—
22	9 37·1	10 0·7	10 30·1	Br.	Niesten	239·1	253·4	271·1	243·3	257·6	275·3
22	—	12 25·1	—	Mo.	Bredichin	—	260·4	—	—	264·6	—
22	9 29	9 33	10 23	Gr.	Backhouse	244·8	260·5	277·4	249·0	264·7	281·6
23	—	8 17·8	—	Mo.	Bredichin	—	261·5	—	—	265·6	—
24	—	11 55	—	Br.	Niesten	—	263·8	—	—	267·6	—
25	—	9 55·7	—	Mo.	Bredichin	—	262·1	—	—	265·7	—
25	—	7 44·5	—	Br.	Niesten	—	263·0	—	—	266·7	—
25	—	17 12·3	—	Gr.	Pritchett	—	256·9	—	—	260·5	—
26	—	13 3·1	—	„	„	—	256·9	—	—	260·4	—

1879.	Observed Passages. Prec. End. h m	Middle. h m	Foll. End. h m	Observer.		Corresponding Longitudes. Rot. 870°·60. Prec. End. °	Middle. °	Foll. End. °	Rot. 870°·43. Prec. End. °	Middle. °	Foll. End. °
Sept. 27	8 44·6	9 11·9	9 41·9	Br.	Niesten	240·6	257·2	275·3	244·0	260·5	278·6
27	9 19·8	9 48·3	10 23·3	Berlin	Lohse	240·1	257·3	278·5	243·5	260·7	281·8
27	—	8 59	—	Gr.	Pratt	—	259·9	—	—	263·3	—
27	—	11 35·6	—	Mo.	Bredichin	—	263·8	—	—	267·1	—
27	—	18 51·3	—	Gr.	Pritchett	—	258·1	—	—	261·3	—
29	—	10 43	—	„	Backhouse	—	264·1	—	—	267·1	—
30	—	16 19·6	—	„	Pritchett	—	258·3	—	—	261·0	—
Oct. 1	—	12 15·5	—	„	Pratt	—	261·3	—	—	264·0	—
2	7 59·4	8 19·4	8 44·8	Br.	Niesten	246·6	258·6	274·0	249·1	261·1	276·5
2	—	10 43·1	—	Mo.	Bredichin	—	265·2	—	—	267·7	—
3	—	6 30·9	—	„	„	—	263·4	—	—	265·7	—
5	—	8 10·8	—	„	„	—	265·0	—	—	267·0	—
5	—	15 26·1	—	Gr.	Pritchett	—	259·0	—	—	261·0	—
6	—	11 22·3	—	„	Pratt	—	262·2	—	—	264·0	—
6	10 59	11 23	—	„	Backhouse	248·2	262·7	—	250·0	264·5	—
6	11 25·7	11 42	12 3	Br.	Niesten	253·7	263·6	276·3	255·5	265·4	278·1
7	7 12·5	7 32·5	7 53·5	„	„	251·3	263·4	273·1	252·9	265·0	274·7
7	—	17 4·3	—	Gr.	Pritchett	—	259·6	—	—	261·2	—
8	—	12 56·7	—	„	„	—	260·5	—	—	262·0	—
11	—	13 7·5	—	Mo.	Bredichin	—	268·0	—	—	269·0	—

	Observed Passages.			Observer.		Corresponding Longitudes. Rot. 870°·60.			Rot. 870°·43.		
	Prec. End.	Middle.	Foll. End.			Prec. End.	Middle.	Foll. End.	Prec. End.	Middle.	Foll. End.
1879.	h m	h m	h m			°	°	°	°	°	°
Oct. 12	—	16 10·2	—	Gr.	Pritchett	—	259·9	—	—	260·7	—
13	—	12 1·0	—	„	„	—	259·8	—	—	260·4	—
14	—	10 32·2	—	Mo.	Bredichin	—	265·9	—	—	266·3	—
14	—	17 50·5	—	Gr.	Pritchett	—	261·6	—	—	262·0	—
15	—	13 41·1	—	„	„	—	261·5	—	—	261·8	—
16	9 32·3	9 51·3	10 12·3	Br.	Niesten	251·1	262·6	275·3	251·2	262·7	275·4
17	—	8 1·8	—	Mo.	Bredichin	—	266·7	—	—	266·6	—
17	—	15 18·9	—	Gr.	Pritchett	—	261·8	—	—	261·7	—
18	—	11 11·7	—	„	„	—	262·9	—	—	262·6	—
19	—	17 0·3	—	„	„	—	264·2	—	—	263·7	—
20	—	5 32·5	—	Mo.	Bredichin	—	268·0	—	—	267·5	—
20	—	12 50·4	—	Gr.	Pritchett	—	263·7	—	—	263·1	—
22	—	14 29·4	—	„	„	—	264·6	—	—	263·7	—
24	—	8 50·3	—	Mo.	Bredichin	—	269·9	—	—	268·6	—
24	—	16 7·4	—	Gr.	Pritchett	—	265·0	—	—	263·7	—
25	—	11 59·6	—	„	„	—	265·7	—	—	264·3	—
26	—	7 50	—	„	Pratt	—	265·3	—	—	263·7	—
27	—	13 38·0	—	„	Pritchett	—	266·2	—	—	264·4	—
28	—	—	9 56	„	Backhouse	—	—	282·6	—	—	280·6
29	—	15 15·3	—	„	Pritchett	—	266·1	—	—	264·0	—

1879.		Observed Passages. Prec. End. h m	Middle. h m	Foll. End. h m	Observer.		Corresponding Longitudes. Rot. 870°·60. Prec. End. °	Middle. °	Foll. End. °	Rot. 870°·43. Prec. End. °	Middle. °	Foll. End. °
Oct.	30	—	11 6·3	—	Gr.	Pritchett	—	266·1	—	—	263·8	—
Nov.	1	—	5 24·6	—	Mo.	Bredichin	—	269·7	—	—	267·1	—
	2	—	8 35·3	—	Gr.	Pratt	—	266·4	—	—	263·6	—
	2	—	8 45	—	„	Green	—	272·2	—	—	269·4	—
	3	—	14 20·8	—	„	Pritchett	—	265·7	—	—	262·9	—
	5	—	6 4·3	—	„	Pratt	—	266·6	—	—	263·3	—
	5	—	16 0·7	—	„	Pritchett	—	267·1	—	—	263·7	—
	6	—	11 53·0	—	„	„	—	267·8	—	—	264·4	—
	7	7 13	7 44	8 13	„	Gledhill	249·1	267·8	285·3	245·5	264·2	281·7
	7	7 15	7 47	—	„	Green	250·3	269·6	—	246·7	266·0	—
	7	—	—	8 17	„	Backhouse	—	—	287·8	—	—	284·1
	10	6 17·9	6 50·5	7 9·6	Ath.	Schmidt	249·8	269·5	281·1	245·7	265·4	277·0
	10	—	—	5 44	Gr.	Backhouse	—	—	286·7	—	—	282·6
	10	—	15 7·2	—	„	Pritchett	—	267·2	—	—	263·0	—
	12	—	6 45	—	„	Green	—	264·6	—	—	260·1	—
	12	7 17·7	8 24·7	8 47·6	Ath.	Schmidt	226·9	267·4	281·2	222·5	263·0	276·8
	12	6 20	6 50	7 20	Gr.	Gledhill	249·4	267·6	285·7	245·0	263·1	281·2
	12	—	6 50	—	„	Pratt	—	267·6	—	—	263·1	—
	12	—	9 25·8	—	Mo.	Bredichin	—	270·9	—	—	266·4	—
	14	7 59	8 25	9 6	Gr.	Gledhill	250·2	265·9	290·7	245·4	261·1	285·9

1879.	Observed Passages. Prec. End. h m	Middle. h m	Foll. End. h m	Observer.		Corresponding Longitudes. Rot. 870°·60. Prec. End. °	Middle. °	Foll. End. °	Rot. 870° 43. Prec. End. °	Middle. °	Foll. End. °
Nov. 15	—	14 13·3	—	Gr.	Pritchett	—	267·0	—	—	261·9	—
17	5 35	—	—	„	Backhouse	254·6			249·3		—
19	—	7 35	—	„	Green	—	268·0	—	—	262·3	
19	—	7 35·5	—	„	Pratt	-	268·3	—	—	262·6	—
22	—	14 58·3	—	„	Pritchett	-	267·3		—	261·1	
23	—	10 52·3	—	„	„	—	269·0	—	—	262·7	—
24	7 52·7	8 23·0	8 50·5	Ath.	Schmidt	253·5	271·8	288·5	247·0	265·3	282·0
25	—	5 7·9	—	Mo.	Bredichin	—	270·9	—	—	264·2	—
25	—	12 32·8	—	Gr.	Pritchett	—	270·6	—	—	263·9	—
26	8 18·9	8 43·9	9 8·9	Br.	Niesten	257·1	272·2	287·3	250·2	265·3	280·4
28	—	10 5·3	—	Gr.	Pritchett	—	272·7	—	—	265·5	—
29	5 39·3	6 4·3	6 34·3	Br.	Niesten	251·8	266·9	285·1	244·5	259·6	277·7
29	5 15	5 50	6 30	Gr.	Gledhill	247·7	268·9	293·0	240·4	261·5	285·7
29	—	5 53	6 21	„	Backhouse	—	270·7	287·6	—	263·3	280·3
29	—	15 49·3	—	„	Pritchett	—	271·1	—	—	263·7	—
30	—	11 40·3	—	„	„	—	271·0		—	263·5	—
Dec. 1	7 3	7 35	—	„	Gledhill	253·9	273·2	-	246·2	265·5	—
2	—	3 53·4	—	Br.	Niesten		279·1		—	271·2	-
3	8 45	9 10	—	Gr.	Gledhill	256·3	271·4		248·3	263·4	—
4	—	7 33·2	—	Mo.	Bredichin		272·5		—	264·3	—

	Observed Passages.			Observer.		Corresponding Longitudes.					
						Rot. 870°·60.			Rot. 870°·43.		
1879.	Prec. End. h m	Middle. h m	Foll. End. h m			Prec. End. °	Middle. °	Foll. End. °	Prec. End. °	Middle. °	Foll. End. °
Dec. 4	6 9·3	6 50·6	7 0·7	Ath.	Schmidt	255·2	280·2	286·3	247·0	272·0	278·1
6	—	6 36	—	Gr.	Pratt	—	269·6	—	—	261·0	—
6	6 28·9	6 54·6	7 23·9	Br.	Niesten	254·7	270·3	288·0	246·2	261·8	279·4
6	6 14	6 40	7 10	Gr.	Backhouse	256·3	272·0	290·1	247·7	263·4	281·6
6	6 5	6 45	7 15	,,	Gledhill	250·8	275·0	293·1	242·3	266·5	284·6
6	—	6 45	7 15	,,	Knott	—	275·0	293·1	—	266·5	284·6
6	7 58·9	8 18·8	8 52·3	Ath.	Schmidt	262·3	274·3	294·6	253·8	265·8	286·1
8	8 2·2	8 29·2	8 55·2	Br.	Niesten	251·9	268·2	283·9	243·0	259·3	275·0
8	7 41	8 21	8 46	Gr.	Gledhill	249·7	273·8	288·9	240·7	264·9	280·0
9	4 2·0	4 22·3	4 56·8	Br.	Niesten	257·1	269·4	290·2	248·1	260·3	281·2
11	5 22	—	—	Gr.	MacCance	256·8	—	—	247·4	—	—
11	—	15 40·6	—	,,	Pritchett	—	270·7	—	—	261·3	—
12	—	11 33·0	—	,,	,,	—	271·5	—	—	261·9	—
13	—	7 35	—	,,	Backhouse	—	278·0	—	—	268·3	—
16	4 49·9	5 16·9	5 44·4	Br.	Niesten	258·8	275·1	291·7	248·6	264·9	281·5
16	—	14 50·3	—	Gr.	Pritchett	—	272·3	—	—	262·0	—
18	6 18·3	6 48·3	7 15·3	Br.	Niesten	253·0	271·1	287·5	242·4	260·6	276·9
20	—	8 30	—	,,	,,	—	273·4	—	—	262·4	—
22	—	9 52·6	—	Gr.	Pritchett	—	274·6		—	263·3	—
23	5 38·4	6 2·9	6 32·9	Br.	Niesten	260·8	275·6	293·7	249·3	264·1	282·3
23	5 18	5 48	6, 15	Gr.	Backhouse	259·0	277·2	293·5	247·6	265·7	282·0

	Observed Passages.			Observer.		Corresponding Longitudes. Rot. 870° 60.			Rot. 870°·43.		
	Prec. End.	Middle.	Foll. End.			Prec. End.	Middle.	Foll. End.	Prec. End.	Middle.	Foll. End.
1879.	h m	h m	h m			°	°	°	°	°	°
Dec. 24	—	11 31·8	—	Gr.	Pritchett	—	275·3			263·7	—
25	7 1	7 27	7 56	"	Backhouse	262·0	277·7	295·3	250·2	266·0	283·5
26	4 35·2	4 51·5	5 14·1	Ath.	Schmidt	266·9	276·7	290·4	255·0	264·8	278·5
26	—	13 10·3	—	Gr.	Pritchett	—	275·6	—	—	263·6	—
1880.											
Jan. 4	5 15	5 42	6 11	"	MacCance	261·6	277·9	295·4	248·1	264·4	282·0
12	—	12 16·3	—	"	Pritchett	—	279·1	—	—	264·2	—
14	—	13 54·3	—	"	"	—	279·0	—	—	263·8	—
17	—	4 5·0	—	Mo.	Bredichin	—	283·0	—	—	267·4	—
17	—	11 23·4	—	Gr.	Pritchett	—	278·8	—	—	263·1	—
19	—	5 43·4	—	Mo.	Bredichin	—	283·2	—	—	267·2	—
19	—	13 0·8	—	Gr.	Pritchett	—	278·4	—	—	262·3	—
26	3 47·9	4 12·9	4 42·9	Br.	Niesten	266·1	281·2	299·3	248·9	264·0	282·1
26	—	—	4 30	Gr.	Gledhill	—	—	302·1	—	—	284·9
28	5 15·3	5 43·3	6 11	Br.	Niesten	259·6	276·5	293·2	242·0	258·9	275·7
28	5 10	5 40	6 15	Gr.	Gledhill	266·9	285·1	306·0	249·4	267·5	288·5
Feb. 4	5 30	—	—	"	Brewin	251·4	—	—	232·7	—	—
4	6 4	6 30	6 58	Br.	Niesten	261·4	277·1	294·0	242·6	258·4	275·3
5	—	12 6·3	—	Gr.	Pritchett	—	281·3	—	—	262·3	—
7	—	13 43·3	—	"	"	—	280·6	—	—	261·4	—
22	6 6	—	—	Cambr.	Trouvelot	271·3	—	—	249·4	—	—

The times of the observed passages contained in the list are taken from the following sources:—

T. W. Backhouse, Sunderland.	*Monthly Notices*, vol. xl. p. 157.
Th. Bredichin, Moscow.	*Annales de l'Observatoire de Moscou*, tom. vi. 2 livraison, p. 105, 106.
T. D. Brewin, Leicester.	*Monthly Notices*, vol. xl. p. 377.
J. Gledhill, Halifax.	*The Observatory*, vol. iii. p. 280 and 355.
N. Green, St. John's Wood.	Private communication.
G. Knott, Cuckfield.	*Astron. Register*, vol. xviii. p. 91.
O. Lohse, Potsdam.	*Astron. Nachr.* No. 2282.
J. L. Mac Cance, Putney.	Private communication.
L. Niesten, Brussels.	*Bulletins de l'Acad. Roy. de Belgique*, t. xlviii. no. 12. [The observations from November 26 to February 4 have been kindly communicated by letter.]
H. Pratt, Brighton.	*Monthly Notices*, vol. xl. p, 154.
C. H. Pritchett, Glasgow, Miss.	From a list kindly communicated in MSS., in which the observed times are already given in Greenwich M. T.
J. Schmidt, Athens.	*Astron. Nachr.* No. 2309.
L. Trouvelot, Cambridge, Mass.	*Observatory*, vol. iii. p. 417, where for G. M. T. must be read Cambr. M. T.

An examination of the longitudes in the three last columns will give at least some preliminary indications of the errors of the observations and of the deviations of the motion of the red spot from regularity. The discrepancies between the observations of different observers and on different days point to the existence of some grave sources of error and show the necessity for greater care and caution. On the other hand, the consideration of the fair agreement frequently found may give some assurance that the observations are worth making, and observers may perhaps be induced thereby to spend sufficient time and patience in watching the passages across the central meridian of all well-marked points on the planet's surface, so that at last proper observations may be available for investigating the various motions which are going on there.

The possible Ten-Month Period of Variation in Latitude.
By A. M. W. Downing, Esq.

If from any cause the axis of rotation of the Earth does not coincide with the axis of figure the former will revolve round the latter in a period which is equal to

$$\frac{A}{C-A},$$

employing the notation usual in works on dynamics. The most probable value of this period appears to be 306 mean solar days. The apparent pole of the Earth would therefore travel round the pole of the adjacent principal axis in this time, and give rise to what may be called a ten-month period of variation in latitude as determined from meridian zenith distances of stars observed above and below the pole. Clerk Maxwell has examined (*Trans. Edin. R. S.* vol. xxi. p. 569) the observations of *Polaris* made at the Royal Observatory, Greenwich, during the years 1851–4 to ascertain whether any period of this kind could be detected in the resulting latitudes, the apparent co-latitude being found from the observations made during each month. There appeared to be a slight indication of a maximum in 1851 March, 1852 Feb., 1852 Dec., 1853 Nov., and 1854 Sept. The author considers, however, that "this result is to be regarded as very doubtful," and that "more observations would be required to establish the existence of so small a variation at all."

In the present paper I have discussed the Greenwich observation of *Polaris* made during the ten years 1868–1877, with the view of determining the amount of this variation (if it exists), and also the time of maximum effect.

The zenith distances of *Polaris*, as extracted from the volumes of *Greenwich Observations*, have been corrected, where necessary, for errors in the screws of the microscope-micrometers by applying the quantities given in the Introduction to the Nine-Year Catalogue; also for error in the Transit-Circle thermometer, a correction of $-0^\circ \cdot 5$ being applied to all readings, and the corresponding correction applied to the zenith distance. The refractions used are those of Bessel's *Tabulæ Regiomontanæ*. The correction for discordance of direct and reflexion observations has been also applied.

The observations have then been grouped—usually by months, except in cases where there is an insufficient number of observations, either above or below the pole. The apparent co-latitude has then been formed from the observations of each group, the effect of uncorrected aberration and of annual parallax being practically eliminated by this method of treating the observations. The following table exhibits the series of co-latitudes

thus formed—the degrees and minutes being 38° 31′ in all cases:—

	1868.	1869.	1870.	1871.	1872.	1873.	1874.	1875.	1876.	1877.
	″	″	″	″	″	″	″	″	″	″
Jan.	22·12	22·28	—	—	21·92	21·96	—	—	—	22·32
Feb.	21·77	—	—	—	—	—	—	—	—	21·62
March	21·99	22·14	22·25	22·61	21·66	21·52	21·65	22·20	21·88	21·97
April	21·45	21·98	21·40	23·29	21·65	21·50	21·21	21·43	21·25	21·65
May	22·10	21·56	21·24	22·14	21·83	21·92	21·69	21·72	21·39	21·86
June	21·84	21·07	21·81	21·53	21·46	21·37	21·12	21·41	21·51	21·39
July	21·55	—	21·38	21·63	—	21·84	20·71	21·76	21·28	20·52
August	21·73	—	21·98	—	—	21·62	—	21·37	21·29	21·70
Sept.	21·48	—	21·55	21·61	21·64	21·59	22·02	21·64	21·50	20·86
Oct.	22·19	21·65	21·16	21·39	21·55	21·57	21·92	22·28	22·08	21·77
Nov.	22·41	21·93	21·70	21·98	21·70	21·87	22·05	22·46	21·81	21·96
Dec.	21·89	22·51	22·96	21·88	22·15	22·36	21·82	22·89	22·50	21·52

There appears, therefore, to be an annual variation in the Greenwich apparent co-latitudes (the possibility of the existence of which was pointed out by me in *M. N.* vol. xl. p. 86), but there is no variation with a ten-month period of any considerable amount.

I proceed now to calculate the amount of the latter variation, assuming its period to be 306 days.

Let r be the angular distance between the moveable and fixed poles, and θ the angle which the great circle joining them makes with the Greenwich meridian, referred to the fixed pole, at the time 1872·0. Also let ϕ be the apparent co-latitude, and ϕ_0 the co-latitude referred to the fixed pole, then we have

$$\begin{aligned}\phi &= \phi_0 + r\cos\{\theta + 429{\cdot}4(t-1872)\}\\ &= \phi_0 + r\cos\theta\cos 429{\cdot}4(t-1872) - r\sin\theta\sin 429{\cdot}4(t-1872).\end{aligned}$$

Let

$$\begin{aligned}x &= r\cos\theta,\\ y &= r\sin\theta,\\ z &= \text{correction to the assumed value of } \phi_0,\ 38^\circ\ 31'\ 21''{\cdot}90,\\ n &= 38^\circ\ 31'\ 21''{\cdot}90 - \phi;\end{aligned}$$

then the equations of condition are of the form

$$ax + by + cz + n = 0.$$

If m and m' be the number of observations above and below the pole respectively in each group, the weight of the resulting

apparent co-latitude, and therefore of the corresponding equation of condition, is

$$\frac{4mm'}{m+m'}$$

Equations of conditions have been formed in this manner for each of the determinations of co-latitude (100 in number) given in the table above; these have been solved by the method of least squares, and the following normal equations obtained:—

$$\begin{aligned} +629{\cdot}66x + 11{\cdot}78y + 10{\cdot}58z + 44{\cdot}08 &= 0, \\ +11{\cdot}78x + 544{\cdot}25y + 7{\cdot}88z + 18{\cdot}76 &= 0, \\ +10{\cdot}58x + 7{\cdot}88y + 1179{\cdot}z + 105{\cdot}86 &= 0. \end{aligned}$$

From these I find

$$\begin{aligned} x &= -0''{\cdot}068 \pm 0''{\cdot}015, \\ y &= -0{\cdot}032 \pm 0{\cdot}016, \\ z &= -0{\cdot}089 \pm 0{\cdot}011. \end{aligned}$$

the probable error of a single determination of co-latitude of weight unity being

$$\pm 0''{\cdot}384.$$

From the above values of x and y we have

$$\begin{aligned} r &= 0''{\cdot}075 \pm 0''{\cdot}015, \\ \theta &= 205^\circ{\cdot}0 \pm 6^\circ{\cdot}7. \end{aligned}$$

Although the value thus found for r is so small, still it will be remarked that it is five times greater than its probable error; so that, supposing we may assume that the variation in question remains constant for ten years, its reality may be asserted with some degree of confidence. But it is evidently much too small to be detected by mere inspection of the observations, which appears to have been the only process resorted to by Clerk Maxwell in the paper referred to above.

The value of θ which I have found indicates that the apparent latitude of Greenwich was a maximum on 1872, Oct. 12. Now, Prof. C. A. F. Peters, in his well-known work *Recherches sur la parallaxe des étoiles fixes*, has investigated this question of variability of latitude, making use of the Pulkowa observations of *Polaris* from 1842 March to 1843 April. He found $r = 0''{\cdot}079 \pm 0''{\cdot}017$, and for the time when the latitude of Pulkowa was a maximum 1842 Nov. 16. If the variation remain constant the latitude should, therefore, again have been a maximum on 1872 Nov. 16; and the longitude of Pulkowa being 2^h east, the latitude of Greenwich would be maximum $\frac{1}{12}$ of the

period, or about 26 days earlier; that is, about 1872, Oct. 21. Prof. Peters' results, therefore, agree remarkably with mine, both as to the magnitude of the variation and the time of its maximum effect.

On the other hand, Dr. Nyrén, from a thorough discussion of three series of Pulkowa observations made by Peters, by Gyldén, and by himself, has arrived at discordant results, and concludes that there is no constancy of magnitude or phase in the variation of apparent latitude. Dr. Nyrén's investigation is contained in his paper entitled *Die Polhöhe von Pulkowa.* There appears, therefore, to be this difficulty about the question—that if a short series of observations is used we cannot be sure that we have satisfactorily got rid of accidental errors, or of errors (possibly) depending on the refraction having periods of twelve months, or of six months; and if we discuss a sufficiently long series of observations to eliminate these errors, we make what may be an illegitimate assumption—viz., that the variation remains constant.

The co-latitude of Greenwich deduced from the ten-years' observation of *Polaris* is

$$38^\circ\ 31'\ 21''\!\cdot\!811 \pm 0''\!\cdot\!011.$$

Blackheath,
1880, *May* 10.

On a Photograph of Jupiter's Spectrum showing Evidence of Intrinsic Light from that Planet. By Henry Draper, M.D.

There has been for some years a discussion as to whether the planet *Jupiter* shone to any perceptible extent by his own intrinsic light, or whether the illumination was altogether derived from the Sun. Some facts seem to point to the conclusion that it is not improbable that *Jupiter* is still hot enough to give out light, though perhaps only in a periodic or eruptive manner.

It is obvious that spectroscopic investigation may be usefully employed in the examination of this question and I have incidentally, in the progress of an allied inquiry,* made a photograph which has sufficient interest to be submitted to the inspection of the Astronomical Society.

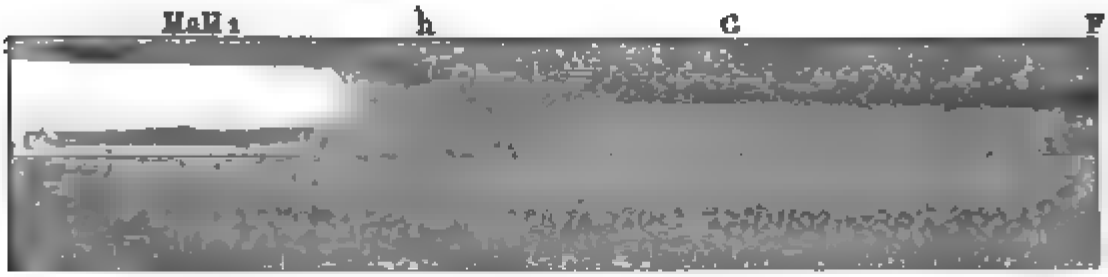
If the light of *Jupiter* is in large part the result of his own incandescence it is certain that the spectrum must differ from that of the Sun, unless the improbable hypothesis is advanced that the same elements, in the same proportions and under the same physical conditions, are present in both bodies. Most of

* See paper *On Photographing the Spectra of the Stars and Planets*, read before the American National Academy of Sciences, Oct. 28, 1879, and published in *Nature*, Nov. 27, 1879, and in the *American Journal of Science*, Dec. 1879.

the photographs I have made of the spectrum of *Jupiter* answer this question decidedly, and from their close resemblance to the spectrum of the Sun indicate that under the average circumstances of observation almost all the light coming to the Earth from *Jupiter* must be merely reflected light originating in the Sun. For this reason I have used the spectrum of *Jupiter* as a reference spectrum on many of my stellar spectrum photographs.

But on one occasion, viz. on September 27, 1879, a spectrum of *Jupiter* with a comparison spectrum of the Moon was obtained which shows a different state of things. Fortunately, owing to the assiduous assistance of my wife, I have a good record of the circumstances under which this photograph was taken, and this will make it possible to connect the aspect of *Jupiter* at the time with the spectrum photograph, though I did not examine *Jupiter* with any care through the telescope that night, and indeed did not have my attention attracted to this photograph till some time afterwards.

I send herewith to the Astronomical Society for examination the original negative, which is just as it was produced, except that it has been cemented with Canada balsam to another piece of glass for protection. Attached to the photograph is an explanatory diagram (*see* figure) intended to point out the peculiarities which are of interest. It will be noticed at once that the main dif-



ference is not due to a change in the number or arrangement of the Fraunhofer lines, but rather to a variation in the strength of the background. In the case of the Moon the background is uniform across the width of the spectrum in any region, but in the case of *Jupiter* the background is fainter in the middle of the width of the spectrum in the region above the line *h*, and stronger in the middle in the region below *h*, especially toward F. The observer must not be confused by the dark portion where the two spectra overlap along the middle of the combined photograph.

In order to interpret this photograph it must be understood that the spectrum of *Jupiter* is produced from an image of the planet thrown upon the slit of the spectroscope by a telescope of 183 inches focal length, the slit being placed approximately in the direction of a line joining the poles of the planet. The spectroscope did not, therefore, integrate the light of the whole disk, but analysed a band at right angles to the equator and

extending across the disk. If either absorption or production of light was taking place on that portion of *Jupiter's* surface, there might be a modification in the intensity of the general background of the photograph spectrum.

A casual inspection will satisfy anyone that such modifications in the intensity of the background are readily perceptible in the original negative. They seem to me to point out two things that are occurring: first, an absorption of solar light in the equatorial regions of the planet; and, second, a production of intrinsic light at the same place. We can reconcile these apparently opposing statements by the hypothesis that the temperature of the incandescent substances producing light at the equatorial regions of *Jupiter* did not suffice for the emission of the more refrangible rays, and that there were present materials which can absorb those rays from the sunlight falling on the planet.

If the spectrum photograph exhibited only the absorption phenomenon above *h* the interest attached to it would not be great, because a physicist will readily admit from theoretical considerations that such might be the case owing to the coloured belts of the planet. But the strengthening of the spectrum between *h* and F in the portions answering to the vicinity of the equatorial regions of *Jupiter* bears so directly on the problem of the physical condition of the planet as to incandescence that its importance cannot be overrated.

The circumstances under which this photograph was taken were as follows:—Longitude of Observatory, $4^h\ 55^m\ 29^s{\cdot}7$ W. of Greenwich. Night not very steady. *Jupiter* and the Moon differed but little in altitude. *Jupiter's* spectrum was exposed to the photographic plate for 50 minutes, the Moon was exposed for 10 minutes. *Jupiter* was near the meridian. The photograph of *Jupiter's* spectrum was taken between $9^h\ 55^m$ and $10^h\ 45^m$ New York mean time, September 27, 1879.

I have suspected that perhaps there may have been an influence produced by the great coloured patch on Jupiter which has made itself felt in this photograph. It may be that eruptions of heated gases and vapours of various composition, colour, and intensity of incandescence are taking place on the great planet; and a spot which would not be especially conspicuous from its tint to the eye might readily modify the spectrum in the manner spoken of above.

Central Solar Eclipses in Great Britain during 1,000 years.
By the Rev. S. J. Johnson.

The following list shows all the central eclipses I have been able to find in Great Britain from the thirteenth to the twenty-third century (inclusive), computed by the same approximate method used for the eclipses in the Saxon Chronicle (6th to 12th century). See *Monthly Notices*, April 1873.

		d	h				d	h	
1263	Aug.	5	1½	annular	1737	March	1		annular
1279	April	12	6¼	annular	1748	July	25		annular
1310	Jan.	31	1½	annular	1764	April	1		annular
1330	July	16	3¾	total	1836	May	15		annular
1411	Aug.	18	6	annular	1847	Oct.	9		annular
1433	June	17	3	total	1858	March	15		annular
1502	Sept.	30	18¾	annular	1927	June	[redacted]	17¼	central
1547	Nov.	12	1¼	annular	1999	Aug.	10	22¼	total
1598	Feb.	25		total	2090	Sept.	23	5½	total
1601	Dec.	24	1¼	annular	2093	July	23	0¼	annular
1621	May	20	20¼	annular	2135	Oct.	6	19¾	total
1652	April	8		total	2151	June	14	6½	total
1715	May	2	21	total	2189	Nov.	7	20¼	total
1724	May	22		total	2200	April	14	5¾	central

Continuing the examination to the year A.D. 2500, or 620 years from the present time, no eclipse appears likely to be quite total at Greenwich, the nearest approaches to this being 2151 June 14, and 2381 July 21, especially in the former instance.

Eclipse of 1279 is, by the tables used, annular at London.

1330. This is apparently total in Scotland for a very short time; and in 1339 July 7 at $\frac{1}{2}^{h}$ the annular phase may have touched the North of Scotland.

1411. Probably annular in Ireland and SW. of England.

On May 8, 1491, the central and annular phase would scarcely escape the mainland of Scotland at the extreme North.

1502. Very soon after sunrise.

1547. Very widely annular, and, by the tables used, London was within the track.

1601. Annular right across England. According to these tables, I also obtain an annular phase for Nidiosia (Drontheim) in Norway, where, in the appendix to Tycho Brahé's *Hist. Cel.* it is said to have been so observed. "Sol ita lunare corpus intra sui complexum comprehenderat, ut lux undiquaque ad marginem diffunderetur."

1621. Narrowly annular in England.

On June 10, 1630, the central phase, which must have been

an extremely narrow track, seems only just to have escaped the SW. coast. (Dr. Bainbridge made this eclipse to begin at Oxford at 5·58 and end at 7·48.)

2135. Seems to be total right across England, and deserves a rigorous computation.

2189. This may be total in the SW. of England.

Abbenhall Rectory, Gloucester,
1880, *May* 10.

On the Variability of B.A.C. 2472. By J. Tebbutt, Esq.

My suspicions as to the variability of this star were founded on an attempt to observe its occultation by the Moon on April 22, 1874. In the Occultation List of the *Nautical Almanac*, the star is set down as one of the sixth magnitude. The earliest recorded estimates of magnitude that I have yet been enabled to find are those by Lalande in his catalogue of 47390 stars for 1800. Two independent estimates of this astronomer assign 8 and 8½ as its magnitude. In Taylor's Madras Catalogue for 1835, Robinson's Armagh Catalogue for 1840, and the new Greenwich Nine Year Catalogue, it is put down as of the 6th magnitude, while 6·5 is the estimate in the Washington Catalogue of 10658 stars for 1860. At the present epoch, however, it certainly does not exceed the 8th magnitude. As seen in the 3-inch Transit instrument, it will scarcely bear the faintest illumination of the wires, while B.A.C. 2469, which may be viewed in the same field with it, is distinct under strong illumination. I cannot find that B.A.C. 2472 has ever been suspected as a Variable, although it is situated far north of the equator, and not very distant from several well-known Variables in the constellation *Gemini*.

In conclusion, I may also record my strong suspicions as to the variable character of the star numbered 14571 in Lalande's Catalogue, which is, in fact, only a few minutes of a degree distant from B.A.C. 2472.

Observatory, Windsor, N.S. Wales,
1880, *January* 12.

Diagrams of the position of the Great Southern Comet, as seen Feb. 5–12, from the ship 'Superb,' were received from Mr. Barker, and a communication from Mr. Tebbutt, who saw the head of the Comet, for a few seconds, between clouds, on the evening of Feb. 14; also a set of elements computed by Mr. H. T. Vivian, from Mr. Gill's observations of Feb. 11, 13, 15, and agreeing tolerably well with those computed at Lord Lindsay's Observatory.

Observations of Comet 1880 *b* (Schäberle), *made at Dun Echt Observatory with the Filar Micrometer of the* 15-*inch Refractor*.

Date. 1880.	Dun Echt Mean Time. h m s	☄ − ✱ Δα m s	☄ − ✱ Δδ ′ ″	α☄ h m s	δ☄ ° ′ ″	Observer.
a Apr. 22	11 57 45·3	+1 49·47	− 9 29·0	6 15 39·57	+72 37 42·3	T. G. L.
b 22	12 55 40·8	+3 55·53	2 19·2	6 15 38·46	72 36 4·6	„
a 22	12 55 40·8	+1 48·12	11 5·6	6 15 38·22	72 36 5·7	„
23	10 58 22·9	− 6·81	7 16·3	6 15 17·54	71 58 15·0	R. C.
25	13 51 15·7	+ 11·80	3 34·4	6 14 47·18	70 32 29·2	T. G. L.
May 8	11 6 48·8	+ 3·76	− 1 26·8	6 17 9·16	+62 45 35·8	R. C.

Adopted Mean Places of Comparison Stars for 1880·0.

	α h m s	Reduction. s	δ ° ′ ″	Reduction. ″	
a Apr. 22	6 13 47·79	+2·31	+72 46 56·6	+14·7	Arg.-Oeltz. 6740
b 22	6 11 40·66	2·27	72 38 9·1	14·7	„ 6695
23	6 15 22·08	2·27	72 5 16·8	14·5	$\frac{\text{3 Bonn. Cat.} + 72^\circ \text{ No. } 317 + \text{Arg.-Oeltz. } 6767}{4}$
25	6 14 33·26	2·12	70 35 49·8	13·8	Radcliffe Cat. No. 1707
May 8	6 17 3·72	+1·68	+62 46 52·6	+10·0	Arg.-Oeltz. $\frac{6815+6816}{2}$

Dun Echt Observatory,
1880, *May* 13.

Elements of Schäberle's Comet. By J. R. Hind, F.R.S.

The following elements of the Comet discovered by Schäberle, on April 6, are calculated from an observation at Strasburg, by Professor Winnecke, on April 11, and observations by Major Tupman, on April 27 and May 8; the latter being, no doubt, a very accurate position, from the circumstance of the Comet having passed nearly over a star of the eighth magnitude, which was twice observed in Argelander's Zones; the angle of position changed more than 100° during Major Tupman's comparisons:—

Perihelion Passage, 1880, July, 1·61467 G.M.T.

π	112° 7′ 42″·4	Apparent Equinox April 25
☊	257 13 42·4	
i	56 55 57·3	
Log q	0·2587892	

For the middle observation (C–O)
$\Delta\lambda \cos\beta = -6''·5$
$\Delta\beta = -4''·0$

Retrograde.

The coordinate constants for equinox of June 1, are

$$x = r[9·7605836] \sin(v + 192° 20' 33'')$$
$$y = r[9·9647468] \sin(v + 138\ 53\ 27)$$
$$z = r[9·9563320] \sin(v + 60\ 19\ 11)$$

From the above elements it appears probable that the Comet, after being invisible for some weeks about the perihelion passage, will again come under observation early in August, in the north-east before sun-rise, the theoretical intensity of light increasing from that time until the end of October or beginning of November. According to these elements, the Comet will not arrive at its least distance from the Earth (1·6) until about November 3, when it will be situated south of the belt of *Orion* not far from Rigel—therefore, passing the meridian about $14^h\ 30^m$ and very favourably situated for observation. In the first week of December the intensity of light will have diminished to the same value that it has in the first week in June, but the Comet will be projected on darker sky-ground, and it seems possible that it may be within reach of the telescope until quite the end of the year.

On the Longitude of the Observatory, Windsor, N.S. Wales. By J. Tebbutt, Esq., Director of the Observatory.

I am indebted to Rear-Admiral Rodgers for the Washington corrections to Hansen's places of the Moon corresponding to occultations observed by me in 1876. I have thus been enabled to apply these corrections, in conjunction with those from Greenwich, to the places corresponding to the occultations of Spica and Antares included in my paper sent to the Society in January last, and also to the places corresponding to three other occultation-phases observed here in 1876. I am desirous that the accompanying table should be regarded as supplementary to that in my previous paper. Due regard being paid to the new longitude-corrections from the occultations of Spica and Antares, it will be found that the mean of the twenty-three occultation results is $+6^s{\cdot}11$ and the longitude of my Observatory becomes $10^h\ 3^m\ 21^s{\cdot}81$, E. This value differs very slightly from that previously forwarded to the Society, and I think it is as satisfactory as any that can be derived from the Moon's motions alone. I shall await with interest its confirmation, or otherwise, by telegraphic connection with Europe.

Date of Occultation 1876	Star Occulted	Windsor Mean Time of Observation	Phase of Occultation	Adopted Corrections to Moon's Tabular Place R.A.	N.P.D.	Resulting Correction to Longitude
June 2	Spica	12 34 7.2	D	−0·62	−4·8	+5·08
2	Spica	13 39 5·2	R	−0·62	−4·8	−0·83
July 3	Antares	12 14 41·0	D	−0·63	−2·1	+7·74
3	Antares	13 7 32·9	R	−0·63	−2·1	+3·91
Oct. 28	λ Aquarii	*7 50 21·6	D	−0·48	+3·4	+5·40
28	λ Aquarii	9 12 51·4	R	−0·48	+3·4	−0·61
28	78 Aquarii	9 30 39·7	D	−0·48	+3·4	+0·74

Errata in my communications in the R.A.S. Monthly Notices.

Vol. xxxvi. page 43. *For* entirely complete *read* certainly complete.
„ xxxvi. „ 100. *Insert* 1875 *at the head of the Observations.*
„ xxxviii. „ 11. *Omit* local *before* mean times.
„ xxxviii. „ 331. *For* lower *read* hour.
„ xxxix. „ 322. *For* east *read* west, *and for* south *read* north.

Observatory, Windsor, N. S. Wales,
1880, *March*, 15.

* Noted rather late.

MONTHLY NOTICES

OF THE

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Francis H. S. Orpen, Esq., F.R.G.S., F.R.C.I., Surveyor-General of Griqualand West, South Africa;

Lieut. Sidney Smith, R.N., H.M.S. *Forester*;

Capt. Robert Wright Sterry, 82 Basinghall Street, E.C.; and

Capt. Peter Thompson, Marine Board, Tower Hill, E.C.;

were balloted for and duly elected Fellows of the Society.

On the Determination of the Solar Parallax by Means of the Parallactic Inequality in the Motion of the Moon. By James Campbell, Esq., and E. Neison, Esq. (Conclusion of the Paper in the May No., pp. 386–411.)

The previous portion of this communication has been devoted to the consideration of the general features of the problem of determining the value of the coefficient of the parallactic inequality, and to the critical examination of the results which have been obtained by previous investigators. The present portion contains a new discussion of the Greenwich Meridian observations of the Moon made during the fifteen years between 1862 and 1876. During this period the Greenwich observations have been systematically reduced and compared with the tabular places of the Moon derived from Hansen's *Tables de la Lune*, and

they form a mass of data far superior to any which have hitherto been employed for the determination of the value of the coefficient of the parallactic inequality.

As a first approximation these observations will be reduced in the manner employed by Mr. Stone and Prof. Newcomb for determining the value of this coefficient, and this step will serve to show the real nature of the assumptions made by them as the basis of their investigations.

Let

δL = the error in the mean longitude of the Moon at a fixed epoch,

E = the error in the tabular longitude of the Moon found by subtracting the observed place from the place assigned by Hansen's tables.

In accordance with the assumption made by Mr. Stone and Prof. Newcomb, that the errors in the tables might be regarded as due to errors in the mean longitude and errors in the parallactic inequality, every observation will furnish an equation of the form

$$\delta L + \delta P \sin \theta = E.$$

Then, in conformity with the same method, every observation was selected for which the value of $\sin \theta$ exceeded 0·90, this being equivalent to taking observations within two days of the first and third Quarters of the Moon. The observations were divided into yearly groups, each group commencing in the October of one year and ending in the October of the next, this division being adopted so as to eliminate as far as possible any discordances due to errors in the annual equation. For each group δL would represent the error of the mean longitude of the Moon on March 31st.

First, all the observations for which $\sin \theta$ was positive were added together and the mean taken. If e' denotes the mean of the errors of the tables and a' the mean of the values of $\sin \theta$, there will result by this means an expression of the form

$$\delta L + a'\delta P = e'.$$

Treating the observations for which $\sin \theta$ was negative in the same manner, and putting e''' and a''' for the corresponding values denoted by e' and a' at the first Quarter, an expression is obtained of the form

$$\delta L - a'''\delta P = e'''.$$

Therefore, eliminating δL, and dividing by the coefficient of δP.

$$\delta P = \frac{e' - e'''}{a' + a'''}.$$

As no correction has been introduced for the variation in the semi-diameter, this result will be really $\delta(P)$, the correction to the apparent coefficient.

In this manner a value for $\delta(P)$ was obtained for each yearly group. These values are

Year.	Number of Observations.	Value of $\delta(P)$.	Difference from the Mean.
1876–1875	35	$\delta(P) = -2''.39$	$-0''.71$
1875–1874	35	$= -1.26$	$+0.42$
1874–1873	52	$= -0.89$	$+0.79$
1873–1872	31	$= -1.00$	$+0.68$
1872–1871	40	$= -1.66$	$+0.02$
1871–1870	38	$= -1.66$	$+0.02$
1870–1869	31	$= -1.97$	-0.29
1869–1868	35	$= -1.42$	$+0.26$
1868–1867	47	$= -2.26$	-0.58
1867–1866	38	$= -1.81$	-0.13
1866–1865	36	$= -1.51$	$+0.17$
1865–1864	37	$= -2.32$	-0.64
1864–1863	38	$= -1.98$	-0.30
1863–1862	34	$= -1.64$	$+0.04$

These results are moderately accordant, the greatest difference from the mean being less than four-fifths of a second, or less than a moiety of the correction, and the mean of all the discordances is only one-third of a second. Dividing the observations into two seven-year groups, in order to detect any systematic variation, the results are:—

From 265 observations during the period 1862–1869

$$\delta(P) = -1''.85 \pm 0''.091.$$

From 262 observations during the period 1869–1876

$$\delta(P) = -1''.50 \pm 0''.138.$$

These results require to be corrected for any error in the value adopted by Hansen for the horizontal semi-diameter of the Moon. This must be determined from the observation of the transits of both limbs made when the Moon is Full. During the period 1862–1877 there have been 38 such observations made, but five of these were observed by computers only occasionally engaged in observation, and are liable to unknown errors. These five must therefore be rejected, and only those observations employed which were made by the astronomers who systematically observe the Moon. Put δS for the excess of the tabular over the observed semi-diameter; then from 22 accordant observations during the period 1862–1869

$$\delta S = +0''.37 \pm 0''.10.$$

Next, from 11 accordant observations during the period 1869–1877

$$\delta S = +0''.10 \pm 0''.08.$$

These corrections are to be applied as they stand to the values obtained above for $\delta(P)$, or the apparent value of the parallactic inequality by the formula

$$(P)+\delta(P)+\delta S = P_1 = -126''.46.$$

Accordingly we have:—For the group 1862–1869

$$(P) = -124''.98 \pm 0''.14.$$

For the group 1869–1876

$$(P) = -125''.08 \pm 0''.16.$$

The two results are beautifully accordant.

Taking, therefore, the entire group in one, we have from 527 observations during 1862–1876

$$(P) = -125''.06 \pm 0''.10.$$

This result does not seem to harmonise with the value found by Prof. Newcomb, which was

$$(P) = -124''.41;$$

but it must be remembered that this result obtained by Prof. Newcomb was founded on the observations made during the years 1862–1865 only, and without applying any correction to Hansen's semi-diameter. From the same data the results obtained above would yield the value

$$(P) = -124''.48,$$

in perfect accord with Prof. Newcomb.

On comparing the above result with that deduced from the observations given by Sir G. B. Airy, which on the hypothesis of a forty-five-year inequality is

$$(P) = -124''.11,$$

it will be seen that they differ by $0''.95$. But if this inequality really exists, and does not arise from errors in Airy's tables which do not exist in Hansen's tables, the value found above will require correction for the effect of this inequality. The true value will be therefore

$$(P) = -125''.06 + 0.80B.$$

If for B be substituted its approximate value $B=1''.20$, we obtain the result

$$(P) = -124''.00,$$

a most remarkable result.

In what precedes we have been engaged in determining the value of the apparent coefficient, (P), but the end in view is really the determination of the true coefficient, P, so as to be able to determine the value of the solar parallax. For this purpose it is necessary to take some steps to determine the difference between the two, arising from the variation in the apparent semi-diameter of the Moon due to difference in the amount of contrast between the lunar limb and the sky.

Let

δI = the difference in the irradiation of the limb of the Full Moon and at its Quarters when seen in bright twilight.

Then the differences in the amount of irradiation at the Moon's limb under different amount of contrast may be expressed by supposing δI to be multiplied by a variable coefficient, i, the value of i being zero for the Full Moon, and gradually increasing up to 1·0 for the time of the Quarter, when the Moon is observed in bright twilight, and up to 1·5 when the Moon is observed in bright daylight. This coefficient i will therefore be a function of the Moon's age or time it transits, and the time of year or brightness of the sky at any hour of the day. The variation in this coefficient is necessarily a little arbitrary, but this is not very important where, as in the present case, the value of δI is to be determined from the observations themselves. The rate of variation which was actually assumed cannot differ much from that assumed by Prof. Newcomb in his memoir on the "Corrections to Hansen's Table," Washington, 1876.

Every observation of the Moon now furnished an equation of the form

$$\delta L + i\delta I + \delta P \sin \theta = E.$$

These equations were now treated exactly as before, but each yearly group was now divided into two portions, a Winter and a Summer portion. In the former the observations were nearly all made on a dark sky, when i was either zero or very small, so that the amount of irradiation was nearly constantly at its maximum. In the latter, the observations were mostly made on a bright twilight sky, and many in broad daylight, when the irradiation was much reduced. By comparing the values found for the parallactic inequality under these different conditions, it would be possible to determine the effect of the change in semi-diameter due to the alteration in the amount of irradiation. By grouping the observations from October to March together, the effect of any error in the annual equation would be materially diminished, as it would be negative from October to December, and positive from January to March, and so disappear in the sum of the two. The same elimination of any error in the annual equation would occur in the summer portion, April to October.

It was found that the mean value of i might approximately be taken as 0·75 in the summer group, and 0·25 in the winter group. These values were adopted.

The results obtained are as follows:—

SUMMER II. DIVISION.

Group.	No. of Observations.	Resulting Value.	Difference from Mean = −2″·15
1876	21	$0{\cdot}75\,\delta I + \delta P$ = −0″·03	+2″·12
1875	22	= −1·79	+ ·36
1874	31	= −2·81	− ·66
1873	17	= −1·98	+ ·17
1872	17	= −3·32	−1·17
1871	16	= −3·05	− ·90
1870	16	= −2·08	+ ·07

SUMMER I. DIVISION.

Group.	No. of Observations.	Resulting Value.	Difference from Mean = −1″·84
1869	15	$0{\cdot}75\,\delta I + \delta P$ = −0″·52	+1″·32
1868	29	= −1·56	+ ·28
1867	18	= −2·72	− ·88
1866	18	= −2·08	− ·24
1865	16	= −2·72	− ·88
1864	20	= −2·25	− ·41
1863	13	= −0·99	+ ·85

WINTER II. DIVISION.

Group.	No. of Observations.	Resulting Value.	Difference from Mean = −0″·93
1876–1875	14	$0{\cdot}25\,\delta I + \delta P$ = −4′·72	−3″·79
1875–1874	13	= −0·73	+ ·20
1874–1873	21	= +1·02	+1·95
1873–1872	14	= −0·01	+ ·92
1872–1871	23	= +0·05	+ ·98
1871–1870	22	= −0·23	+ ·70
1870–1869	15	= −1·86	− ·93

WINTER I. DIVISION.

Group.	No. of Observations.	Resulting Value.	Difference from Mean = −1″·87
1869–1868	20	$0{\cdot}25\,\delta I + \delta P$ = −2″·35	− ″·48
1868–1867	18	= −3·02	−1·15
1867–1866	20	= −0·90	+ ·97
1866–1865	18	= −0·95	+ ·92
1865–1864	21	= −1·93	− ·06
1864–1863	18	= −1·72	+ ·15
1863–1862	21	= −2·25	− ·38

Treating the deviations as due to accidental errors of observation, these results yield the following values :—

Summer Division II. 140 observations $0.75\,\delta I + \delta P = -2''.15 \pm 0''.28$,
Summer Division I. 129 observations $0.75\,\delta I + \delta P = -1''.84 \pm 0''.21$,

or uniting the two

Summer 269 observations $0.75\,\delta I + \delta P = -2''.00 \pm 0''.17$.

Next,

Winter Division II. 122 observations $0.25\,\delta I + \delta P = -0''.93 \pm 0''.47$,
Winter Division I. 136 observations $0.25\,\delta I + \delta P = -1''.87 \pm 0''.16$,

or uniting the two

Winter 258 observations $0.25\,\delta I + \delta P = -1''.40 \pm 0''.26$.

Considering the total groups first, by subtracting the winter results from the summer results we have

$$+0.50\,\delta I = -0''.60 \pm 0''.32,$$

or

$$\delta I = -1''.20 \pm 0''.64.$$

That is to say, the Moon's semi-diameter is decreased in bright twilight by $1''.2$ from its value at Full Moon, when it is seen against a black sky. Using this value, we obtain for the value of the true coefficient of the parallactic inequality,

$$\begin{aligned} P &= (P) + 0.50\,\delta I, \\ &= -125''.06 - 0''.60 \\ &= -125.66 \pm 0.34. \end{aligned}$$

It is obvious, however, that the large probable error for the value found for δI, renders this result most unsatisfactory. Considering the two divisions into which the summer and winter results are each divided, we obtain the values

For the II. Divisions $\delta I = -2''.44 \pm 1''.10$,
For the I. Divisions $\delta I = +0.06 \pm 0.54$.

It is evident that there is a systematic difference arising from some cause which is not apparent.

The uncertainty of the value found for δI is best seen by eliminating in the preceding manner the value of δP from each yearly group, and determining the value of δI from the resulting equation. The results are

Year.	Value of δI.
1876–1875	$\delta I = +11''.92$
1875–1874	$= -2.76$
1874–1873	$= -7.96$
1873–1872	$= -3.61$
1872–1871	$= -5.22$
1871–1870	$= -4.75$
1870–1869	$= -0.56$
1869–1868	$= +4.35$
1868–1867	$= +5.06$
1867–1866	$= -4.03$
1866–1865	$= -1.88$
1865–1864	$= -1.68$
1864–1863	$= -0.74$
1863–1862	$= -2.22$

These results are extremely discordant; neglecting the first value, we obtain from the rest the result

$$\delta I = -1''.66 \pm 0''.69.$$

Including all, the result is

$$\delta I = -0''.68 \pm 0''.92.$$

It is obvious that any results based on such wild values must be fallacious. It shows the misleading character of values obtained from the means of large numbers of observations without examining the separate results to see if they are really accordant.

The result of the preceding investigation might be regarded as discouraging and as showing that it would not be practicable to determine in this manner the effect of the variation in the amount of irradiation at the limb of the Moon. If this were really the case, it would be most unsatisfactory and throw grave doubt on the possibility of determining the true value of the parallactic inequality from observations of the transit of the lunar limb. For the only permissible method of ascertaining the amount of this variation in the irradiation is by this process of comparing the summer with the winter observations. If this prove to be impracticable, it involves as a necessary consequence the failure of the whole method of determining the solar parallax from these observations.

Further consideration shows that these results are too discordant for this view to be adopted. These extreme discordances can only arise from systematic errors due to imperfections in the tables, or method of reducing the observations. These discordances must be due to the failure of the assumption, which

we have adopted, in common with Mr. Stone and Prof. Newcomb, and taken as the basis of our work—the assumption that in the mean of a number of observations all errors except those of the parallactic inequality can be neglected and be trusted to destroy one another.

To determine a more trustworthy value of the coefficient of the parallactic inequality a more elaborate investigation must be made. We proceed, therefore, to this.

Let Hansen's tables be assumed to be affected by the following errors:—

δL = an error in the mean longitude at a given epoch,

δl = an error in the motion of the mean longitude,

δP = an error in the coefficient of the parallactic inequality,

δV = an error in the coefficient of the variation,

δM = an error in the coefficient of the annual equation,

δA = an error in the coefficient of the term with the argument $4\theta - 2\beta$.

Further, suppose Hansen's tables imperfect from not containing terms of the following nature:—

$\delta K \sin k$ = a term with the argument $k = \alpha + 2A - 2J$,

$\delta F \sin f$ = a term with the argument $f = 2\theta - \alpha + \mu - N$,

$\delta F' \sin f'$ = a term with the argument $f' = \alpha - \mu + N$,

where

A = mean longitude of perigee of the lunar orbit,

J = mean longitude of Jupiter,

N = an argument which increases only a few degrees per annum.

Lastly put

$i\delta I$ = the variation in the amount of irradiation at the limb of the Moon,

$t\delta T$ = the effect of the systematic errors in the Greenwich clock star places,

$s\delta S$ = the apparent error of Hansen's value of the horizontal semidiameter of the Moon,

and

E = apparent error of Hansen's tables.

Then every observation will give an equation of the form

$$\delta L + l\delta l + \delta P \sin p + \delta V \sin v + \delta M \sin \mu + \delta A \sin \alpha + \delta K \sin k + \delta F \sin f + \delta F' \sin f' + i\delta I + t\delta T + s\delta S = E,$$

a system of equations with twelve unknown quantities, or rather variable corrections.

Although in this manner the labour required by the investigation is enormously increased, it has the advantage of furnishing results in which some confidence may be felt, and derived by a process having some pretence to refinement and precision.

The term

$$\delta A \sin a = \delta A \sin (4\theta - 2\beta)$$

introduces a correction necessitated by an error contained in Hansen's tables owing to which the term

$$+0.335 \sin (4\theta - 2\beta)$$

has been employed instead of the correct term

$$-0''.285 \sin (4\theta - 2\beta).$$

It follows, therefore, that every one of the tabular places given by Hansen's tables requires the correction

$$-0''.62 \sin (4\theta - 2\beta).$$

The necessity for this correction admits of no dispute, and was pointed out by Hansen in a note in his *Darlegung*. Its actual value has been applied to every observation with the effect of markedly improving the accord between the separate observations. As far as the results in the present investigation are concerned, the effect of this correction proves to be trifling.

For the moment passing over the new terms to be introduced, it will be well to consider the effect of the other three corrections.

The first of these is that denoted by

$$i\delta I,$$

which allows for the variation in the amount of irradiation at the limb. The manner in which the coefficient i of this term is supposed to vary has been already explained. The term has been introduced as an indeterminate correction to every observation.

The second of these corrections is that denoted by

$$t\delta T,$$

and arising from the systematic errors in the places of the clock stars used at Greenwich in determining the place of the Moon. The method of applying this correction is as follows. In the Introduction to the Greenwich Nine Year Catalogue is to be found the list of the corrections required by the Right Ascensions of the Greenwich clock stars. From the different volumes of Greenwich Observations it would be possible to ascertain the stars from which the assumed clock-error was derived which was used in each observation, and thus ascertain the exact correction necessary. This would be a work of enormous labour to gain a very small result, for it is only the systematic effect of these errors which need be considered. A sufficiently accurate result was obtained, therefore, by the following method. An examination was made of the actual stars employed for each

observation of the Moon made in the years 1864, 1870, 1877, and it was assumed that the method employed in these years would be also the method employed in the intervening years. It was found that the clock-error used in determining the place of the Moon was derived as a rule mainly from stars which transit within two or three hours of the time of the transit of the Moon. In fact, it was found that the mean place of the Moon for a lunation would be affected by an error closely approximating to the mean error of the clock stars transiting within four hours on either side of ten o'clock. In the summer the stars are within closer and in the winter within rather wider limits than these. Then from the known systematic errors of the clock stars transiting within this period a correction could be found with some approximation to be applied to the mean error of the Moon's place. This only served for correcting the errors of the lunar tables of long period. The effect of these errors on the terms of short period will be insensible, except for terms like the parallactic inequality, whose arguments have always the same value when they transit at the same hour, and, consequently, in which these errors accumulate. The only term really needing correction is the parallactic inequality. The examination of the observations made during 1864, 1870, and 1877, showed that the clock-errors used for determining the place of the Moon at the transit of the limb at the time of first Quarter were derived from stars which mainly transited six hours earlier than the stars from which was derived the clock-error used for observations of the Moon at the time of third Quarter. The interval was found to be rather less in summer and rather more in winter. Therefore, if during this interval the systematic errors in the Right Ascensions varied, the effect of this variation would be thrown on the apparent value of the parallactic inequality; knowing the amount of the systematic errors in the places of the clock stars, it was easy to calculate their effect on the parallactic inequality, on the annual equation, on the motion in mean longitude, and on the error in mean longitude.

During the years 1862 to 1869, the star places used were derived from the First Seven Year Catalogue, and the values of the systematic errors of this catalogue give the corrections

$$\begin{aligned}\delta L &= +0''.15\\ \delta(l) &= -0.62\\ \delta M &= +0.15\\ \delta P &= +0.06\end{aligned}$$

During the years 1870—1877, the Second Seven Year Catalogue was used, and the corrections are:—

$$\begin{aligned}\delta L &= +0''.06\\ \delta(l) &= -0.33\\ \delta M &= +0.11\\ \delta P &= +0.04\end{aligned}$$

These are the quantities which must be added to those derived from the observations in order to correct the observed values for the effect of these systematic errors in the places of the Greenwich clock stars. It will be observed that the most important effect is produced on the apparent motion of the mean longitude of the Moon, an effect amounting to nearly two-thirds of a second during the earlier period. This correction is distinguished as $\delta(l)$, for it is not truly a correction to the motion in mean longitude, but is really a periodical yearly inequality affecting it, so that, though it will be confounded with the apparent motion in mean longitude when this is derived separately for each year, it will vanish from it if the correction to the motion in mean longitude be derived from two or more years combined; but in this case the observations must be corrected for this inequality separately. The effect on the parallactic inequality is insignificant when the observations of a whole year are summed. It is, however, necessary to determine the effect for each quarter of the year, in order to ascertain whether they give rise to different errors in summer and winter. The result is as follows:—

Mean Period for which the correction is derived.	First Seven Year Catalogue.	Second Seven Year Catalogue.
January, February, March,	$\delta P = -0''.09$	$\delta P = -0''.05$
April, May, June,	$= -0.09$	$= -0.08$
July, August, September,	$= -0.35$	$= -0.18$
October, November, December,	$= +0.77$	$= +0.47$

Consequently

Correction to the mean Winter observations	$= +0''.34$	$= +0''.21$
Correction to the mean Summer observations	$= -0.22$	$= -0.13$

The very important correction due to this cause is apparent, and it will give rise to differences in the apparent value of the variation in the irradiation amounting to the difference between the above values multiplied by two, or

$$\delta I = -1''.12 \qquad = -0''.68$$

The values thus found for these corrections may be regarded as close approximations to the truth, as making any permissible variations in the assumed basis of the investigation will not alter the value found for the correction by more than a very small amount.

In dealing with these corrections, it is proposed to apply them to the final result for each quarter, and not to the separate observations. This method will effect considerable saving in labour, and give results sensibly as correct.

The third correction to be considered is that denoted by

$$s\delta S$$

the correction to the assumed semi-diameter. This is scarcely a correction to the tables, as every Observatory will have a systematically different value for the apparent semi-diameter, and the correction, like the last two, is better applied to the observed place than to the tabular place of the Moon. The correction for the assumed error in the tabular place of the Moon will really only affect the parallactic inequality, and will be applied to the results, and not to the separate observations themselves. No sensible error can be introduced by this means. The value of δS adopted will be taken from the observed times of transit of the Moon's diameter when both limbs are observed at Full. The corrections are

From 11 observations between	1862 and 1865	δS =	$+0''.42$
„ 11 „	1866 „ 1869	=	$+0.33$
„ 11 „	1870 „ 1877	=	$+0.10$

The effect of personality in the transit diameter of the Moon seems to have been over-estimated. The results for the principal Greenwich observers are as follows:—

Dunkin	4 observations	δS =	$+0''.09$
Ellis	7 „	=	$+0.10$
Criswick	11 „	=	$+0.25$
Lynn	3 „	=	$+0.22$
J. Carpenter	3 „	=	$+0.92$
Thackeray	2 „	=	$+0.75$

The two last are considerably greater that the others, it is true, but the results rest on so very few observations—three and two—that these differences may be said to be accidental. The actual effects on the observed values of the inequalities amount of course to only a small fractional part of these differences.

The principal new term to be introduced is that denoted by

$$\delta K \sin k.$$

It is the inequality due to the direct perturbing action of *Jupiter*, and corresponding to the inequality arising from the action of the Sun known as the Evection. It may be termed, therefore, the Jovian Evection. It is an inequality which has a most important effect on the apparent value of the parallactic inequality, though now for the first time taken into consideration.

This term was discovered in the year 1876 by Prof. Newcomb

as an empirical inequality existing in the mean longitude of the Moon, and arising from an unknown source. Its origin was identified by Mr. Neison, and its value independently calculated by him. Although its existence has long been known, its value seems to have first been calculated by Prof. Hansen, who published his results in his *Darlegung*. He found it to possess a very small coefficient, and, regarding it as insensible, did not include it in his tables. Its true value seems to have been first discovered and made known by the Rev. W. E. Penny, and to this gentleman belongs all the credit attaching to priority. Although the first to be made known, it is not the only term with a sensible coefficient arising from the action of the planets which do not find the place they ought in Hansen's tables. In this respect the lunar tables are decidedly faulty.

The argument of this term can be written in the form

$$k = a + 20^\circ\!\cdot\!655[\mathrm{Y} - 1873{\cdot}25],$$

so that it may be regarded as a long inequality—with a period of 18 years—in the eccentricity and longitude of the perigee of the lunar orbit. It was in this form that it was discovered by Prof. Newcomb. But it also takes the form of an apparent inequality in the mean longitude of the Moon, and therefore of a yearly inequality in the value of the parallactic inequality. For

$$a = l - \mathrm{A} = l - 40^\circ\!\cdot\!675[\mathrm{Y} - 1865{\cdot}25],$$

or

$$k = l - 20^\circ\!\cdot\!020[\mathrm{Y} - 1875{\cdot}00].$$

Now, the argument of the parallactic inequality may be written in the form

$$\theta = l - (\mu + \mathrm{A}) = l - \mu - 281^\circ\!\cdot\!2;$$

consequently by substitution

$$k = \theta + \mu - 20^\circ\!\cdot\!020[\mathrm{Y} - 1872{\cdot}05].$$

This term may now be regarded as an inequality in the value of the parallactic inequality with the value

$$\delta\mathrm{P} = -\delta k \cos\{\mu - 20^\circ\!\cdot\!020[\mathrm{Y} - 1872{\cdot}05]\}.$$

When the results for a whole year are taken, and the observations are equally distributed over the lunation, the effects of this term will vanish as the positive and negative values will destroy

each other. In practice they will never quite destroy each other, as the observation can only be equally spread over two-thirds of the lunation, so that for some years this term will make the parallactic inequality slightly too large, and for another similar period slightly too small.

But when the observations are divided into half-yearly groups, this will no longer be the case. The value of μ may be taken at 0° at the beginning of the year, and uniformly increased nearly 1° per day, until it becomes 360° on December 31. Now, in 1872 the value of the second term may be considered as zero. Consequently from October to March the argument of the inequality in the value of the parallactic inequality will be between 270° and 90°, or the inequality will be constantly positive, and will make the value of the parallactic inequality too large by $0{\cdot}70\,\delta k$; but from April to September exactly the reverse will happen, and the parallactic inequality be $0{\cdot}70\,\delta k$ too small. When, therefore, these two values are subtracted to obtain the supposed effect of the variation in semi-diameter, the variation will be too small by $1{\cdot}40\,\delta k$, a quantity of more than $2''{\cdot}0$. On the other hand, in the year 1863, when the slowly varying factor has increased by 180°, these things will be exactly reversed. There will result, therefore, a periodical variation in the winter and summer values of the parallactic inequality, with a period of eighteen years; and consequently a similar inequality will appear in the value of the irradiation of the limb. The importance of this term, and the fallacy of assuming that the errors of the lunar tables must eliminate themselves in the mean results of a few years, are thus strikingly illustrated.

There only remains to be considered the pair of terms given under the form

$$\delta F \sin f = \delta F \sin \{2\theta - a + \mu - N\},$$
$$\delta F' \sin f' = \delta F' \sin \{a - \mu + N\};$$

where N is supposed to be an argument of long period increasing by only a few degrees per annum, or else differing in its period from that of the annual equation by only a few degrees per annum.

These two terms are intimately connected with each other and with the parallactic inequality. For the arguments of these terms may be written in the form

$$f = (2l - 2l') - (l - A) + (l' - A') - N = (l - l') + (A - A') - N = \theta + (A - A' - N),$$
$$f' = (l - A) + l' - A') - N \qquad = (l - l') - (A - A') + N = \theta - (A - A' - N).$$

Therefore, putting for brevity

$$f'' = A - A' - N,$$

these three terms may be written in the form

$$\delta P \sin p = \delta P \sin \{\theta\}$$
$$\delta F \sin f = \delta F' \sin \{\theta + f''\}$$
$$\delta F' \sin f' = \delta F' \sin \{\theta - f''\}.$$

Consequently,

$$\delta P \sin p + \delta F \sin f + \delta F' \sin f'$$
$$= \{\delta P + (\delta F + \delta F') \cos f''\} \sin \theta + \{(\delta F - \delta F') \sin f''\} \cos \theta.$$

When, therefore, the coefficient of the argument $\sin \theta$ is determined from the observations, instead of obtaining P, the correction to the parallactic inequality, there will be obtained the quantity

$$\delta P + (\delta F + \delta F') \cos f''.$$

Consequently, if f'' be an argument of long period, there will arise in the value of δP a variation with a period equal to that of f'', but this variation will not affect in any sensible manner the difference between the winter and summer values of the parallactic inequality, or, therefore, the correction for the variation in irradiation. If f'' be an argument of shorter period and differing little from the period of the annual equation, so that $f''=\mu \pm f'''$—where f''' is an argument of long period—then it will give rise to no sensible variation in the annual values for the correction δP, but will give rise to systematic differences between the winter and summer values, and so produce an apparent inequality of the long period f''' in the value found for the variation in the irradiation.

There exist a number of these terms in the analytical expression for the true longitude of the Moon, all due to the perturbing action of the planets, and many having coefficients which may prove to be of sensible amount. It is obvious, however, that there will be no need to introduce these terms into the equations for the separate observations, when it is intended merely to determine the value of the correction to the parallactic inequality, for it will be quite sufficient to apply to the value found the proper corrections arising from these terms. The application of these corrections must be deferred, moreover, until the values of these coefficients have been more accurately determined.

The principal term in the equation of the centre, or the term with the argument α, will be a term in which any error in the coefficient will be thrown as a systematic variation on the value found for the irradiation. For the argument of this term can be written in the form

$$\alpha = \theta + \mu - (A - A').$$

Strictly, therefore, a correction should be applied to the observation for the effect of any error in this term. But we have already introduced a term with a very similar argument—namely,

$$k = \theta + \mu + (A + A' - 2J),$$

which does not differ much, as

$$A - 2J = -\frac{1}{2}A$$

very nearly. If, therefore, we employ for the coefficient of the argument k the value resulting from observation, it will embody in it very nearly the combined effect of the proper coefficient of the term with the argument k and the correction required by any error in the value of the equation of the centre. If, therefore, this coefficient be derived from observation and substituted in the equation to δP, it will serve, not only to correct the value for the missing term $\delta K \sin k$, but for any error in the value assigned by Hansen to the lunar eccentricity and longitude of perigee. On the other hand, the coefficient δK will not be constant from year to year. The same remark applies likewise to terms having very nearly the same period as the variation and annual equation.

The entire number of nearly 600 observations was now reduced to the form given above and was divided into yearly groups in the same manner as before. Each group was in turn divided into a summer and winter group, with the view of enabling the effect of the variation in irradiation to be determined. The resulting equations were then summed up in the usual manner, and after eliminating δL, the error in mean longitude, the value of δP was determined in terms of the errors of the tables and the indeterminate corrections δV, δM, δK, δl, δI, and the known corrections derived from δA, δT, δS. The results are given below.

In the expressions for δP, the first term is that due solely to the error of the tables; then follows the correction to this quantity required by the error in the term $\delta A \sin a$, and expressed in seconds of arc, so that (δA) is to be regarded as unity; next in order follow the corrections due to the terms $t\delta T$ and $s\delta S$, also expressed in seconds of arc, so that in these terms likewise (δT) and (δS) are to be considered unity. After these follow the corrections depending on δV, δM, δK, δI, δl. It must be remembered that though the coefficients of these indeterminate quantities may seem small, they multiply quantities which may exceed $2''\cdot 0$.

In the expressions for δI, a similar arrangement is followed.

SUMMER II. DIVISION.

1876	21 Obs.	δP = −0″·03	−0″·10 (δA)	−0″·12 (δT)	+0·10 (δS)	+0·04 δV	−0·29 δM	−0·34 δK	−0·73 δI	−0·11 δl
1875	22	= −1·79	−0·06	−0·12	+0·10	+0·27	−0·11	−0·15	−0·66	−0·02
1874	31	= −2·81	+0·10	−0·12	+0·10	+0·07	−0·17	+0·02	−0·78	−0·17
1873	17	= −1·98	−0·12	−0·14	+0·10	+0·09	−0·28	+0·35	−0·76	−0·10
1872	17	= −3·32	+0·06	−0·13	+0·10	+0·23	−0·27	+0·65	−0·77	−0·06
1871	16	= −3·05	−0·03	−0·15	+0·10	+0·10	−0·06	+0·74	−0·82	−0·08
1870	16	= −2·08	−0·02	−0·14	+0·10	+0·40	−0·07	+0·64	−0·70	−0·01
		Mean = −2·15	−0·02	−0·13	+0·10	+0·17	−0·18	+0·27	−0·75	−0·08

SUMMER I. DIVISION.

1869	15	= −0·52	+0·23	−0·30	+0·33	+0·25	−0·41	+0·27	−0·58	−0·13
1868	29	= −1·56	+0·14	−0·24	+0·33	+0·16	−0·13	+0·22	−0·66	−0·10
1867	18	= −2·72	+0·04	−0·25	+0·33	+0·17	−0·04	+0·05	−0·65	+0·02
1866	18	= −2·08	−0·16	−0·21	+0·33	−0·03	−0·33	+0·02	−0·79	−0·14
1865	16	= −2·72	+0·02	−0·22	+0·42	+0·18	−0·06	+0·02	−0·75	−0·02
1864	20	= −2·25	−0·14	−0·23	+0·42	−0·05	−0·14	−0·51	−0·78	−0·14
1863	13	= −0·99	+0·19	−0·23	+0·42	+0·22	−0·37	−0·34	−0·75	−0·21
		Mean = −1·84	+0·05	−0·24	+0·37	+0·13	−0·21	−0·05	−0·71	−0·10

WINTER II. DIVISION.

1876–1875	14 Obs.	δP = −4″·72	+0″·33 (δA)	+0″·25 (δT)	+0·10 (δS)	+0·00 δV	−0·16 δM	+0·34 δK	−0·32 δI	−0·35 δl
1875–1874	13	= −0·73	+0·23	+0·23	+0·10	−0·07	−0·01	−0·23	−0·29	−0·10
1874–1873	21	= +1·02	−0·04	+0·17	+0·10	−0·00	−0·17	−0·40	−0·29	−0·17
1873–1872	14	= −0·01	+0·02	+0·25	+0·10	+0·10	−0·44	−0·43	−0·21	−0·22
1872–1871	23	= +0·05	+0·08	+0·17	+0·10	+0·03	−0·07	−0·78	−0·15	+0·00
1871–1870	22	= −0·23	−0·14	+0·33	+0·10	+0·03	−0·07	−0·53	−0·24	−0·09
1870–1869	15	= −1·86	+0·08	+0·50	+0·10	−0·10	+0·20	−0·77	−0·32	+0·03
		Mean = −0·93	+0·08	+0·27	+0·10	−0·00	−0·07	−0·40	−0·26	−0·13

WINTER I. DIVISION.

1869–1868	20	= −2·35	−0·13	+0·34	+0·33	+0·22	−0·13	−0·58	−0·16	−0·10
1868–1867	18	= −3·02	+0·03	+0·34	+0·33	+0·18	−0·09	−0·24	−0·37	−0·05
1867–1866	20	= −0·90	−0·03	+0·47	+0·33	+0·13	+0·14	−0·24	−0·20	−0·25
1866–1865	18	= −0·95	+0·04	+0·39	+0·33	+0·27	−0·15	−0·10	−0·21	−0·03
1865–1864	21	= −1·93	+0·09	+0·40	+0·42	−0·10	+0·08	+0·30	−0·27	+0·03
1864–1863	18	= −1·72	−0·01	+0·34	+0·42	+0·10	−0·16	+0·58	−0·13	+0·13
1863–1862	21	= −2·25	−0·06	+0·46	+0·42	+0·19	−0·35	+0·49	−0·19	−0·36
		Mean = −1·87	−0·01	+0·39	+0·37	+0·14	−0·07	+0·03	−0·22	−0·09

IRRADIATION II. DIVISION.

1876–1875	35 Obs.	δI = + 11″·92	− 1″·09 (δA)	− 0″·96 (δT)	+ 0·09 δV	− 0·32 δM	− 1·73 δK	+ 0·70 δl
1875–1874	35	= − 2·76	− 0·85	− 0·94	+ 0·88	− 0·37	+ 0·20	+ 0·20
1874–1873	52	= − 7·96	+ 0·30	− 0·62	+ 0·14	+ 0·06	+ 0·87	+ 0·02
1873–1872	31	= − 3·61	− 0·27	− 0·74	− 0·04	− 0·08	+ 1·40	+ 0·22
1872–1871	40	= − 5·22	− 0·03	− 0·50	+ 0·30	− 0·33	+ 2·29	− 0·10
1871–1870	38	= − 4·75	+ 0·19	− 0·80	+ 0·13	+ 0·03	+ 2·12	+ 0·02
1870–1869	31	= − 0·56	− 0·27	− 1·76	+ 1·37	− 0·72	+ 3·82	− 0·12
		Mean = − 1·85	− 0·29	− 0·90	+ 0·42	− 0·25	+ 1·29	+ 0·14

IRRADIATION I. DIVISION.

1869–1868	35	= + 4·35	+ 0·86	− 1·60	+ 0·04	− 0·61	+ 2·05	+ 0·09
1868–1867	47	= + 5·06	+ 0·40	− 2·20	− 0·07	− 0·12	+ 1·67	− 0·20
1867–1866	38	= − 4·03	+ 0·16	− 1·74	+ 0·09	− 0·41	+ 0·66	+ 0·66
1866–1865	36	= − 1·88	− 0·33	− 1·06	− 0·51	− 0·39	+ 0·21	− 0·20
1865–1864	37	= − 1·68	− 0·15	− 1·36	+ 0·60	− 0·32	− 0·59	− 0·12
1864–1863	38	= − 0·74	− 0·20	− 0·84	− 0·24	− 0·19	− 1·64	− 0·42
1863–1862	34	= + 2·22	+ 0·44	− 1·26	+ 0·04	− 0·04	− 2·01	+ 0·26
		Mean = + 0·47	+ 0·17	− 1·44	− 0·01	− 0·30	+ 0·05	− 0·02

In the equations for determining δI, if each year be given equal weight as in the preceding, it is obvious that the years for which the coefficient of δI is least when the winter values of δP are subtracted from the summer values will have greater weight really assigned to them than those for which the coefficient of δI is greatest. But this is exactly the reverse to what is desirable. For this reason, instead of using the mean of the result for each year, it will be more satisfactory to group the whole seven years together and determine δI from the resulting equation. In this manner each year will have its proper weight assigned to it. The results of this solution are

II. Division.

$$1876\text{–}1869 \quad 262 \text{ Obs.} \quad \delta I = -2''.53 \quad -0''.20\,(\delta A) \quad -0''.86\,(\delta T) \quad +0.35\,\delta V \quad -0.22\,\delta M \quad +1.39\,\delta K \quad +0.10\,\delta l$$

I. Division.

$$1869\text{–}1862 \quad 265 \text{ Obs.} \quad \delta I = +0''.07 \quad +0''.11\,(\delta A) \quad -1''.33\,(\delta T) \quad +0.04\,\delta V \quad -0.30\,\delta M \quad -0.23\,\delta K \quad -0.03\,\delta l$$

Uniting the summer and winter portions of each division together, the value for the correction to the parallactic inequality becomes

II. Division.

$$1876\text{–}1869 \quad 262 \text{ Obs.} \quad \delta P = -1''.54 \quad +0''.03\,(\delta A) \quad +0''.07\,(\delta T) \quad +0''.10\,(\delta S) \quad +0.08\,\delta V \quad -0.12\,\delta M \quad -0.06\,\delta K \quad -0.50\,\delta I \quad -0.10\,\delta l$$

I. Division.

$$1869\text{–}1862 \quad \delta P = -1''.85 \quad +0''.02\,(\delta A) \quad +0''.07\,(\delta T) \quad +0''.37\,(\delta S) \quad +0.14\,\delta V \quad -0.14\,\delta M \quad -0.15\,\delta K \quad -0.46\,\delta I \quad -0.10\,\delta l$$

An examination of the coefficient of the different corrections to the coefficient of the various inequalities shows the important part they take in creating apparent discordances in the value found for the coefficient of the parallactic inequality, and the necessity which exists for their being taken into account in any investigations. It so happens that in the two seven-year groups these corrections to a great extent destroy each other. Yet this is merely accidental, and the mere shifting of a year from one group to another would make important alterations. Had the period of fifteen years been divided into three five-year groups the resulting values would be utterly discordant unless these terms were introduced which arise from the errors in Hansen's tables. As previously pointed out, the most important term is that due to the disturbing action of the planet *Jupiter*, and known as the Jovian Evection. In the expression for the variation in the irradiation the coefficient multiplying this correction varies between $-2{\cdot}0$ and $+3{\cdot}8$, and as the correction amounts to $1''{\cdot}5$, this corresponds to a variation in the apparent value of the irradiation of from $-3''{\cdot}0$ to $+5''{\cdot}7$, or a range of $8''{\cdot}7$.

It remains to consider the value of the corrections denoted by δV, δK, δK, δl, which must be employed for determining the true value of the coefficient of the parallactic inequality. The values employed will be derived from a discussion of the entire mass of meridian observations made at Greenwich between 1862 and 1877. Only provisional corrections have as yet been deduced but it is most unlikely that any changes which may be required when the theory of the terms of long period is placed on a satisfactory footing will affect in any important degree the value of the parallactic inequality in so far as it depends on these terms. These provisionally adopted values are

$$\begin{aligned} \delta V &= +0''{\cdot}25 \\ \delta M &= -1{\cdot}20 \\ \delta K &= +1{\cdot}50 \\ \delta l &= +1{\cdot}00 \end{aligned}$$

Employing these results, we have

$$\begin{aligned} &\text{II. Division} \quad \delta I = -1''{\cdot}06 \pm 0''{\cdot}25, \\ &\text{I. Division} \quad \delta I = -1{\cdot}18 \pm 0{\cdot}20; \end{aligned}$$

whence, uniting the two groups,

$$\delta I = -1''{\cdot}12 \pm 0''{\cdot}14.$$

Next, considering the value of the apparent coefficient of the parallactic inequality, we have

$$\begin{aligned} &\text{II. Division} \quad \delta(P) = -1''{\cdot}37 \pm 0''{\cdot}06, \\ &\text{I. Division} \quad \delta(P) = -1{\cdot}33 \pm 0{\cdot}08; \end{aligned}$$

the mean of the two divisions is

$$\delta(P) = -1''.35 \pm 0''.05.$$

Lastly, taking the value of the true coefficient of the parallactic inequality, we have

$$\text{II. Division} \quad \delta P = -1''.37 - 0''.50\,\delta I = -0''.84 \pm 0''.15,$$
$$\text{I. Division} \quad \delta P = -1.33 - 0.46\,\delta I = -0.80 \pm 0.12.$$

The mean of these two very accordant values gives us the final value for the real coefficient of the parallactic inequality,

$$\delta P = -0''.82 \pm 0''.09.$$

From these corrections we deduce the results:—

True value of the apparent coefficient

$$(P) = -125''.11 \pm 0''.05.$$

True value of the real coefficient

$$P' = -125''.64 \pm 0''.09.$$

Comparing these with the results obtained by the previous approximate solution, which were

$$(P) = -125''.06 \pm 0''.10,$$
$$P = -125.66 \pm 0.34,$$

it will be seen that they do not differ materially, but the weights to which each is entitled are incomparably different.

The above value for the real coefficient of the parallactic inequality we present as the final result of the investigation, and we regard it as the most satisfactory result which has as yet been obtained, it being the only value which has been derived strictly from the observations without the introduction of any empirical correction.

It must be admitted, however, that this result has still to be corrected for the influence of any inequality of long period which may exist in the apparent value of the parallactic inequality and arise from the perturbations produced by the planets. It has already been shown that if N be an argument of long period increasing by only a few degrees per annum, there will be two classes of terms which will seriously affect the apparent value of the coefficient of the parallactic inequality—those with the argument

$$\theta + N,$$

which will produce directly a long inequality in the value of P, but will not sensibly affect the apparent value of δI, the coeffi-

cient of the variation in irradiation, and those with the argument

$$\theta+\mu+N,$$

which will only indirectly affect the value of P by producing an inequality of long period in the apparent value of δI, and thus to a smaller degree in P.

As far as the first class of terms are concerned : if the observations be spread over many years they could be determined from observation without much difficulty, as they will affect the value of (P) and of P in the same manner If, therefore, they were otherwise than small, it might be anticipated that they would be revealed by an examination of the values derived from the Greenwich observations made between 1750 and 1851. With regard to the second class: the only material which exists for their investigation is the values found above for δI from the observations between 1862 and 1877. These are insufficient; they would suffice to detect any inequality of eight or nine years' period, but such a term is unimportant, as its effect would have been eliminated by the method adopted for determining P. A term of much longer period may exist, but the materials do not exist for recognising it.

The proper method for determining the effect of such terms is unquestionably by a thorough investigation of the theory of the perturbations in the motion of the Moon due to the disturbing action of the planets—a work now far advanced in the hands of one of us. Beyond the analytical labour there is no difficulty in the determination of the value of every inequality due to the action of the planets which *could* possibly exceed $0''\cdot01$. This effected, there will vanish the last theoretical difficulty in the way of determining the solar parallax by means of the parallactic inequality. Until it be done, some uncertainty must still remain.

When the values assigned to the correction $\delta V, \delta M, \delta K, \delta l$ come to be substituted in the expressions for δI during the different years, it will be found that there still remain considerable outstanding errors. These are partly due to the fact that constant values have been assigned to these corrections, whereas the observations show that they are subject to some variation due to the fact that Hansen's tables involve one or two more corrections not included in the preceding expression. Strictly speaking, in each year should have been used the values found for $\delta V, \delta M, \delta K, \delta l$ from the observations of that year, as in that manner these neglected corrections would have been practically eliminated, The method actually adopted for determining the value of P gives results which are sensibly as accurate as if the stricter method had been employed, for the corrections are multiplied by such small coefficients that slight variations may be neglected. When the expressions for δI are considered the use of the stricter method markedly reduces the values of the outstanding errors in the years 1876, 1869, 1868. But it must be remembered

that the apparent errors of these years are much exaggerated by their being derived from equations in which δI is multiplied by a small coefficient, so that in dividing by this coefficient the apparent errors are increased to double or triple or even quadruple their true amount. In the value for the irradiation which has been derived from the observations and actually employed, all the equations are grouped together and appear with their proper weight according to the largeness of their coefficients of δI. The result, therefore, is entitled to far more weight than may appear from a consideration of the outstanding discordances, so many of which have been thus exaggerated.

It remains to compare together the different results, with the view of seeing how far they harmonise with each other. However, it will first be necessary to consider whether the values found by Airy, Stone, and Newcomb require any further corrections to those already deduced.

The term $\delta K \sin k$ requires first consideration. As this term produces an eighteen-year inequality in the value of P, it may be expected to produce a periodical variation in the values found by Sir G. Airy from his nine-year groups. This anticipation proved true, and it was found that the existence of this term in the Moon's true longitude will necessitate a correction approximately amounting to $0''\cdot20$ being applied with alternate signs to every pair of values in Sir G. Airy's list, commencing with the groups for which the mean years are 1847 and 1843. If this correction be applied, it will be found to markedly reduce the outstanding residuals in the comparison between observation and the theory of a forty-five-year inequality. This is shown by the following figures, where in the first line are placed the uncorrected results for the last ten groups, and in the second line the corrected results, found by applying $\pm 0''\cdot20$ to every alternate pair:—

$$-0''\cdot24+0''\cdot10-0''\cdot28-0''\cdot25-0''\cdot32+0''\cdot35+0''\cdot06-0''\cdot03-0''\cdot21+0''\cdot31$$
$$-0\cdot04-0\cdot10+0\cdot08-0\cdot05-0\cdot12+0\cdot15-0\cdot14+0\cdot17-0\cdot01+0\cdot11.$$

The sum of the residuals in the first line is $2''\cdot15$, whilst those in the second line only amount to $0''\cdot97$, showing a quadruple weight. The existence of this term $\delta K \sin k$ does not seem to otherwise affect Sir G. Airy's values.

From the omission of this term $\delta K \sin k$ from Hansen's tables, and from the existence in the tables of an error of the form $\delta A \sin a$, it is obvious that the results obtained by Mr. Stone and by Prof. Newcomb will require correction for these imperfections in the tables. For Mr. Stone's value the corrections can be only approximately formed by assuming that the distribution over the lunation of the observations employed by him did not materially differ from the results for the Greenwich observations during the last fifteen years. From this basis it is

easily shown by the variation in the value of K, that Mr. Stone's result will require the correction

$$+\frac{1}{5}\delta K+0{\cdot}05\,\delta A.$$

The correction required by Mr. Newcomb's result can be determined with more certainty by a comparison of the Washington Observations with the results obtained above from the discussion of the Greenwich Observations. This comparison indicates that Prof. Newcomb's value requires the correction

$$-\frac{1}{9}\delta K-0{\cdot}15\,\delta A.$$

The result obtained by Mr. Stone will probably also require correction for the systematic errors in the Greenwich Twelve Year Catalogue. This correction will amount to

$$+0''{\cdot}18.$$

Employing these results, the different values which have been obtained for the apparent coefficient of the parallactic inequality may be written in the form

Airy	$(P) = -124''{\cdot}53+\frac{1}{3}B,$
Stone	$(P) = -124{\cdot}55-\frac{1}{7}B+\frac{1}{9}\delta V-\frac{1}{7}\delta M+\frac{1}{5}\delta K+0{\cdot}05\,\delta A+0{\cdot}18,$
Newcomb	$(P) = -124{\cdot}36+\frac{2}{5}B+\frac{1}{9}\delta V-\frac{1}{7}\delta M-\frac{1}{9}\delta K-0{\cdot}15\,\delta A,$
C. & N.	$(P) = -125{\cdot}03+\frac{4}{5}B.$

Substituting for the different corrections, except B, the values which have been assigned to them, we have

Airy	$(P) = -124''{\cdot}53+\frac{1}{3}B,$
Stone	$(P) = -123{\cdot}84-\frac{1}{7}B,$
Newcomb	$(P) = -124{\cdot}42+\frac{2}{5}B,$
C. & N.	$(P) = -125{\cdot}11+\frac{4}{5}B.$

It is obvious that if B be neglected, or supposed zero, which corresponds to assuming that the hypothetical forty-five-year inequality is non-existent, the resulting values for the apparent coefficient of the parallactic inequality are far from accordant.

If, however, for B be substituted the value derived from Sir G. Airy's investigations,

$$B = +1''{\cdot}20 \pm 0''{\cdot}20,$$

the different values become

Airy	(P) =	$-124''{\cdot}53 + 0''{\cdot}40$	=	$-124''{\cdot}13$,
Stone	(P) =	$-123{\cdot}84 - 0{\cdot}17$	=	$-124{\cdot}01$,
Newcomb	(P) =	$-124{\cdot}42 + 0{\cdot}48$	=	$-123{\cdot}96$,
C. & N.	(P) =	$-125{\cdot}11 + 1{\cdot}00$	=	$-124{\cdot}11$.

These results are extremely accordant, probably accidentally so, the mean of the four being

$$(P) = -124''{\cdot}05.$$

It is obvious that the altogether unexpected coincidence of the values tends to very materially strengthen the hypothesis of the real existence of this inequality,

$$1''{\cdot}20 \sin \{\theta - 8{\cdot}0(Y - 1825{\cdot}5)\},$$

which so markedly renders them accordant. But for one circumstance, this marvellous accord would render the existence of this inequality unquestionable. This circumstance is the fact that it is the apparent values which are thus rendered so accordant, and there is no reason why these values should be accordant. For they depend on the variation of the irradiation at the limb, and the amount of this variation may differ in different instruments and with different observers, so that, although the real values of the coefficient of the parallactic inequality must agree, there is no such reason why the apparent values should thus agree. There would be nothing remarkable in the values for different observations differing by a considerable fraction of a second. This consideration must prevent too great reliance being placed on this accord.

The final conclusion of our investigation must be stated as follows. That the real value of the parallactic inequality in the motion of the Moon is either

$$-125''{\cdot}64 \pm 0''{\cdot}09$$

or

$$-124''{\cdot}64 \pm 0''{\cdot}25.$$

Which value is correct will depend on whether we admit or do not admit the existence of this forty-five-year inequality. Now, does theory admit of the existence of such a term? Sir G. Airy replies in the negative. But this is, we conceive, an oversight.

In the theory of the perturbation of the Moon produced by the action of the planets, there unquestionably exist terms with arguments of this form and, moreover, with coefficients which *may* amount to a second of arc. There is, therefore, nothing inconsistent with theory in the existence of such a term. The question, Does it exist? is one which cannot be answered until the theory of these terms has been further advanced, and this does not come within the scope of our present paper.

It must be admitted that this uncertainty is not satisfactory; but, on the other hand, it is not discouraging. In the present paper it has been shown that it is possible to derive a fairly satisfactory result for the value of the parallactic inequality from the existing observations. It only remains to effect an auxiliary investigation to enable the true value of the parallactic inequality to be deduced from the results obtained in the present paper. This is the investigation of those neglected inequalities in the motion of the Moon due to the action of the planets. This effected—and it is merely a work of moderate labour, and does not present any particular difficulty—the results just obtained will enable us to definitely determine the value of the solar parallax from the parallactic inequality.

In the meantime we think it well to submit the present investigation to the Society, as it is really a work complete in itself, and one which has led to novel and important results.

It remains to determine the value of the solar parallax from the value found for the coefficient of the parallactic inequality. For this purpose Newcomb's data will be taken. Let

π = constant of solar parallax,

Π = constant of lunar parallax = $3422''.7$,

μ = mass of the Moon = $\frac{1}{81.5}$.

m = the ratio of the mean motion of the Sun and Moon,

f = value of analytical coefficient of parallactic inequality.

Then

$$\sin \pi = -\sin \Pi\left(1 - \frac{1}{6}m^2\right)\frac{1+\mu}{1-\mu}f \,.\, P.$$

The values of the coefficient of P are as follows:—

Hansen	$= 1 \div 14.189$,
Delaunay	$= 1 \div 14.198$,
Neison	$= 1 \div 14.201$.

The value adopted will be

$$1 \div 14.200,$$

Taking merely, as it stands, the value derived from the Greenwich observations between 1862 and 1877, or

$$P = -125''\cdot64 \pm 0''\cdot09,$$

it gives the value for the solar parallax

$$\pi = 8''\cdot848 \pm 0''\cdot007.$$

If, however, the assumed existence of the forty-five-year inequality be admitted, then the value of the parallactic inequality is

$$P = 124''\cdot64 \pm 0''\cdot25,$$

or the solar parallax is

$$\pi = 8''\cdot778 \pm 0''\cdot018.$$

Which of these is the correct value must be deferred for future investigation.

In the preceding portions of this communication, attention has been confined to the determination of the value of the parallactic inequality, by the discussion of meridian observations of the transit of the lunar limb. It has there been shown that, apart from some remediable imperfections in the theory of the action of the planets upon the motion of the Moon, the whole difficulty of the problem lies in determining the effect of the variations in the apparent semi-diameter of the Moon arising from the variations in the irradiation due to the varying contrast between the brightness of the limb and the darkness of the sky. If this uncertainty were only eliminated, there would remain no difficulty in the way of determining the value of the parallactic inequality to within 0''·05 of the truth. This would correspond to determining the value of the solar parallax to within 0''·003.

As long as the observations to be discussed are observations of the limb of the Moon, this difficulty about the determination of the variation in the semi-diameter must be faced. It may be more or less successfully overcome, and we may reduce the uncertainty to within far smaller limits than was heretofore thought possible, but even under the best conditions it will always be a source of some uncertainty. It has long appeared to us, that the right method of facing this problem of determining the true value of the parallactic inequality was to avoid this difficulty altogether, by entirely abandoning the use of the limb. Let the object observed be a small conspicuous craterlet, near the centre of the nearer hemisphere of the Moon, and the entire difficulty about the semi-diameter and its variation utterly vanishes.

Suppose a series of transits of such a craterlet be taken with

a high-class Meridian instrument like the Transit Circles at Greenwich or Washington. These observations can be reduced to the centre of the Moon, with as much ease and far greater certainty than any observation of the limb. It is only necessary to apply the correction depending on the libration of the Moon. This involves no difficulty, for the apparent libration of the Moon can be easily calculated to within $0''\cdot02$. For this libration is simply the difference between the true and mean places of the Moon, and an error of $1''$ in the position of the Moon in the heavens, corresponds to an error of only $0''\cdot005$ in the apparent libration. The only difficulty which could be supposed to exist lies in the uncertainty as to the amount of any real libration of the Moon. But this cannot exceed $2''\cdot0$ for any terms with periods shorter than a year, and is probably far smaller. Even were it much larger, as the period of its variation must differ materially from that of the parallactic inequality, it could not give rise to any sensible error in determining the value of this term. The theory of the motion of the real libration of the Moon is so well known that its effects could be eliminated from the values deduced for all the terms of short period in the lunar theory. The effect of any real libration with a period greater than a year could not affect the determination of the errors in the inequalities of short period. It would, moreover, be easy to make the same series of observations which served to determine the value of the coefficient of the parallactic inequality serve to determine the value of this real libration. The libration of the Moon does not present, therefore, any difficulty in the employment of this method of observation, a method which offers the great advantage of rendering us independent of all the difficulties involved in the uncertainty as to the real semi-diameter of the Moon, or of its variations due to differences in irradiation, or to the existence of irregularities at the limb.

The accuracy of the observations would also be materially improved, for it is far easier to observe a sharply-marked craterlet than to observe the irradiated limb. A craterlet could be selected which would present the appearance of a sharply-defined white ring, about $4''$ in diameter, surrounding a grey interior about $2''$ in diameter. Such an object can be observed with fully twice the accuracy of the limb. The probable error of a single observation of the lunar limb with the fine Transit Circles at Greenwich and Washington has been found to be $2''\cdot0$; whereas, even with a small Transit instrument, a four-feet Transit, the probable error of a single observation of a craterlet on the surface is only $\pm 1''\cdot00$. In a high-class instrument it would be less. The systematic errors involved in the observation of the transit of such an object would be far less important than those in the case of the Moon's limb, for the observation would be made under practically the same conditions day after day throughout the entire semi-lunation, and so disappear from the result; whereas those of the observation of

the lunar limb change their character at Full Moon, when the limb observed is changed. It would be only at the time of first and third Quarters that any shadows would be present; during the rest of the period, the craterlet would preserve unchanged its appearance as a white circlet with a grey interior on a yellowish surface.

Even apart from the greater accuracy with which the observation could be made, the great advantage of being independent of any uncertainty about the lunar semi-diameter and its variation would render this very method far superior to that founded on observations of the limb.

We believe that five hundred such observations of the Moon spread over eight or nine years would enable us to deduce the true value of the parallactic inequality to within far less than $0''\cdot10$.

To properly carry out this method, it is essential that a permanently mounted Transit Circle of good design should be used. But until this be done, much good might result from the employment of a less powerful instrument. In this manner not only might the advantages of the method be demonstrated but much valuable time be saved. For this reason, therefore, this work has been undertaken at the Arkley Observatory, belonging to Mr. Campbell.

The craterlet selected for observation is that known as Murchison A, a small craterlet about $5°$ of selenographical arc from the mean centre of the visible hemisphere, and very approximately in $+ 1°0'\cdot1$ selenographical longitude and $+ 4°4'\cdot0$ selenographical latitude. Each day's work, when possible, comprises a transit of Murchison A and the observation of ten or twelve clock stars including four Moon culminating stars. The instrument employed is a Transit instrument of good quality, of $2\frac{3}{4}$ feet focal length and $2\cdot7$ inches' aperture, with a massive iron stand resting on a substantial stone pier. Each observation is reduced by an expansion of the method detailed in chapter xxix. of Neison's work on the Moon. Every observation is then carefully compared with the tabular place of the Moon, and the apparent error of the tables is equated to the assumed error in the parallactic inequality and other terms which may be supposed to need correction. These equations will, when a sufficient number is obtained, serve for determining the different errors in the tabular values assigned by Hansen to the different inequalities in the motion of the Moon.

Owing to the remarkably bad weather of the past year the number of observations obtained have been fewer than was anticipated. The work was commenced in April 1879. During the last year only 21 observations were obtained. All these have been reduced. During the first four months of this year no less than 27 additional observations were obtained. Most of these have been reduced.

It is intended that the results obtained from the discussion

of the observations now in progress at the Arkley Observatory should form a sequel to the present communication, and thus afford a complete solution of the great problem of determining the solar parallax from the parallactic inequality in the motion of the Moon.

Arkley Observatory: Barnet, Herts,
1880, May 13.

Investigation of the Secular Acceleration of the Moon's Mean Motion, caused by the Secular Change in the Eccentricity of the Earth's Orbit; taking into Account Terms of the Order of m^4, but neglecting the Eccentricity and Inclination of the Moon's Orbit. By Professor J. C. Adams, M.A., F.R.S.

As the question of the Moon's secular acceleration has lately been again brought before the Society, I have thought that it might not be useless or without interest to communicate an investigation of the two leading terms of that acceleration which I gave many years ago in my lectures on the lunar theory.

1. Let r, θ be the polar coordinates of the Moon at time t, $u=\frac{1}{r}$, $\mathrm{H}=r^2\frac{d\theta}{dt}$, μ the sum of the masses of the Earth and Moon; also let m' be the mass of the Sun, r', θ' its polar coordinates, a' the Sun's mean distance, n' its mean motion, and e' the eccentricity of its orbit, $\lambda'=n't+\epsilon'$ its mean longitude, and $\phi'=n't+\epsilon'-\varpi'$ its mean anomaly.

Then the equations to be satisfied are

$$\frac{d^2u}{d\theta^2}+u=\frac{\mu}{\mathrm{H}^2}-\frac{1}{2}\,\frac{m'}{\mathrm{H}^2u^3r'^3}-\frac{3}{2}\,\frac{m'}{\mathrm{H}^2u^3r'^3}\cos 2(\theta-\theta')$$
$$+\frac{3}{2}\,\frac{m'}{\mathrm{H}^2u^4r'^3}\,\frac{du}{d\theta}\sin 2(\theta-\theta'),$$

and

$$\frac{d(\mathrm{H}^2)}{d\theta}=-\frac{3m'}{u^4r'^3}\sin 2(\theta-\theta').$$

Also, by the formulæ of elliptic motion

$$\frac{m'}{r'^3}=\frac{m'}{a'^3}\left(\frac{a'}{r'}\right)^3=n'^2\left\{1+\frac{3}{2}e'^2+3e'\cos\phi'+\frac{9}{2}e'^2\cos 2\phi'\right\},$$

$$\frac{m'}{r'^3}\cos 2(\theta-\theta')=\frac{m'}{a'^3}\left(\frac{a'}{r'}\right)^3\cos 2(\theta-\theta')$$

$$=n'^2\left\{\left(1-\frac{5}{2}e'^2\right)\cos 2(\theta-\lambda')+\frac{7}{2}e'\cos(2\theta-2\lambda'-\phi')\right.$$
$$-\frac{1}{2}e'\cos(2\theta-2\lambda'+\phi')$$
$$\left.+\frac{17}{2}e'^2\cos(2\theta-2\lambda'-2\phi')\right\};$$

and

$$\frac{m}{r'^3}\sin 2(\theta-\theta') = \frac{m'}{a'^3}\left(\frac{a'}{r'}\right)^3 \sin 2(\theta-\theta')$$

$$\begin{aligned} = n'^2\Big\{\left(1-\frac{5}{2}e'^2\right)\sin 2(\theta-\lambda') &+ \frac{7}{2}e'\sin(2\theta-2\lambda'-\phi') \\ &-\frac{1}{2}e'\sin(2\theta-2\lambda'+\phi') \\ &+\frac{17}{2}e'^2\sin(2\theta-2\lambda'-2\phi')\Big\}. \end{aligned}$$

The angles involved in these expressions are formed by combining the angle $2\theta-2\lambda'$ with multiples of ϕ'.

For our present purpose we may omit the terms which involve $2\phi'$. Also, for the sake of brevity we may write

$n't$ instead of $n't+\epsilon'-\varpi'$ or ϕ',

$2\theta-2n't$ instead of $2\theta-2(n't+\epsilon')$ or $2\theta-2\lambda'$,

$2\theta-3n't$ instead of $2\theta-2\lambda'-\phi'$,

$2\theta-\ \ n't$ instead of $2\theta-2\lambda+\phi'$,

since no ambiguity can arise from this abbreviation.

Hence our equations become

$$\begin{aligned} \frac{d^2u}{d\theta^2}+u = \frac{\mu}{H^2} &- \frac{1}{2}\frac{n'^2}{H^2u^3}\left\{1+\frac{3}{2}e'^2+3e'\cos n't\right\} \\ &-\frac{3}{2}\frac{n'^2}{H^2u^3}\Big\{\left(1-\frac{5}{2}e'^2\right)\cos(2\theta-2n't)+\frac{7}{2}e'\cos(2\theta-3n't) \\ &\qquad\qquad -\frac{1}{2}e'\cos(2\theta-n't)\Big\} \\ &+\frac{3}{2}\frac{n'^2}{H^2u^4}\frac{du}{d\theta}\Big\{\left(1-\frac{5}{2}e'^2\right)\sin(2\theta-2n't)+\frac{7}{2}e'\sin(2\theta-3n't) \\ &\qquad\qquad -\frac{1}{2}e'\sin(2\theta-n't)\Big\}, \end{aligned}$$

and

$$\begin{aligned} \frac{d(H^2)}{d\theta} = -\frac{3n'^2}{u^4}\Big\{\left(1-\frac{5}{2}e'^2\right)\sin(2\theta-2n't)&+\frac{7}{2}e'\sin(2\theta-3n't) \\ &-\frac{1}{2}e'\sin(2\theta-n't)\Big\}. \end{aligned}$$

2. After these preliminaries, it will be convenient to begin by finding the relations between the actual mean motion n of the Moon and the constant parts of u and H^2 when these quantities are developed in the form we have adopted, carrying the approximation as far as terms involving $m^4e'^2$, on the supposition that e' and therefore also that n is constant.

For this purpose it is sufficient to take

$$nt+\epsilon = \theta + 3me' \sin n't - \frac{11}{8}m^2\left(1-\frac{5}{2}e'^2\right)\sin(2\theta-2n't)$$

$$-\frac{77}{16}m^2e' \sin(2\theta-3n't) + \frac{11}{16}m^2e' \sin(2\theta-n't),$$

$$u = \frac{1}{a}\left\{1 - \frac{3}{2}m^2e' \cos n't + m^2\left(1-\frac{5}{2}e'^2\right)\cos(2\theta-2n't)\right.$$

$$\left.+\frac{7}{2}m^2e' \cos(2\theta-3n't) - \frac{1}{2}m^2e' \cos(2\theta-n't)\right\},$$

which are readily derived from the equations of motion.

Differentiate the first of these equations and put

$$\frac{n'}{n} = m,$$

$$\therefore \frac{ndt}{d\theta}\left\{1 - 3m^2e' \cos n't - \frac{11}{4}m^2\left(1-\frac{5}{2}e'^2\right)\cos(2\theta-2n't) - \frac{231}{16}m^2e' \cos(2\theta-3n't)\right.$$

$$\left.+\frac{11}{16}m^2e' \cos(2\theta-n't)\right\}$$

$$= 1 - \frac{11}{4}m^2\left(1-\frac{5}{2}e'^2\right)\cos(2\theta-2n't) - \frac{77}{8}m^2e' \cos(2\theta-3n't) + \frac{11}{8}m^2e' \cos(2\theta-n't),$$

or

$$\frac{ndt}{d\theta} = 1 + \frac{9}{2}m^4e'^2 + 3m^2e' \cos n't - \frac{11}{4}m^2\left(1-\frac{5}{2}e'^2\right)\cos(2\theta-2n't)$$

$$-\frac{77}{8}m^2e' \cos(2\theta-3n't) + \frac{11}{8}m^2e' \cos(2\theta-n't),$$

since the other terms only give rise to terms of higher orders than we have here taken into account.

Hence

$$H^2 = \left(\frac{d\theta}{u^2dt}\right)^2 = u^{-4}\left(\frac{dt}{d\theta}\right)^{-2}$$

$$= n^2a^4\left\{1 + 5m^4(1-5e'^2) + \frac{45}{4}m^4e'^2 + \frac{245}{4}m^4e'^2 + \frac{5}{4}m^4e'^2 + 6m^2e' \cos n't\right.$$

$$\left.-4m^2\left(1-\frac{5}{2}e'^2\right)\cos(2\theta-2n't) - 14m^2e' \cos(2\theta-3n't) + 2m^2e' \cos(2\theta-n't)\right\}$$

$$\times\left\{1 - 9m^4e'^2 + \frac{27}{2}m^4e'^2 + \frac{363}{32}m^4(1-5e'^2) + \frac{17787}{128}m^4e'^2 + \frac{363}{128}m^4e'^2\right.$$

$$-6m^2e' \cos n't + \frac{11}{2}m^2\left(1-\frac{5}{2}e'^2\right)\cos(2\theta-2n't) + \frac{77}{4}m^2e' \cos(2\theta-3n't)$$

$$\left.-\frac{11}{4}m^2e' \cos(2\theta-n't)\right\};$$

or, by actual multiplication,

$$H^2 = n^2a^4\left\{1 + \frac{523}{32}m^4(1-5e'^2) + \frac{14083}{64}m^4e'^2 - 18m^4e'^2 - 11m^4(1-5e'^2)\right.$$
$$-\frac{539}{4}m^4e'^2 - \frac{11}{4}m^4e'^2 + \frac{3}{2}m^2\left(1-\frac{5}{2}e'^2\right)\cos(2\theta-2n't)$$
$$\left.+\frac{21}{4}m^2e'\cos(2\theta-3n't) - \frac{3}{4}m^2e'\cos(2\theta-n't)\right\}$$
$$= n^2a^4\left\{1 + \frac{171}{32}m^4(1-5e'^2) + \frac{4131}{64}m^4e'^2 + \frac{3}{2}m^2\left(1-\frac{5}{2}e'^2\right)\cos(2\theta-2n't)\right.$$
$$\left.+\frac{21}{4}m^2e'\cos(2\theta-3n't) - \frac{3}{4}m^2e'\cos(2\theta-n't)\right\}.$$

Hence the constant part of H^2 is

$$n^2a^4\left\{1 + \frac{171}{32}m^4 + \frac{2421}{64}m^4e'^2\right\},$$

n being the actual mean motion.

Hence

$$\frac{\mu}{H^2} = \frac{\mu}{n^2a^4}\left\{1 - \frac{171}{32}m^4(1-5e'^2) - \frac{4131}{64}m^4e'^2 + \frac{9}{8}m^4(1-5e'^2) + \frac{441}{32}m^4e'^2 + \frac{9}{32}m^4e'^2\right.$$
$$-\frac{3}{2}m^2\left(1-\frac{5}{2}e'^2\right)\cos(2\theta-2n't) - \frac{21}{4}m^2e'\cos(2\theta-3n't)$$
$$\left.+\frac{3}{4}m^2e'\cos(2\theta-n't)\right\}$$
$$= \frac{\mu}{n^2a^4}\left\{1 - \frac{135}{32}m^4(1-5e'^2) - \frac{3231}{64}m^4e'^2 - \frac{3}{2}m^2\left(1-\frac{5}{2}e'^2\right)\cos(2\theta-2n't)\right.$$
$$\left.-\frac{21}{4}m^2e'\cos(2\theta-3n't) + \frac{3}{4}m^2e'\cos(2\theta-n't)\right\},$$

and therefore the constant part of $\frac{\mu}{H^2}$ is

$$\frac{\mu}{n^2a^4}\left\{1 - \frac{135}{32}m^4 - \frac{1881}{64}m^4e'^2\right\}.$$

3. Also

$$\frac{du}{d\theta} = \frac{1}{a}\left\{\frac{3}{2}m^2e'\sin n't - 2m^2\left(1-\frac{5}{2}e'^2\right)\sin(2\theta-2n't)\right.$$
$$\left.-7m^2e'\sin(2\theta-3n't) + m^2e'\sin(2\theta-n't)\right\},$$

and

$$\frac{d^2u}{d\theta^2} = \frac{1}{a}\left\{-4m^2\left(1-\frac{5}{2}e'^2\right)\cos(2\theta-2n't) - 14m^2e'\cos(2\theta-3n't)\right.$$
$$\left.+2m^2e'\cos(2\theta-n't)\right\};$$

also

$$\frac{n'^2}{H^2u^3} = \frac{m^2}{a}\left\{1 + \frac{9}{2}m^2e' \cos n't - \frac{9}{2}m^2\left(1 - \frac{5}{2}e'^2\right)\cos(2\theta - 2n't)\right.$$
$$\left. - \frac{63}{4}m^2e' \cos(2\theta - 3n't) + \frac{9}{4}m^2e' \cos(2\theta - n't)\right\},$$

and

$$\frac{n'^2}{H^2u^4}\frac{du}{d\theta} = \frac{m^2}{a}\left\{-2m^2\left(1 - \frac{5}{2}e'^2\right)\sin(2\theta - 2n't) - 7m^2e' \sin(2\theta - 3n't)\right.$$
$$\left. + m^2e' \sin(2\theta - n't)\right\}.$$

Hence, substituting in the first differential equation and transposing, we find the quantity which is to be equated to $\frac{\mu}{H^2}$ to be

$$\frac{1}{a}\left\{1 + \left(\frac{1}{2} + \frac{3}{4}e'^2\right)m^2 + \frac{27}{8}m^4e'^2 - \frac{27}{8}m^4(1 - 5e'^2) - \frac{1323}{32}m^4e'^2 - \frac{27}{32}m^4e'^2\right.$$
$$+ \frac{3}{2}m^4(1 - 5e'^2) + \frac{147}{8}m^4e'^2 + \frac{3}{8}m^4e'^2$$
$$\left. - \frac{3}{2}m^2\left(1 - \frac{5}{2}e'^2\right)\cos(2\theta - 2n't) - \frac{21}{4}m^2e' \cos(2\theta - 3n't) + \frac{3}{4}m^2e' \cos(2\theta - n't)\right\}$$
$$= \frac{1}{a}\left\{1 + \frac{1}{2}m^2\left(1 + \frac{3}{2}e'^2\right) - \frac{15}{8}m^4(1 - 5e'^2) - \frac{321}{16}m^4e'^2\right.$$
$$- \frac{3}{2}m^2\left(1 - \frac{5}{2}e'^2\right)\cos(2\theta - 2n't) - \frac{21}{4}m^2e' \cos(2\theta - 3n't)$$
$$\left. + \frac{3}{4}m^2e' \cos(2\theta - n't)\right\}.$$

Comparing this with the former expression and observing that $\frac{\mu}{n^2a^3}$ is nearly $= 1$, we see that the periodic terms agree, and by equating the non-periodic parts, we have

$$\frac{\mu}{n^2a^3}\left\{1 - \frac{135}{32}m^4(1 - 5e'^2) - \frac{3231}{64}m^4e'^2\right\}$$
$$= 1 + \frac{1}{2}m^2\left(1 + \frac{3}{2}e'^2\right) - \frac{15}{8}m^4(1 - 5e'^2) - \frac{321}{16}m^4e'^2,$$

or

$$\frac{\mu}{n^2a^3} = 1 + \frac{1}{2}m^2\left(1 + \frac{3}{2}e'^2\right) + \frac{75}{32}m^4(1 - 5e'^2) + \frac{1947}{64}m^4e'^2$$
$$= 1 + \frac{1}{2}m^2\left(1 + \frac{3}{2}e'^2\right) + \frac{75}{32}m^4 + \frac{1197}{64}m^4e'^2,$$

which gives the relation between n and a.

4. In the above, e' is considered constant throughout; if now we consider e' to be variable, we may choose n and a so that the

constant (or rather the non-periodic) parts of u and of H^2 may have the same forms as before, and in this case we shall find the same relation between n and a as that which has just been found, and n will continue to signify the *actual mean motion* at the time to which θ belongs, but n and a will now become variable quantities, and, in order to satisfy our equations, it will be necessary to add certain periodic terms to u and H^2 which would not exist if e' were constant.

Suppose then that

$$u = \frac{1}{a}\left\{1 + \delta v - \frac{3}{2}m^2e'\cos n't + m^2\left(1 - \frac{5}{2}e'^2\right)\cos(2\theta - 2n't) + \frac{7}{2}m^2e'\cos(2\theta - 3n't) - \frac{1}{2}m^2e'\cos(2\theta - n't)\right\},$$

and

$$H^2 = n^2a^4\left\{1 + 2\delta\eta + \frac{171}{32}m^4 + \frac{2421}{64}m^4e'^2 + \frac{3}{2}m^2\left(1 - \frac{5}{2}e'^2\right)\cos(2\theta - 2n't) + \frac{21}{4}m^2e'\cos(2\theta - 3n't) - \frac{3}{4}m^2e'\cos(2\theta - n't)\right\}.$$

We will suppose e' to vary uniformly with the time, and very slowly, or, in other words, we will suppose

$$\frac{de'}{dt} \text{ to be constant, so that } \frac{d^2e'}{dt^2} = 0,$$

and we will neglect

$$\left(\frac{de'}{dt}\right)^2.$$

We must therefore recollect that $\frac{de'}{d\theta}$ is not constant, but is equal to

$$\frac{de'}{dt}\cdot\frac{dt}{d\theta} = \frac{1}{Hu^2}\cdot\frac{de'}{dt}$$

$$= \frac{de'}{ndt}\left\{1 + 3m^2e'\cos n't - \frac{11}{4}m^2\cos(2\theta - 2n't) - \frac{77}{8}m^2e'\cos(2\theta - 3n't) + \frac{11}{8}m^2e'\cos(2\theta - n't)\right\}.$$

5. In consequence of the variability of e', $\frac{du}{d\theta}$ will contain the additional terms

$$\frac{1}{Hu^2}\cdot\frac{1}{a}\left\{-\frac{da}{adt} - \frac{3}{2}m^2\frac{de'}{dt}\cos n't - 5m^2e'\frac{de'}{dt}\cos(2\theta - 2n't) + \frac{7}{2}m^2\frac{de'}{dt}\cos(2\theta - 3n't) - \frac{1}{2}m^2\frac{de'}{dt}\cos(2\theta - n't)\right\}$$

$$+ \frac{1}{a}\cdot\frac{d\,.\,\delta v}{d\theta},$$

or

$$\frac{1}{an}\left\{-\frac{da}{adt}-\frac{3}{2}m^2\frac{de'}{dt}\cos n't-5m^2e'\frac{de'}{dt}\cos(2\theta-2n't)+\frac{7}{2}m^2\frac{de'}{dt}\cos(2\theta-3n't)\right.$$
$$\left.-\frac{1}{2}m^2\frac{de'}{dt}\cos(2\theta-n't)\right\}$$
$$+\frac{1}{a}\cdot\frac{d\,.\,\delta v}{d\theta},$$

to the order of approximation required.

Therefore also $\dfrac{d^2u}{d\theta^2}$ will contain the additional terms

$$\frac{1}{an}\left\{10m^2e'\frac{de'}{dt}\sin(2\theta-2n't)-7m^2\frac{de'}{dt}\sin(2\theta-3n't)+m^2\frac{de'}{dt}\sin(2\theta-n't)\right.$$
$$\left.+10m^2e'\frac{de'}{dt}\sin(2\theta-2n't)-7m^2\frac{de'}{dt}\sin(2\theta-3n't)+m^2\frac{de'}{dt}\sin(2\theta-n't)\right\}$$
$$+\frac{1}{a}\frac{d^2\,.\,\delta v}{d\theta^2}$$

neglecting

$$\frac{d^2a}{dt^2},\ \left(\frac{da}{dt}\right)^2,\ \text{and also } m^2\frac{de'}{dt}$$

in the coefficients of the periodic terms.

Hence

$$\frac{d^2u}{d\theta^2}+u$$

contains the additional terms

$$\frac{1}{a}\left\{\frac{d^2\,.\,\delta v}{d\theta^2}+\delta v\right\}$$

$$+\frac{1}{an}\left\{20m^2e'\frac{de'}{dt}\sin(2\theta-2n't)-14m^2\frac{de'}{dt}\sin(2\theta-3n't)+2m^2\frac{de'}{dt}\sin(2\theta-n't)\right\}.$$

Also $\dfrac{\mu}{H^2}$ contains the additional term

$$\frac{\mu}{n^2a^4}\Big[-2\delta\eta\Big].$$

The other terms which enter into the first differential equation receive no additional terms of the order to which we restrict ourselves.

6. Also differentiating the expression for H^2, and including terms of the order

$$m^4e'\frac{de'}{dt}$$

in the non-periodic part, but only those of the orders

$$m^2\frac{de'}{dt}\ \text{ and }\ m^2e'\frac{de'}{dt}$$

in the periodic part, we have the following additional terms in $\frac{d(\mathrm{H}^2)}{d\theta}$, viz.

$$n^2a^4\frac{1}{\mathrm{H}u^2}\left\{\frac{2dn}{ndt}+\frac{4da}{adt}+\frac{2421}{32}m^4e'\frac{de'}{dt}-\frac{15}{2}m^2\frac{e'de'}{dt}\cos(2\theta-2n't)\right.$$
$$\left.+\frac{21}{4}m^2\frac{de'}{dt}\cos(2\theta-3n't)-\frac{3}{4}m^2\frac{de'}{dt}\cos(2\theta-n't)\right\}$$
$$+n^2a^4\left\{2\frac{d\,.\,\delta\eta}{d\theta}\right\}.$$

Also the right-hand side of the second differential equation contains the following additional quantity:—

$$m^2n^2a^4[4\delta v]\left\{3\sin(2\theta-2n't)+\frac{21}{2}e'\sin(2\theta-3n't)-\frac{3}{2}e'\sin(2\theta-n't)\right\},$$

which, as we shall immediately find, contains non-periodic terms of the order

$$m^4e'\frac{de'}{dt}.$$

Hence, taking the periodic parts of this equation, we have

$$2\frac{d\,.\,\delta\eta}{d\theta}=\frac{1}{n}\left\{\frac{15}{2}m^2e'\frac{de'}{dt}\cos(2\theta-2n't)-\frac{21}{4}m^2\frac{de'}{dt}\cos(2\theta-3n't)\right.$$
$$\left.+\frac{3}{4}m^2\frac{de'}{dt}\cos(2\theta-n't)\right\};$$

$$\therefore\quad 2(\delta\eta)=\frac{1}{n}\left\{\frac{15}{4}m^2e'\frac{de'}{dt}\sin(2\theta-2n't)-\frac{21}{8}m^2\frac{de'}{dt}\sin(2\theta-3n't)\right.$$
$$\left.+\frac{3}{8}m^2\frac{de'}{dt}\sin(2\theta-n't)\right\}.$$

7. Substitute this in the first equation, putting

$$\frac{\mu}{n^2a^3}=1$$

in the coefficients of the periodic terms, as these are only required to the order of m^2, and we obtain

$$\frac{d^2\,.\,\delta v}{d\theta^2}+\delta v=-\frac{1}{n}\left\{20m^2e'\frac{de'}{dt}\sin(2\theta-2n't)-14m^2\frac{de'}{dt}\sin(2\theta-3n't)\right.$$
$$+2m^2\frac{de'}{dt}\sin(2\theta-n't)+\frac{15}{4}m^2e'\frac{de'}{dt}\sin(2\theta-2n't)$$
$$\left.-\frac{21}{8}m^2\frac{de'}{dt}\sin(2\theta-3n't)+\frac{3}{8}m^2\frac{de'}{dt}\sin(2\theta-n't)\right\}$$
$$=-\frac{1}{n}\left\{\frac{95}{4}m^2e'\frac{de'}{dt}\sin(2\theta-2n't)-\frac{133}{8}m^2\frac{de'}{dt}\sin(2\theta-3n't)\right.$$
$$\left.+\frac{19}{8}m^2\frac{de'}{dt}\sin(2\theta-n't)\right\}.$$

$$\therefore\qquad \delta v=\frac{1}{n}\left\{\frac{95}{12}m^2e'\frac{de'}{dt}\sin(2\theta-2n't)-\frac{133}{24}m^2\frac{de'}{dt}\sin(2\theta-3n't)\right.$$
$$\left.+\frac{19}{24}m^2\frac{de'}{dt}\sin(2\theta-n't)\right\}.$$

Substitute this value of δv, and also the value of $\frac{1}{\mathrm{H}u^2}$, viz.

$$\frac{1}{n}\left\{1+3m^2e'\cos n't-\frac{11}{4}m^2\cos(2\theta-2n't)-\frac{77}{8}m^2e'\cos(2\theta-3n't)\right.$$
$$\left.+\frac{11}{8}m^2e'\cos(2\theta-n't)\right\},$$

for that quantity in the second differential equation, and equate the non-periodic parts which result from this substitution,

$$\therefore\quad \frac{2dn}{ndt}+\frac{4da}{adt}+\frac{2421}{32}m^4e'\frac{de'}{dt}+\frac{165}{16}m^4e'\frac{de'}{dt}-\frac{1617}{64}m^4e'\frac{de'}{dt}-\frac{33}{64}m^4e'\frac{de'}{dt}$$
$$=\frac{95}{2}m^4e'\frac{de'}{dt}-\frac{931}{8}m^4e'\frac{de'}{dt}-\frac{19}{8}m^4e'\frac{de'}{dt},$$

or

$$\frac{2dn}{ndt}+4\frac{da}{adt}+\frac{963}{16}m^4e'\frac{de'}{dt}=-\frac{285}{4}m^4e'\frac{de'}{dt},$$

$$\therefore\quad \frac{dn}{ndt}+2\frac{da}{adt}=-\frac{2103}{32}m^4e'\frac{de'}{dt}.$$

8. The substitution of the values of δv and $\delta\eta$ in the first differential equation introduces no non-periodic terms depending on $\frac{de'}{dt}$; consequently the value of $\frac{\mu}{n^2a^3}$ remains of the same form as before.

Hence

$$\log\left(\frac{\mu}{n^2a^3}\right)=\frac{1}{2}m^2\left(1+\frac{3}{2}e'^2\right)+\frac{75}{32}m^4+\frac{1197}{64}m^4e'^2-\frac{1}{8}m^4(1+3e'^2$$
$$=\frac{1}{2}m^2\left(1+\frac{3}{2}e'^2\right)+\frac{71}{32}m^4+\frac{1173}{64}m^4e'^2;$$

$$\therefore\quad \frac{2dn}{ndt}+3\frac{da}{adt}=-\frac{3}{2}m^2e'\frac{de'}{dt}-\frac{1173}{32}m^4e'\frac{de'}{dt}-m^2\left(\frac{dm}{mdt}\right)$$
$$=-\left(\frac{3}{2}m^2+\frac{1173}{32}m^4\right)e'\frac{de'}{dt}+m^2\left(\frac{dn}{ndt}\right),$$

since

$$m=\frac{n'}{n},\ \text{and}\ \therefore\ \frac{dm}{mdt}=-\frac{dn}{ndt},$$

n' being constant.

Hence

$$(4-2m^2)\frac{dn}{ndt}+6\frac{da}{adt}=-\left(3m^2+\frac{1173}{16}m^4\right)e'\frac{de'}{dt},$$

also from above

$$3\frac{dn}{ndt}+6\frac{da}{adt}=\qquad -\frac{6309}{32}m^4e'\frac{de'}{dt};$$

$$\therefore\quad (1-2m^2)\frac{dn}{ndt}=-\left(3m^2-\frac{3963}{32}m^4\right)e'\frac{de'}{dt}.$$

and

$$\frac{dn}{ndt} = -\left(3m^2 - \frac{3771}{32}m^4\right)e'\frac{de'}{dt};$$

$$\therefore \qquad 2\frac{da}{adt} = \left(3m^2 - \frac{3771}{32}m^4\right)e'\frac{de'}{dt} - \frac{2103}{32}m^4e'\frac{de'}{dt}$$

$$= \left(3m^2 - \frac{2937}{16}m^4\right)e'\frac{de'}{dt},$$

or

$$\frac{da}{adt} = \left(\frac{3}{2}m^2 - \frac{2937}{32}m^4\right)e'\frac{de'}{dt}.$$

9. These equations give the rate of variation of the quantities n and a. We will now show that n denotes the actual mean motion, as it did when e' was constant.

From the values of u and H^2 we find

$$\frac{dt}{d\theta} = \frac{1}{Hu^2} = \frac{1}{n}\left\{1 - 2\delta v - \delta\eta + \frac{9}{2}m^4e'^2 + 3m^2e'\cos n't - \frac{11}{4}m^2\left(1 - \frac{5}{2}e'^2\right)\cos(2\theta - 2n't)\right.$$
$$\left. - \frac{77}{8}m^2e'\cos(2\theta - 3n't) + \frac{11}{8}m^2e'\cos(2\theta - n't)\right\},$$

or

$$\frac{ndt}{d\theta} = 1 + \frac{9}{2}m^4e'^2 + 3m^2e'\cos n't - \frac{11}{4}m^2\left(1 - \frac{5}{2}e'^2\right)\cos(2\theta - 2n't)$$
$$- \frac{77}{8}m^2e'\cos(2\theta - 3n't) + \frac{11}{8}m^2e'\cos(2\theta - n't)$$
$$- \frac{425}{24}m^2e'\frac{de'}{ndt}\sin(2\theta - 2n't) + \frac{595}{48}m^2\frac{de'}{ndt}\sin(2\theta - 3n't)$$
$$- \frac{85}{48}m^2\frac{de'}{ndt}\sin(2\theta - n't).$$

Divide by

$$1 + \frac{9}{2}m^4e'^2 + 3m^2e'\cos n't$$

and take into account $m^4e'^2$ in the non-periodic term,

$$\therefore \frac{ndt}{d\theta}\left\{1 - 3m^2e'\cos n t\right\} = 1 - \frac{11}{4}m^2\left(1 - \frac{5}{2}e'^2\right)\cos(2\theta - 2n't)$$
$$- \frac{77}{8}m^2e'\cos(2\theta - 3n't) + \frac{11}{8}m^2e'\cos(2\theta - n't)$$
$$- \frac{425}{24}m^2e'\frac{de'}{ndt}\sin(2\theta - 2n't)$$
$$+ \frac{595}{48}m^2\frac{de'}{ndt}\sin(2\theta - 3n't) - \frac{85}{48}m^2\frac{de'}{ndt}\sin(2\theta - n't),$$

Substitute this value of δv, and also the value of $\frac{1}{\mathrm{H}u^2}$, viz.

$$\frac{1}{n}\left\{1+3m^2e'\cos n't-\frac{11}{4}m^2\cos(2\theta-2n't)-\frac{77}{8}m^2e'\cos(2\theta-3n't)\right.$$
$$\left.+\frac{11}{8}m^2e'\cos(2\theta-n't)\right\}$$

for that quantity in the second differential equation, and equa[te] the non-periodic parts which result from this substitution,

$$\therefore\quad \frac{2dn}{ndt}+\frac{4da}{adt}+\frac{2421}{32}m^4e'\frac{de'}{dt}+\frac{165}{16}m^4e'\frac{de'}{dt}-\frac{1617}{64}m^4e'\frac{de'}{dt}-\frac{33}{64}m^4e'\frac{de'}{dt}$$
$$=\frac{95}{2}m^4e'\frac{de'}{dt}-\frac{931}{8}m^4e'\frac{de'}{dt}-\frac{19}{8}m^4e'\frac{de'}{dt},$$

or

$$\frac{2dn}{ndt}+4\frac{da}{adt}+\frac{963}{16}m^4e'\frac{de'}{dt}=-\frac{285}{4}m^4e'\frac{de'}{dt},$$
$$\therefore\qquad \frac{dn}{ndt}+2\frac{da}{adt}=-\frac{2103}{32}m^4e'\frac{de'}{dt}.$$

8. The substitution of the values of δv and $\delta\eta$ in the first diff[er]ential equation introduces no non-periodic terms depending [on] $\frac{de'}{dt}$; consequently the value of $\frac{\mu}{n^2a^3}$ remains of the same form [as] before.

Hence

$$\log\left(\frac{\mu}{n^2a^3}\right)=\frac{1}{2}m^2\left(1+\frac{3}{2}e'^2\right)+\frac{75}{32}m^4+\frac{1197}{64}m^4e'^2-\frac{1}{8}m^4(1+3e'^2$$
$$=\frac{1}{2}m^2\left(1+\frac{3}{2}e'^2\right)+\frac{71}{32}m^4+\frac{1173}{64}m^4e'^2;$$
$$\therefore\qquad \frac{2dn}{ndt}+3\frac{da}{adt}=-\frac{3}{2}m^2e'\frac{de'}{dt}-\frac{1173}{32}m^4e'\frac{de'}{dt}-m^2\left(\frac{dm}{mdt}\right)$$
$$=-\left(\frac{3}{2}m^2+\frac{1173}{32}m^4\right)e'\frac{de'}{dt}+m^2\left(\frac{dn}{ndt}\right),$$

since

$$m=\frac{n'}{n},\ \text{and}\ \therefore\ \frac{dm}{mdt}=-\frac{dn}{ndt},$$

n' being constant.

Hence

$$(4-2m^2)\frac{dn}{ndt}+6\frac{da}{adt}=-\left(3m^2+\frac{1173}{16}m^4\right)e'\frac{de'}{dt},$$

also from above

$$3\frac{dn}{ndt}+6\frac{da}{adt}=\qquad -\frac{6309}{32}m^4e'\frac{de'}{dt};$$
$$\therefore\qquad (1-2m^2)\frac{dn}{ndt}=-\left(3m^2-\frac{3963}{32}m^4\right)e'\frac{de'}{dt}.$$

and therefore

$$\int^{\theta} n dt = \theta + 3me' \sin n't - \frac{11}{8} m^2 \left(1 - \frac{5}{2} e'^2\right) \sin (2\theta - 2n't)$$

$$- \frac{77}{16} m^2 e' \sin (2\theta - 3n't) + \frac{11}{16} m^2 e' \sin (2\theta - n't)$$

$$+ 3 \frac{de'}{ndt} \cos n't + \frac{295}{24} m^2 e' \frac{de'}{ndt} \cos (2\theta - 2n't) - \frac{413}{48} m^2 \frac{de'}{ndt} \cos (2\theta - 3n't)$$

$$+ \frac{59}{48} m^2 \frac{de'}{ndt} \cos (2\theta - n't).$$

Hence θ differs from $\int ndt$ by periodic terms only, which proves the proposition.

The value of $\frac{dn}{ndt}$ above found agrees with that found in my paper published in the 'Philosophical Transactions' for 1853.

Note on the Constant of Lunar Parallax. By Professor J. C. Adams, M.A., F.R.S.

From the report of a discussion which took place at a late meeting of the Society, I have reason to believe that an explanation of the apparent discrepancy between the value of the constant of parallax given by me in the Appendix to the *Nautical Almanac* for 1856, and in the *Monthly Notices*, vol. xiii. p. 263, and the value of the constant found by Hansen in the Introduction to his Lunar Tables, may not be unacceptable to some of our members.

It will be proper to begin this explanation by recalling to mind that my formula, in the article of the *Monthly Notices* above referred to, does not represent the parallax itself, but rather the sine of that quantity converted into seconds of arc by dividing by sin 1″ or, which is the same thing, by multiplying by the number of seconds in the arc equal to the radius. The employment of the sine of the parallax instead of the parallax itself appears to be desirable both on theoretical as well as practical grounds.

In the first place, the sine of the parallax, being proportional to the reciprocal of the radius vector, is the quantity given directly by the lunar theory, and, in the next place, it is the same quantity which is wanted in the reduction of lunar observations.

What I have called the constant of parallax in the papers above referred to is, then, the constant term in the expression for the converted sine of the parallax, supposing the periodic

terms to be expressed in cosines of angles which increase in proportion to the time. The value found for this constant was $3422''\cdot325$.

This quantity may also be called very appropriately the mean sine of the parallax, although I do not use the term in the papers referred to.

The value of the corresponding constant in the expression of the parallax itself is $0''\cdot157$ greater than this, or $3422''\cdot48$, which may appropriately be called the mean parallax.

The formula in the Introduction to Hansen's Lunar Tables does not give the sine of the parallax, but the *logarithm* of the sine of the parallax, and the constant which Hansen calls C is a quantity such that the constant term in his expression for the logarithm of sine of the parallax is

$$\log \sin C.$$

Now, it is plain that the constant term in the development of log sin parallax is a different quantity from the logarithm of the constant term of the sine of the parallax, and hence my constant of parallax differs from Hansen's quantity

$$\frac{\sin C}{\sin 1''}.$$

We may readily express the relation between these two constants in the case in which the orbit is supposed to be an undisturbed ellipse.

In this case, if the reciprocal of the radius vector, which is proportional to the sine of the parallax, be developed in terms of cosines of multiples of the mean anomaly,

then, a being the semi-axis major,
and e the eccentricity of the orbit,

the constant term in the development will be $\frac{1}{a}$.

In the same case, the constant term in the development of the logarithm of the reciprocal of the radius vector, expressed in terms of the same form as before, will be

$$\log \frac{1}{a}\left(1-\frac{1}{4}e^2\right)$$

very nearly, instead of $\log \frac{1}{a}$; so that if c denote the constant term in the former development, and $\log c'$ the constant term in the latter, we shall have

$$\frac{c'}{c} = 1-\frac{1}{4}e^2 \text{ very nearly.}$$

[illegible]is relation will still be approximately though not exactly [illegible] when the Moon's perturbations are taken into account. [illegible]n himself, in a paper in the 17th volume of the *Astro-* [illegible] *Nachrichten*, p. 299, in which he gives the results which

he had obtained in a preliminary investigation of the lunar perturbations, finds that the number corresponding to the constant term in the logarithm of the sine of the parallax requires to be augmented by $2''\cdot71$ in order to reduce it to the constant term in the sine of the parallax itself.

Calling the parallax p, Hansen finds that the value of the constant term in $\log\left(\frac{\sin p}{\sin 1''}\right)$ is

$$\log(3419''\cdot35),$$

and hence he concludes that the constant term in $\left(\frac{\sin p}{\sin 1''}\right)$ is

$$3422''\cdot06.$$

By repeating Hansen's calculation and taking into account some small terms omitted by him, I find the amount of the reduction to be slightly less than the above, viz. $2''\cdot67$, so that the constant term in

$$\frac{\sin p}{\sin 1''}$$

according to Hansen's preliminary theory would be $3422''\cdot02$.

This value, however, is not immediately comparable with my own, being founded on different elements.

Both values are purely theoretical, depending on the ratio of the Moon's mass to that of the Earth, the ratio of the Earth's equatorial and polar axes, and the ratio of the Earth's radius to the length of the seconds' pendulum in a given latitude.

If M denote the mass of the Earth,
m that of the Moon,
A the Earth's equatorial radius,
R the Earth's radius at a point of which the sine of the latitude is $\frac{1}{\sqrt{3}}$,
P the length of the seconds' pendulum at the same point;

then the constant term of the sine of the horizontal parallax corresponding to the latitude just specified may be represented by

$$\left(\frac{M}{M+m}\cdot\frac{R}{P}\right)^{\frac{1}{3}}F,$$

and therefore the constant term of the sine of the equatorial horizontal parallax may be represented by

$$\frac{A}{R}\left(\frac{M}{M+m}\cdot\frac{R}{P}\right)^{\frac{1}{3}}F=\left(\frac{M}{M+m}\cdot\frac{A^3}{R^2P}\right)^{\frac{1}{3}}F,$$

where F is a factor which may be found by theory from elements which may be considered as known with all desirable accuracy.

The values of $\frac{M}{m}$, A, R and P employed in finding my constant are the following:—

$$\frac{M}{m} = 81{\cdot}5,$$

which corresponds very nearly to Dr. Peters' constant of Nutation;

$$A = 20923505 \text{ English feet}$$
$$R = 20900320 \quad ,,$$
$$P = 3{\cdot}256989 \quad ,,$$

R and P belong to a point the sine of the *geographical* latitude of which is $\frac{1}{\sqrt{3}}$.

A and R are the quantities found from Bessel's latest determination of the figure and dimensions of the Earth as given in *Astron. Nachr.* vol. xix., p. 216, supposing that

$$1 \text{ Toise} = 6{\cdot}394564 \text{ English feet.}$$

P is found thus: according to the formula given in p. 94 of Baily's Report on Foster's Pendulum experiments, *Mem. of the R.A.S.* vol. vii., the square of the number of vibrations made in a mean solar day, at a point the sine of whose geographical latitude is $\frac{1}{\sqrt{3}}$, by a pendulum which vibrates seconds in London is

$$7441625711 + \frac{1}{3}(38286335) = 7454387823.$$

Also Captain Kater's determination of the length of the seconds' pendulum in London is

$$39{\cdot}13929 \text{ inches} = 3{\cdot}2616075 \text{ feet.}$$

Hence as the square of the number of vibrations made at a given place in a given time varies inversely as the length of the pendulum, we derive the value above given for P.

The values of the fundamental elements employed by Hansen are the following:—

$$\frac{M}{m} = 80$$
$$A = 6377157 \text{ metres}$$
$$R_1 = 6370063 \quad ,,$$
$$P_1 = 0{\cdot}992666 \quad ,,$$

and R_1 and P_1 belong to a point the sine of the *geocentric* latitude of which is $\frac{1}{\sqrt{3}}$.

The corresponding values of R and P for a point the sine of whose geographical latitude is $\frac{1}{\sqrt{3}}$ are the following:—

$$R = 6370126 \text{ metres}$$
$$P = 0{\cdot}992651 \quad ,,$$

And the constant term of the sine of the equatorial horizontal parallax may be represented either by

$$\left(\frac{M}{M+m}\,\frac{A^3}{R^2P}\right)^{\frac{1}{3}}F, \quad \text{or by} \quad \left(\frac{M}{M+m}\,\frac{A^3}{R_1{}^2P_1}\right)^{\frac{1}{3}}F_1.$$

In my calculation of the factor F, I took into account terms of the order of the square of the Earth's compression. It would otherwise have been useless to distinguish between R^2P and $R^2{}_1P_1$ or between F and F_1.

At the time when Hansen's paper appeared in the *Astron. Nachr.* Bessel's latest determination of the figure and dimensions of the Earth was not available. Hansen employed an earlier determination given by Bessel in *Astron. Nachr.* vol. xiv., p. 344, in which the results were affected by an error in the calculation of the French arc of the meridian which was discovered later.

Hence the corrections to be applied to the logarithms employed by Hansen in order to make them agree with those employed by me are the following, expressed in units of the 7th decimal:—

	Correction.
$\log\left(\frac{M}{M+m}\right)$	$+987$
$\log\left(\frac{A}{R}\right)$	$+\ 25$
$\log\left(\frac{R}{P}\right)$	-150

The correction to be applied to Hansen's value of the logarithm of the constant term in the sine of the parallax is therefore

$$25 + \frac{1}{3}(987 - 150) = 304 \quad \text{of the same units.}$$

And the corresponding correction of the constant term of the sine of the parallax will be $0''{\cdot}24$, and therefore according to Hansen's preliminary theory, employing my system of fundamental data, the value of this constant term will be $3422''{\cdot}26$.

In my independent transformation of Hansen's expression I found the rather more precise value $3422''{\cdot}264$.

This is less than my own value of the same constant by $0''{\cdot}06$ nearly, as stated in my paper in the Appendix to the *Nautical Almanac* for 1856.

I there intimated my belief that Hansen's definitive theory would probably be found to introduce a correction to his former value of the constant term in question, and this turns out to be the case.

In *Astron. Nachr.* vol. xvii., p. 298, the constant term in $-w$ which denotes the perturbations of the natural logarithm of the reciprocal of the radius vector, divided by sin 1″, is given as 1345″·281, but in the introduction to Hansen's Lunar Tables this same quantity is given as 1348″·840. Hence, the correction to the former value is 3″·559, and multiplying this by sin 1″ and by 3422″ we find the corresponding correction of the constant of parallax to be 0″·059, so that this constant becomes 3422″·323, a result which agrees perfectly with my own.

In this connection it may be worth mentioning that the only periodic term in which I found any difference much exceeding 0″·01 between my coefficients of parallax and those obtained by a transformation of the results of Hansen's preliminary theory was that which has the argument denoted by $t+z$ in Damoiseau's notation.

The corresponding term in $-w$ is in Hansen's preliminary theory

$$10''\!\cdot\!92 \cos(t+z),$$

whereas in the Introduction to the Lunar Tables this term is

$$8''\!\cdot\!73 \cos(t+z);$$

the correction to the coefficient is $-2''\!\cdot\!19$, and multiplying this as before by sin 1″ and by 3422″ we find the correction to the corresponding term of the sine of the parallax to be

$$-0''\!\cdot\!036 \cos(t+z),$$

and if this be applied to the value of this term in the preliminary theory, viz.

$$0''\!\cdot\!181 \cos(t+z),$$

the result is

$$0''\!\cdot\!145 \cos(t+z),$$

which agrees perfectly with my own.

It should be remarked that, in the Introduction to his Lunar Tables, Hansen still continues to use the same fundamental data as he had done in his earlier paper, so that the value of the constant term in the sine of the parallax according to the data adopted in the Tables is 3422″·08.

Note added June 17, 1880.

In Professor Newcomb's valuable transformation of Hansen's Lunar Theory which I have just received, it is wrongly assumed

that I employed the same data as Hansen for the figure and dimensions of the Earth, and that my value of P, viz. 3·256989 feet, relates like Hansen's to a point the sine of whose *geocentric* latitude is $\frac{1}{\sqrt{3}}$, whereas it should be the *geographical* latitude, as that is the latitude which enters into Baily's formula from which my value of P is deduced.

In consequence of this, Professor Newcomb finds a discrepancy of 0″·03 between Hansen's value of the constant of parallax and mine when both are derived from the same system of fundamental data; but it has been shown above that no such discrepancy exists.

By a typographical error, the value of P which Professor Newcomb quotes from me is printed as 3·256 89 feet, instead of 3·256989 feet.

Ephemeris for finding the Positions of the Satellite of Neptune, 1880–81. By A. Marth, Esq.

P, angle of position of the minor axis of the apparent orbit in the direction of superior conjunction.

a, *b*, major and minor semi-axis of the apparent orbit.

Long. = longitude of the satellite in its orbit, reckoned from the point which is in superior conjunction with the planet.

Gr. Noon.	P.	*a*.	*b*.	Log. *a*.	Log. *b*.	Long.	Diff.
1880.	°	″	″			°	°
Sept. 1	314·30	16·66	6·51	1·2216	0·8136	112·48	612·51
11	314·24	16·74	6·53	·2238	·8150	4·99	·46
21	314·14	16·81	6·54	·2257	·8156	257·45	·41
Oct. 1	314·01	16·88	6·54	1·2273	0·8156	149·86	·37
11	313·87	16·93	6·53	·2285	·8149	42·23	·34
21	313·71	16·96	6·51	·2294	·8135	294·57	·32
31	313·55	16·97	6·48	1·2298	0·8115	186·89	612·30
Nov. 10	313·38	16·97	6·44	·2297	·8089	79·19	·30
20	313·22	16·95	6·40	·2292	·8059	331·49	·31
30	313·07	16·91	6·35	1·2282	0·8025	223·80	·32
Dec. 10	312·93	16·86	6·29	·2268	·7989	116·12	·35
20	312·82	16·79	6·24	·2251	·7952	8·47	·38
30	312·74	16·71	6·19	1·2231	0·7917	260·85	612·43
1881.							
Jan. 9	312·68	16·63	6·14	·2208	·7884	153·28	·48
19	312·66	16·53	6·10	·2183	·7854	45·76	·53
29	312·67	16·44	6·06	1·2158	0·7828	298·29	·58
Febr. 8	312·72	16·34	6·04	·2133	·7808	190·87	·64
18	312·79	16·25	6·02	·2109	·7795	83·51	612·69
28	312·90	16·17	6·01	1·2088	0·7790	336·20	

These values are to be interpolated for the times for which the apparent places of the satellite are required, and the position-angles p and distances s are then found by

$$s \sin (\mathrm{P}-p) = a \sin \text{long.}$$
$$s \cos (\mathrm{P}-p) = b \cos \text{long.}$$

The satellite moves in the direction of decreasing position angles, and will be at its greatest elongations ("nf." in posit P+90 and distance a, "sf." in posit. P−90) and at its conjunctions ("sup." in posit. P and distance b, "inf." in posit. P−180°) at the following hours, Gr. M.T.—

"Sp." Elong.		"Inf." Conj.		"Nf." Elong.		"Sup." Conj.	
1880.	h		h		h		h
Aug. 31	15·2	Sept. 2	2·4	Sept. 3	13·7	Sept. 5	1·0
Sept. 6	12·2	7	23·5	9	10·8	10	22·0
12	9·3	13	20·6	15	7·8	16	19·1
18	6·4	19	17·7	21	4·9	22	16·2
24	3·5	25	14·7	27	2·0	28	13·3
30	0·5	Oct. 1	11·8	Oct. 2	23·1	Oct. 4	10·4
Oct. 5	21·6	7	8·9	8	20·2	10	7·4
11	18·7	13	6·0	14	17·3	16	4·5
17	15·8	19	3·1	20	14·4	22	1·6
23	12·9	25	0·2	26	11·5	27	22·7
29	10·0	30	21·3	Nov. 1	8·6	Nov. 2	19·9
Nov. 4	7·1	Nov. 5	18·4	7	5·7	8	17·0
10	4·2	11	15·5	13	2·8	14	14·1
16	1·3	17	12·6	18	23·9	20	11·2
21	22·5	23	9·7	24	21·0	26	8·3
27	19·6	29	6·8	30	18·1	Dec. 2	5·4
Dec. 3	16·7	Dec. 5	3·9	Dec. 6	15·2	8	2·5
9	13·8	11	1·0	12	12·3	13	23·6
15	10·9	16	22·1	18	9·4	19	20·7
21	8·0	22	19·2	24	6·5	25	17·8
27	5·0	28	16·3	30	3·6	31	14·9
1881.							
Jan. 2	2·1	Jan. 3	13·4	Jan. 5	0·7	Jan. 6	11·9
7	23·2	9	10·5	10	21·7	12	9·
13	20·3	15	7·5	16	18·8	18	6·1
19	17·3	21	4·6	22	15·9	24	3·1
5	14·4	27	1·7	28	12·9	30	0·2
1	11·4	Feb. 1	22·7	Feb. 3	10·0	Feb. 4	21·2

"Sp." Elong. 1881.	h	"Inf." Conj.	h	"Nf." Elong.	h	"Sup." Conj.	h
Feb. 6	8·5	Feb. 7	19·7	Feb. 9	7·0	Feb. 10	18·3
12	5·5	13	16·8	15	4·0	16	15·3
18	2·5	19	13·8	21	1·1	22	12·3
23	23·6	25	10·8	26	22·1	28	9·3

Addition to the Ephemeris for Physical Observations of Jupiter. By A. Marth, Esq.

Though the Ephemeris published on pp. 416–418 of the *Monthly Notices* contains the data necessary for the reductions of observations, it gives them only so far as they do not involve an assumed value of the planet's ellipticity, and supplementary computations are consequently needed.

Bessel has treated the questions connected with observations of an incompletely illuminated planetary disk in the sixth paper of his *Astron. Untersuchungen*, but he considers only observations made with a heliometer or double-image micrometer, so that some additions referring to observations made by means of webs are required.

If $e = \sin \epsilon_0$ is the eccentricity of the planet's spheroidal surface or $\cos \epsilon_0$ the proportion of the polar axis to the equatorial diameter, P the angle of position of the polar axis, B the latitude of the earth above the planet's equator, and $\sin \epsilon = \sin \epsilon_0 \cos B$, the outline of the apparent planetary disk will be an ellipse, the minor axis of which, in position-angle P, is $= 2a \cos \epsilon$, if the major axis or the equatorial diameter is $= 2a$, and the radius of the disk in position-angle p will be

$$= a \cos \epsilon', \text{ if } \tan \epsilon' = \tan \epsilon \, . \cos (p - P).$$

Provided that the limit of illumination passes through those points on the planet's surface which have the Sun's centre in their true horizon, the projection of the limit of illumination upon the disk will be an ellipse, the radius of which in position-angle p is $= a \cos \epsilon' \cos d'$, if

$$\tan \psi = \tan (p - P) \cos \epsilon, \quad \text{and} \quad \tan d' = \tan d \cos (\psi - w),$$

where d and w have the same significance as in Bessel's paper.

The visible disk of the planet is generally composed of two half-ellipses, the direction of their common diameter or of the line of cusps being in position-angle

$$P + w_1 \mp 90^\circ, \quad \text{where} \quad \tan w_1 = \tan w \cos \epsilon.$$

The limit of illumination is visible in the two quadrants adjoin-

ing the position-angle $P+w_1$, while the rim of the disk is visible in the two opposite quadrants adjoining the position-angle

$$P+w_1+180°.$$

A web or a line being moved to or from the centre of the disk in the direction of the position-angle p or $p+180°$ will be a tangent to the rim of the disk when its distance from the centre is

$$= a\sqrt{1-\sin^2\epsilon\cos^2(p-P)};$$

and it will be a tangent to the curve formed by the limit of illumination when its distance from the centre is

$$= a\sqrt{1-\sin^2\epsilon\cos^2(p-P)}\,.\,\sqrt{1-\sin^2 d\cos^2(p_1-w)},$$

where

$$\tan p_1 = \tan(p-P)\sec\epsilon.$$

Hence, by putting

$$\sin\nu = \sin\epsilon\cos(p-P) \text{ and}$$
$$\sin\delta = \sin d\cos(p_1-w),$$

the distance of the two tangents which touch the two curves and enclose the visible disk is found

$$= 2a\cos\nu\cos^2\frac{1}{2}\delta, \text{ or}$$

minus the defect of illumination, $2a\cos\nu\sin^2\frac{1}{2}\delta$,

$$= 2a\cos\nu.$$

In observations of right ascension and declination the values of p are

$$p = 90° \text{ and } p = 0°.$$

Hence, by putting

$$\sin\nu = \sin\epsilon\sin P, \qquad \text{and } \sin\nu' = \sin\epsilon\cos P,$$
$$\tan P' = \tan P\cos\epsilon, \qquad \tan P' = \tan P\sec\epsilon,$$
$$\sin\delta = \sin d\sin(P'+w), \qquad \sin\delta' = \sin d\cos(P'+w),$$

the differences of the limbs in right ascension and declination are found to be

$$\frac{2a\cos\nu}{15\cos D}\,.\,\cos^2\frac{1}{2}\delta \quad\text{and}\quad 2a\cos\nu'\,.\,\cos^2\frac{1}{2}\delta'$$

(D being the planet's declination), or

$$\frac{2a\cos\nu}{15\cos D} \quad\text{and}\quad 2a\cos\nu'$$

when corrected for the defects of illumination

$$\frac{2a \cos \nu}{15 \cos D} \cdot \sin^2 \frac{1}{2} \delta \quad \text{and} \quad 2a \cos \nu' \cdot \sin^2 \frac{1}{2} \delta'.$$

These latter values will be found further on in the table.

In case the declination-webs are not perpendicular upon the transit-webs, but are so inclined that the corresponding p is not $=0°$, but $=i$, the values

$$\sin \nu' = \sin \epsilon \cos (P - i)$$

and

$$\tan P'' = \tan (P - i) \sec \epsilon$$

must be substituted for those before given.

In measurements of the equatorial and polar diameters

$$p \text{ is } = P + 90 \text{ and } = P.$$

Hence, if

$$\sin \delta = \sin d \sin w \quad \text{and}$$
$$\sin \delta' = \sin d \cos w,$$

the measured distances are

$$2a \cos^2 \frac{1}{2} \delta \quad \text{and} \quad 2a \cos \epsilon \cos^2 \frac{1}{2} \delta',$$

or the defect of illumination of the equatorial diameter

$$2a \text{ is } 2a \sin^2 \frac{1}{2} \delta,$$

and that of the polar diameter

$$2a \cos \epsilon \text{ is } 2a \cos \epsilon \sin^2 \frac{1}{2} \delta'.$$

If, in observing a planetary disk with a double-image micrometer, the images are separated in the direction p, the amount of separation which produces external contact of the two curves is, according to Bessel's investigation,

$$= 2a \sqrt{1 - \sin^2 \epsilon \cos^2 \psi} \cdot \left(1 - \sin^2 \frac{1}{2} d \cos^2 (\psi - w)\right),$$

where

$$\tan \psi = \tan (p - P) \cos \epsilon.$$

Hence measurements in the two directions for which $\psi - w$ is $=0°$ and $=90°$, and which have been specially suggested, would give values of

$$2a \cos^2 \frac{1}{2} d \cdot \sqrt{1 - \sin^2 \epsilon \cos^2 w}$$

and of

$$2a\sqrt{1 - \sin^2 \epsilon \sin^2 w}.$$

The corresponding position-angles are

$$P + w' \text{ and } P + w_1 \pm 90°,$$

where

$$\tan w' = \tan w \sec \epsilon, \quad \text{and} \quad \tan w_1 = \tan w \cos \epsilon,$$

and these angles would have to be known beforehand. The latter direction is that of the line of cusps.

Double-image measurements in the directions P+90 and P of the chief axes give values of

$$2a\left(1 - \sin^2\frac{I}{2} d \sin^2 w\right)$$

and of

$$2a \cos \epsilon \left(1 - \sin^2\frac{I}{2} d \cos^2 w\right)$$

and it appears preferable to take the measures in these directions.

In the following Ephemeris, which will require little further explanation, I give now some additional data for the proper reduction of observations. The defect of illumination of the polar diameter is insensible, and that of the equatorial diameter does not differ sensibly from the "greatest phase" given on page 416 ff.

The assumed value of *Jupiter's* equatorial diameter is still Bessel's 37″·60 at the distance 5·20273, corresponding to the semi-diameter 97″·81 at the distance 1. The assumed value of $\cos \epsilon_0$ or of the proportion of the polar axis to the equatorial diameter is 0·9363. It would be very desirable that these values should be substantially improved by modern observations; but, of course, only the best observations would serve the purpose.

The last column of the Ephemeris gives the elongations O–L of the point O of *Jupiter's* vernal equinox from the point L in the plane of his equator, which is in superior conjunction in reference to the Earth.

Greenw. Noon 1880		Polar Diam.	Difference between limbs. in A.R.	in Decl.	Defect of illumination. in A.R.	in Decl.	*d.*	*w.*	O–L.
June	18	″ 35·42	″ 2·505	″ 35·84	prec. l.	south l.	° 11·25	° 271·34	° 119·638
	23	35·94	2·543	36·36	″ 0·022	″ 0·06	11·49	271·42	118·939
	28	36·48	2·582	36·90	·023	·06	11·67	271·50	118·299
July	3	37·04	2·623	37·47	·024	·06	11·79	271·59	117·721
	8	37·62	2·665	38·06	·025	·06	11·85	271·68	117·208
	13	38·22	2·708	38·66	·025	·07	11·84	271·77	116·764
	18	38·84	2·753	39·28	·025	·06	11·75	271·88	116·391

Greenw. Noon		Polar Diam.	Difference between limbs. in A.R.	in Decl.	Defect of illumination. in A.R.	in Decl.	d.	w.	O—L.
1880.		"	"	"	"	"	°	°	°
July	23	39·47	2·798	39·92	·025	·06	11·59	271·99	116·092
	28	40·11	2·844	40·57	·024	·06	11·35	272·12	115·872
Aug.	2	40·76	2·890	41·22	·024	·06	11·04	272·26	115·732
	7	41·41	2·936	41·88	·022	·05	10·64	272·42	115·673
	12	42·05	2·982	42·53	·021	·05	10·16	272·61	115·697
	17	42·68	3·026	43·16	·019	·04	9·59	272·82	115·804
	22	43·28	3·068	43·77	·017	·04	8·95	273·08	115·993
	27	43·86	3·108	44·36	·014	·03	8·23	273·38	116·262
Sept.	1	44·40	3·145	44·90	·012	·03	7·42	273·74	116·607
	6	44·89	3·178	45·40	·009	·02	6·54	274·20	117·025
	11	45·32	3·208	45·84	·007	·01	5·60	274·84	117·510
	16	45·69	3·233	46·22	·005	·01	4·60	275·78	118·052
	21	45·99	3·252	46·52	·003	·00	3·56	277·14	118·643
	26	46·20	3·266	46·74	·001	·00	2·48	279·72	119·272
Oct.	1	46·33	3·274	46·88	·000	·00	1·39	286·39	119·929
	6	46·38	3·276	46·93	foll l.	north l.	0·42	329·70	120·601
	11	46·33	3·271	46·89	·000	·00	0·98	70·95	121·275
	16	46·20	3·260	46·75	·001	·01	2·07	82·24	121·937
	21	45·98	3·244	46·53	·002	·01	3·15	85·76	122·575
	26	45·67	3·222	46·23	·004	·01	4·21	87·47	123·177
	31	45·29	3·194	45·85	·005	·02	5·22	88·53	123·732
Nov.	5	44·85	3·162	45·40	·008	·03	6·17	89·25	124·229
	10	44·34	3·126	44·89	·010	·03	7·06	89·77	124·660
	15	43·78	3·086	44·33	·012	·04	7·87	90·17	125·017
	20	43·19	3·044	43·72	·015	·05	8·61	90·49	125·295
	25	42·56	3·000	43·09	·017	·05	9·27	90·76	125·491
	30	41·91	2·954	42·43	·018	·06	9·83	90·98	125·601
Dec.	5	41·25	2·907	41·76	·020	·06	10·31	91·17	125·624
	10	40·58	2·860	41·08	·021	·06	10·71	91·34	125·559
	15	39·91	2·813	40·40	·022	·07	11·02	91·48	125·408
	20	39·24	2·766	39·73	·023	·07	11·24	91·62	125·173
	25	38·59	2·721	39·07	·023	·07	11·38	91·74	124·856
	30	37·95	2·676	38·42	·023	·06	11·45	91·85	124·460
1881. Jan.	4	37·33	2·633	37·79	·023	·06	11·44	91·96	123·987
	9	36·74	2·592	37·19	·022	·06	11·35	92·05	123·443
	14	36·17	2·553	36·61	·021	·06	11·20	92·14	122·831
	19	35·62	2·515	36·05	·020	·05	10·98	92·22	122·155
	24	35·10	2·480	35·53	·019	·05	10·70	92·30	121·419

Greenw. Noon	Polar Diam.	Difference between limbs. in A.R.	in Decl.	Defect of illumination. in A.R.	in Decl.	d.	w.	O−L.
1881	"	"	"	"	"	°	°	°
Jan. 29	34·62	2·447	35·03	·017	·05	10·37	92·39	120·627
Feb. 3	34·16	2·416	34·56	·016	·04	9·99	92·48	119·783
8	33·73	2·387	34·12	·015	·04	9·55	92·57	118·892
13	33·33	2·360	33·71	·013	·03	9·07	92·67	117·957
18	32·96	2·336	33·34	·012	·03	8·56	92·77	116·982
23	32·62	2·314	32·99	·010	·02	8·01	92·89	115·971
28	32·31	2·294	32·67	0·009	0·02	7·42	93·02	114·927

The inclinations γ and the ascending nodes Γ of the orbits of the four satellites in reference to the plane of *Jupiter's* equator are the following, the nodes being reckoned from O, the point of the vernal equinox of *Jupiter's* northern hemisphere:—

1880	Sat. I. γ_1	Γ_1	Sat. II. γ_2	Γ_2	Sat. III. γ_3	Γ_3	Sat. IV. γ_4	Γ_4
Feb. 14	0°·0103	3°·7	0°·4654	13°·20	0°·1867	265°·97	0°·3272	329°·50
April 14	·0105	1·8	·4665	11·21	·1858	265·64	·3277	329·56
June 13	·0106	0·0	·4676	9·23	·1849	265·31	·3281	329·64
Aug. 12	·0108	358·4	·4687	7·26	·1839	264·99	·3284	329·75
Oct. 11	·0109	356·9	·4697	5·30	·1827	264·66	·3286	329·86
Dec. 10	·0111	355·5	·4708	3·35	·1815	264·33	·3286	329·99
1881								
Feb. 8	·0112	354·2	·4718	1·40	·1802	263·98	·3286	330·13
April 9	0·0112	352·9	0·4727	359·46	0·1789	263·61	0·3284	330·28

If these values of Γ are added to the elongations O − L of the point O from superior conjunction, given in the last column of the preceding table, the angles Γ + O − L are the longitudes of the ascending nodes of the orbits reckoned from that line in the plane of the equator which is in superior conjunction in reference to the Earth. Hence the latitudes of the satellites above the plane of *Jupiter's* equator are easily found, if their elongations are known.

I have received from Mr. Trouvelot some observations of passages of the red spot made in 1878. The time of the passage of the preceding end on Sept. 25 is only inferred from an estimate made at $6^h\ 30^m$ Camb. M.T.: "preceding extremity of red spot has passed central meridian 10 or 15 minutes." I retain it, however, in the additional list, and will now also not omit the result of Prof. Pritchett's own inference from his observations made on July 9, 1878, an account of which with an illustration is to be found in *The Observatory*, vol. ii., p. 308. Prof. Pritchett writes:—"I am very sure of the time of the transit of the centre of the spot over the Central Meridian on that day within 10 minutes—it must have occurred between $19^h\ 15^m$ and $19^h\ 25^m$ Gr. M. T."

1873.		Observed Passages. Prec. End. h m	Middle. h m	Foll. End. h m		Corresponding Longitudes. Rot. 870°·60. Prec. End. °	Middle. °	Foll. End. °	Rot. 870°·43. Prec. End. °	Middle. °	Foll. End. °
		Pritchett, Glasgow, Missouri.									
July	9	between	19 15	—	Gr.	—	245·6	—	—	324·5	—
			19 25	—	"	—	251·7	—	—	330·6	—
		Trouvelot, Cambridge, Massachusetts.									
Sept.	25	6 15	—	—	Cambridge	176·9	—	—	242·6	—	—
		6 20	—	—	"	179·9	—	—	245·6	—	—
	30	—	6 35	—	"	—	221·5	—	—	286·4	—
Oct.	2	—	7 45	—	"	—	204·8	—	—	269·3	—
Nov.	12	—	—	6 50	"	—	—	219·3	—	—	276·8
	24	6 10	—	—	"	199·7	—	—	255·2	—	—
	29	5 15	—	—	"	198·2	—	—	252·9	—	—
Dec.	16	—	6 35	—	"	—	210·1	—	—	261·9	—
	23	—	5 33	—	"	—	217·6	—	—	268·2	—
	28	—	4 45	—	"	—	220·3	—	—	270·1	—

I can also add the first observation made by Prof. Pritchett since *Jupiter's* reappearance in the present season.

1880											
May	25	—	23 12	—	Gr.	—	306·6	—	—	268·8	—

Prof. Pritchett writes:—"The observation was made after sunrise, but was entirely satisfactory. The spot has changed very little since the autumn of last year."

It is to be hoped that proper observations have been made and may soon be forthcoming to clear up the early history of the spot in 1878.

An Examination of the Double-Star Measures of the Bedford Catalogue. By S. W. Burnham, Esq.

The micrometrical measures of double stars by Admiral Smyth, recorded in the "Bedford Catalogue," may, for the purpose of this examination be considered in two distinct classes, which are unlike in respect to the character of the measures and the order of stars observed. This classification is based wholly upon the fact whether or not other measures of these stars were made prior to 1844.

Class I. Double stars accurately measured by Struve, Sir William Herschel, South, and others, before the epoch of Smyth.

Class II. Double stars, and stars with distinct companions, which had not been carefully measured by any other observer up to the time of the publication of the "Cycle of Celestial Objects."

A cursory examination will show that the quality of the measures as to accuracy is very different. The measures of Class I. are, in the main, uniformly correct. This is shown by comparing them with earlier and later measures by other observers which are known to be correct. The only exceptions are in instances when the prior observations were palpably erroneous. In such cases the Cycle measures are equally in error.

The measures of Class II. are quite as uniformly either roughly approximate or grossly incorrect. The greater portion of the objects in this class are not double stars in any proper sense of the term, but are simply small stars in the same field with larger and generally prominent naked-eye stars. The distances are very great, and the objects themselves are devoid of any interest to a double-star observer. Some of the stars catalogued by Sir John Herschel which have been measured by Smyth belong to this class, as the distances were all estimated by Herschel, and the angles generally given from single readings. A few of the Struve stars, not measured at Dorpat, and rejected in *Mensuræ Micrometricæ*, are also thus inserted. The larger part of the objects in this class would be of easy measurement with a 5·9-inch aperture, and much easier than many in class I.

My attention was first called to the doubtful accuracy of the observations of the Bedford Catalogue by communications relating to the alleged disappearance of the companions to certain

stars, as, for instance, Procyon, 61 *Geminorum*, and others. The reliable catalogues and measures do not furnish a single instance of such changes, and it seemed improbable that these mysterious disappearances, resting entirely upon the unsupported evidence of a single observer, should be real. It appeared much easier to explain the alleged disappearances by assuming errors on the part of the observer, than by assuming the sudden extinction of suns and their possibly attendant worlds. A slight acquaintance with the Bedford Catalogue confirmed the correctness of this view. So many discrepancies were found, it seemed singular that so few of them had been noticed, and that so much time had been wasted in endeavouring to find stars which obviously never existed in the places assigned to them by Smyth. The accompanying observations furnish many additional and similar instances of the disappearance or displacement of companion stars.

It is not clear why most of the stars in Class II. were ever observed at all. In one place Smyth alludes to the determination of proper motion, but even for this purpose, whatever may be said of a careful and elaborate series of observations, a single micrometrical measure, even of the accuracy of ordinary observers, could be of but little value other than to aid in determining whether the principal star had a proper motion. As these stars are for the most part prominent naked-eye stars, the work for ascertaining proper motion has been done by meridian observations. By reason of the uninteresting character of the objects in Class II., very few of them have been measured, or have received any attention from observers since Smyth.

Several years ago I called attention to the character of the double-star measures of Smyth, but the subject attracted no general comment till the communication of Mr. Herbert Sadler was printed in *Monthly Notices*, R.A.S., for January, 1879. The subject matter of the discussion in the Society that followed is so fresh in the minds of the members that it is not necessary that I should allude to it further than to say that in response to urgent requests of several English observers and others I have made the series of observations of the Smyth stars which is herewith submitted.

In investigating the double-star measures of the Cycle, the first step is to examine the means and methods employed for the observations. Admiral Smyth used a refracting telescope of 5·9 inches' aperture, which he speaks of as being perhaps "the finest specimen of the late Mr. Tully's skill." Again he says:—"On repeated trials I find the instrument bears its highest magnifiers [22 to 1200] with remarkable distinctness, as is especially evinced by the roundness of small discs, the dark increase of vacancy between close double stars, and from particular portions of the Moon when dichotomised. I have, therefore, reason to presume that the curves of the lenses are in exact chromatic and spherical aberration throughout; and the focal distance accurately proportioned to the densities of the flint and crown glasses.

..... Nothing can exceed the definition and sharpness with which the telescope comes to focus, even under low powers, and different sized apertures."

Certainly no higher tribute could be paid to the most perfect productions of the best modern optician. Indeed, few of them would warrant such an encomium, if the use of the "highest magnifiers" means that under any circumstances a power of 1200 on such an aperture would be practically an advantage. It is apparent that the accuracy of the work would not be necessarily affected by the size of the instrument when we recall the first-class measures of Dawes with 3·8 inches, and of Herschel and South with 3·75 inches, and other observers with apertures under 6 inches.

With regard to the driving-clock, Smyth says, "the performance is simple and effective," and as he nowhere charges any errors to this cause, we must presume that it was satisfactory.

The next, and perhaps the most important appliance of all in this connexion, is the micrometer. It is described as "a fine double-line position micrometer, whose screws have been rigidly examined under every revolution," and furnished with powers from 62 to 850. Again he says:—"My double-wired micrometer is one of the very best which the skill and practice of Mr. Dollond was able to produce, and is really a charming instrument to use."

The method of making the measures is described as follows:—"In measuring distances the alternate + and − reading of the micrometer was practised to get rid of the zero correction. The position, or angle made by the line joining the two stars with the direction of diurnal rotation at the meridian, was mostly taken by placing the stars between two dark parallel lines, and was altered many degrees backwards or forwards to prevent the eyes being biassed." He also says: —"It is proper to add that I seldom observed but on the finest nights, and that my objects, with rare exceptions, were limited to 30° South Declination, which from my observatory was well clear of vaporous addenda to the refraction."

The present investigation relates only to the stars in Class II. There is little to be said about the other class. As already stated, the Cycle measures of nearly all stars, difficult and otherwise, which had been previously measured, are in harmony with the earlier observations. A working list was made of all stars belonging to Class II. where both distances and angles are given by Smyth, and they have been observed from time to time during the past year. The results now found, together with the observations of Smyth, and other measures, where any have been made, are given in the following pages.

Perhaps comment on the results of this examination is not necessary, as the observations will speak for themselves.

The discrepancies are so great, and so constantly recurring that it is obviously impossible to explain them on the theory of the ordinary errors of observation. From the entire list of stars, rejecting a few where the identity of the particular companion is

uncertain, and those only partially measured by Smyth, we find from the 126 stars remaining that the mean difference between the present measures and those recorded in the Bedford Catalogue is 25″ in distance, and 7° in position-angle. That the measures of Smyth are but little, if any, better than mere eye-estimates, made without wires in the field, cannot be seriously doubted, to say nothing of the measures of stars which, from their occupying entirely different places, could not have been seen at all. All my early catalogues of new stars were published with estimated distances and angles, and I have given the result of a comparison of these estimates with the subsequent measures of Baron Dembowski, in the *Astronomical Register* for January 1876, from which it appears that the average error in the estimate of stars less than 1″ apart was 10°·2 in angle and 0″·11 in distance; and for stars separated from 1″ to 6″ the corresponding errors were 8°·4 in angle, and 0″·45 in distance. It will be remembered that not only were some of these stars exceedingly close, but nearly all of them difficult objects, and hard to see under any circumstances. It is evident that the angle of two distant stars could be more accurately estimated than that of a close pair. This comparison is perhaps not applicable to the probable error in estimating distances as large as those of the stars in Class II.; but I think it will be found that the estimated distances of wide pairs in the catalogue of Sir John Herschel are as near the truth as the distances in the Bedford Catalogue.

It is obvious that these observations are not micrometrical measures at all in the usual sense of the term, when used in reference to the work of all other observers from the time of Sir William Herschel to the present day. That such approximate results do not belong necessarily to the age in which they are made is apparent from the work of several contemporary observers using telescopes much inferior in point of light. The measures of Dawes with a 3·8-inch aperture, and of Herschel and South with 3¾-inch, will compare favourably with any modern work. It is true that many of Smyth's observations have "weight 1" attached to them; and in some cases that is described as being little or no better than an estimate; but it is also true that the same weight is given to a large number of measures in Class I., where the results are in harmony with other measures which are unquestionably accurate, and that these measures of the Cycle have been used for comparison and computation by all the authorities who dealt with the subject.

The intrinsic accuracy of the measures in Class II. cannot be inferred from the weight thus assigned. It is a noticable fact, however, that larger weights generally are attached to measures in Class I., although the stars are much more difficult, and to the measures of the components of a group which are not measured for the first time. For instance, in β *Lyræ*, to the measure of B is given "weight 9," while C and D, practically not more difficult and observed the same night, have each "weight 1." The result

is that all the Cycle observations, without reference to weight, have been uniformly regarded as measures, and not as mere estimates.

It could hardly be supposed that anyone would give the result of guesswork in tenths of a degree in angle, and tenths of a second in distance, in stars separated from 25″ to 200″ and more, however accurate his judgment might be.

It remains to seek for the most probable explanation of the extraordinary character of the "Cycle" measures. If those in Class I. were of the same character, as to apparent accuracy, as those in Class II., it would be difficult to suggest any explanation other than a general uncertainty about the whole work. That they were made in some way, and in good faith, will not now be and has not at any time been questioned by me.* We know that the observations in the Bedford Catalogue, which, so far as the double stars are concerned, could have been easily made in one year, are scattered over a series of years. It may, I think, be fairly assumed that they were made in leisure moments, without that care which a more zealous and experienced observer would bestow; with no definite idea of their publication and use; and as an amusement rather than as a serious astronomical work. If we assume that at the beginning the observer made it a practice, in measuring double stars, of setting the micrometer wires in accordance with the previous measure of other observers, for the purpose of identification, or for some other reason, and with the intention of making such changes in the wires as the appearance of the object seemed to warrant, we have at once a complete explanation of the very close agreement with other measures. Besides, this theory answers, as perhaps no other will, in accounting in the most satisfactory manner for the equally close agreement when the prior measures were widely in error. In such a star, for instance, as δ *Cancri*, where the companion would be a faint star in the Smyth telescope, it is very easy to see how, with the wire of the micrometer placed where the small star was expected to be found, the observer might imagine he saw the companion, and so record the position as a measure. There are many instances of large errors of this kind thus readily accounted for, which cannot be explained by the chapter of accidents, nor by any other theory of actual observation. Such a method of observing would hardly be expected to give accurate results in the measure of stars in Class II.

Before too severely condemning such course, it should be borne in mind that not only may the observations, as already suggested, have been made without any expectation of their future publication, and without any attempt at special and independent accuracy, but that the manner of observing may have

* It is due to Mr. Proctor, in this connection, to say that his explanation of certain statements under discussion respecting the Bedford Catalogue at the meeting of the Society before referred to was in exact accordance with the facts, and with the meaning of the writer.

been forgotten, or to some extent lost sight of, when the Bedford Catalogue was prepared.

With few exceptions, all the stars in the list prepared for this purpose have been observed, and the measures carefully made with the 18·5-inch refractor of the Dearborn Observatory. I have observed 148 stars, and made altogether more than 350 measures. The stars cited in the paper by Mr. Sadler before referred to were not put in the list except in a few cases where no recent measure had been made. In submitting these measures it is proper to say that generally they were made under unfavourable atmospheric conditions, and when double stars, properly speaking, could not have been observed. In another view it is important to remark the circumstances under which the observations were made.

It is not improbable that some of these stars may prove to be really double, and although no very close new components have been detected in the course of this work, they may hereafter be found, and perhaps with smaller apertures than that used here. Feeling that the measures had no scientific interest or value aside from their bearing upon the question at issue, I did not feel justified in devoting time to the work which could be more profitably spent in other directions. The present examination relates only to the micrometrical work of the Bedford Catalogue, and no attempt has been made to detect or correct errors in other directions. The magnitudes and colours of small stars are, to say the least, of very questionable accuracy in many instances.

In the preceding remarks I have assumed that my measures are substantially correct. These stars can be re-observed, and the accuracy of my work tested at any time, by anyone interested in the matter. There is no danger of these companion stars disappearing, and but little probability of any appreciable change except the trifling variation which, in a few instances, may result from proper motion.

Chicago,
1880, *February* 21.

MICROMETRICAL MEASURES.

β *Cassiopeæ* (p. 2).

Sm	P = 339°·6	D = 201″·0	1838·65
β	324·5	286·	1879·47
β	324·6	297·	1879·54

In this, and all other cases of single distances, only the nearest whole second is given. There are perhaps a dozen faint stars nearer the principal star than the one measured.

147 ℍ. III. (p. 3).

Sm	P = 120°·0	D = 28″·0	1836·81	10 . . 11
β	142·9	63·53	1879·57	9 . . 10

There is a small star near B, 105° 12″.

γ *Pegasi* (p. 5).

Sm	P = 300°·2	D = 181″·0	1835·07	13
β	285·7	161·35	1879·47	10·5
β	285·5	162·29	1879·54	11·0

There is a small star near B, 199°·3 20″±.

ι *Ceti* (p. 6).

H	P = 14°·2	D = 45″ est.	1828	12
Sm	12·0	45·0	1838·82	15
β	16·5	61·89	1877·94	...
β	15·2	62·02	1879·87	11

12 *Ceti* (p. 8).

AB	H	P = 170°· est.	D = 8″ est.	1825 ±
	Sm	180·8	6·5	1837·89
	De	185·2	8·66	1866·8
	β	186·0	8·63	1879·87
	β	187·2	8·76	1879·89
AC	Sm	113·4	201·0	1837·89
	β	110·4	211·84	1879·87
	β	110·2	212·98	1879·89

52 *Piscium* (p. 10).

H	P = 309°·6	D = 25″ est.	1828	14
Sm	311·0	25·0	1836·92	14
β	306·2	38·38	1879·54	11·3
β	305·8	38·14	1879·57	11·5

The companion is called "deep blue."

δ *Andromedæ* (p. 11).

Sm	P = 208°·3	D = 122″·0	1833·54	11½
β	194·4	228·63	1879·47	10
β	194·4	229·13	1879·54	10

α Cassiopeæ (p. 12).

H	P = 275°·4	D = 56″·17	1781·97
Sm	278·4	96·9	1831·86
Ja	279·8	61·19	1854·8
β	280·0	62·25	1877·57
β	279·8	62·51	1878·65

P O. 146 (p. 14).

Sm	P = 289°·9	D = 57″·9	1837·87	9
β	289·8	65·12	1879·87	8·5
β	290·0	65·07	1879·89	...

H 1046. (p. 17).

H	P = 63°·8	D = 15′ est.	1825 ±
Sm	70·0	12·0	1835·68
β	68·0	16·38	1879·49

σ² Piscium (p. 24).

AB	H	P = 285°·5	D = 48″·13	1780·59
	Sm	293·5	56·0	1832·81
	β	293·5	55·79	1878·98
	β	293·9	56·12	1879·57
AC	Sm	235·0	140·0	1832·81
	β	235·4	138·69	1878·98
	β	233·4	138·14	1879·57

This was examined by Mr. Challis with the Northumberland Equatoreal, but no measures are given with that instrument. Sm. has the extraordinary conclusion that 'an inference of binarity is deducible from a comparison of the registered epochs of H and myself, at the rate of 0°·15 per annum, in a *nf* direction, indicating a highly elongated ellipse, with a period of upwards of 2000 years."

η Ceti (p. 27).

Sm	P = 310°·7	D = 239″·10	1838·93	10
β	304·5	224·86	1879·87	9·0
β	304·5	225·59	1879·89	9·5

β Andromedæ (p. 27).

Sm	P = 299°·0	D = 225″·0	1839·54	12

There is nothing in Sm.'s place. I have measured all the stars within 300″. If there is any choice, G or H probably come the nearest to the Cycle star, and agree best in magnitude. It might be, however, any one except B and C, which are too faint to have been seen with the Bedford glass.

AB	β	P = 269°·0	D = 84″·79	1878·82	12–13
		268·7	85·06	1879·56	12·5
AC		141·1	90·88	1878·82	11·5
		140·3	90·64	1879·56	12·0
AD		305·0	126·42	1879·49	10·5
		304·4	126·22	1879·56	11·2
		304·1	125·39	1879·58	11·0
AE		87·3	159·	1878·82	11·0
		87·3	157·66	1879·56	11·0
AF		207·7	210·	1878·82	11·0
AG		217·5	225·	1878·82	10·0
AH		293·6	305·	1878·82	10·5
		293·5	305·	1879·49	10·0
		294·0	304.	1879·55	...

Where only single distances were taken, the nearest second is given.

H 634 (p. 31).

H	P = 295° est.	D = 30″ est.	1825 ±	13–14
Sm	291·0	35·0	1836·89	14
β	271·6	35·44	1878·86	12
β	270·3	35·26	1879·87	11

The agreement would be of more consequence but for the statement of Smyth that his "angle and distance are mere estimations."

42 H VII. (p. 33).

Sm	P = 315°·0	D = 15″·0	1837·66	9 .. 10
β	324·5	13·31	1879·49	8·5 .. 8·5
β	324·5	13·80	1179·54	8·7 .. 8·7

40 *Cassiopeæ* (p. 37).

H	P = 241°·5	D = 45″ est.	1830	11–12
Sm	240·5	42·0	1834·95	12
β	236·9	53·08	1879·33	11
β	237·2	53·49	1879·49	10·8

76 *M. Persei* (p. 40).

Sm	P = 217°·0	D = 45″·0	1837·79	9 .. 14
β	229·6	33·80	1879·49	9·8 .. 10·5
β	228·7	33·76	1879·54	9·5 .. 10·0

The small star is described as "dusky."

107 *Piscium* (p. 41).

H	P = 316°·5	D = 60″ est.	1830	13
Sm	318·3	55·0	1837·03	14
β	340·9	77·95	1879·87	10
β	340·7	77·71	1879·89	10

H speaks of "a third 14–15 m." I have measured a 12 m. star nearer than this. This is not mentioned by Sm., and is probably too faint to have been seen with his aperture, as he speaks of the other as very minute.

β	P = 223°·2	D = 38″·45	1879·87
β	221·7	38·51	1880·01

ζ *Ceti* (p. 43).

Sm	P = 40°·4	D = 165″·0	1835·87	9
β	40·8	185·46	1879·87	9
β	40·6	185 88	1879·89	9·5

55 *Andromedæ* (p. 43).

H	P = 356°·5	D = 20″ est.		14
Sm	350·0	25·0	1832·95	16
β	355·1	60·00	1879·91	12·5
β	355·4	60·27	1879·84	11·5

I could not perceive any nebulous surrounding to the large star as noted by H. Sm. says the small star "was only caught by intense attention," and yet calls it "bluish."

α *Trianguli* (p. 44).

Sm	P = 179°·0	D = 110″·0	1834·67	11

There is no star in Sm.'s place. I have measured three distant stars from which to select the "Cycle" companion.

AB	P = 124°·1	D = 80″·62	1879·85	11·0
AC	182·3	227·90	1879·54	11·0
	182·3	228·	1879·85	11–12
		229·	1879·86	...
AD	172·8	307·77	1879·57	10·5
	172·8	309·	1879·85	...
	172·8	308·	1879·86	...

There is a 10·5 m. star in the direction of 160°·7. In identifying the Sm. star there is apparently but little choice. As a matter of fact, I have no doubt the star D is the one in question.

P I. 222. (p. 47).

AD	Sm	P = 359°·2	D = 165″·0	1834·99
	P. Sm	360·3	180·4	1862·74
	D [?]	361·6	182·5	1862·93
	Kn	360·8	183·68	1862·95
	β	360·7	183·86	1880·03

P I. 227. (p. 48).

Sm	P = 113°·9	D = 43″·6	1834·87
β	117·3	45·49	1879·87
β	117·2	45·44	1879·89

χ *Persei* (p. 57).

AB	Sm	P = 354°·3	D = 65″·0	1835·82	12
	β	352·8	70·57	1879·54	10·5
	β	352·4	70·37	1879·56	10·5
AC	Sh	136·5	124·53	1824·99	
	Sm	43·2	122·0	1835·82	
	β	136·3	122·66	1879·54	

Sm. gives the angle of AC by Sh. as 43°·5, evidently mistaking *nf* for *sf*, from which the "Cycle" angle differs only 0°·3. This is not an error of 90°. The angle recorded by Sh. of 46°·5 *sf* is (90+46·5)=136°·5, and 46°·5 *nf* is (90 − 46·5)=43°·5. It is not easy to see how Sm. could make such an error in his own observation, as there is no such mistake found anywhere else in the work, and he expressly says, "the readings are reckoned from the north, in the direction *nfspn*, a method which Sir J. Herschel proposed for adoption in 1830, as well for its convenience as 'its avoidance of the continual and most annoying mistakes of the method introduced by his father, of reading by quadrants.'"

P II. 61 (= H 1123) (p. 66).

H	P = 252°·0	D = 15″ est.	1828–9
Sm	250·8	14·0	1837·78
β	248·5	20·00	1879·54
β	249·3	20·20	1880·05

θ Persei (p. 67).

Sm	P = 219°·0	D = 27″·0	1833·65
Fl	218·5	68·0	1877·63
β	218·5	68·87	1879·54

γ′ Fornacis (p. 72).

H	P = 169°·4	D = 45″ est.	1830·
Sm	171·0	45·0	1837·94
β	157·5	49·15	1879·95

γ Persei (p. 75).

H	P = 224°·9	D = 60″ est.	1830·
Sm	226·0	55·0	1837·65
β	323·9	57·72	1879·54
β	323·6	57·72	1879·56

β Persei (p. 77).

The companions in order of distance are as follows :—

AB	β	P = 154°·8	D = 59″·37	1877·74	13
	β	155·8	58·21	1878·65	...
	β	155·2	59·59	1880·05	12·5
AC	β	144·2	67·83	1877·74	13 +
	β	145·2	67·62	1878·65	...
	β	144·9	68·77	1880·05	11·8
AD	Sm	195·0	55·0	1835·63	11
	β	192·7	81·99	1877·7410	
	β	192·4	81·86	1878·65	...
	β	192·9	81·86	1880·05	10
DE	β	114·6	10·51	1877·74	12·5
	β	115·6	10·77	1878·65	...
	β	118·4	11·11	1880·05	12·5

52 *Arietis* (p. 77).

Sm	P = 85°·0	D = 105″·0	1835·88
β	82·9	101·56	1879·54
β	82·5	102·23	1879·89
β	82·5	102·82	1880·03

τ^1 *Eridani* (p. 79).

Sm	P = 240°·8	D = 150″·0	1836·90
β	236·0	160·45	1880·00
β	236·2	159·89	1880·02

There are four stars nearer than this, all readily seen with a 6-inch aperture.

AB	Cin[4]	P = 287°·1	D = 4″·77	1877·86
AC	β	99·3	39·97	1877·81
	β	100·0	39·99	1880·00
AD	β	293·3	123·08	1880·00
AE	β	275·9	130·05	1880·00

a Persei (p. 80).

Sm	P = 206°·0	D = 75″·0	1837·64	9
β	195·6	166·42	1879·54	11·0
β	195·4	165·68	1879·56	11·0

δ Persei (p. 82).

Sm	P = 315°·0	D = 140″·0	1833·74	11
β	313·3	97·20	1878·78	10·5
β	312·5	97·27	1879·54	10·0
β	312·0	97·86	1879·56	10·5

19 *Pleiadum* (p. 84).

H	P = 332°·8	D = 45″ est.	1831
Sm	335·0	45·0	1835·01
Rad	331·1	64·61	1863·1
β	329·3	66·52	1879·85
β	329·8	66·71	1879·86

The third measure is from *Radcliffe Observations*, vol. xxiii.

P. IV. 257 (p. 108).

AB	Sh	P = 304°.4	D = 38″.48	1822·09
	Sm	303·8	38·9	1831·95
	De	304·9	39·20	1874·4
	β	304·4	39·02	1880·06
AC	Sm	88·3	70·0	1831·95
	De	88·4	54·68	1874·4
	β	87·4	54·16	1880·00
	β	88·3	54·24	1880·06

The difference in the accuracy of the distances of the old and new stars is noticeable.

66 *Eridani* (p. 111).

Sm	P = 13°.8	D = 47″.0	1832·01	11
β	9·3	52·25	1879·91	9
β	9·4	52·74	1880·00	9·5

β *Eridani* (p. 111).

Sm	P = 147°.5	D = 120″.0	1830·98	12
β	143·9	116·60	1879·91	10·5
β	143·3	116·76	1880·00	10·8

H 3269 (p. 112).

H	P = 60°.4	D = 20″ est.	1831
Sm	60·7	25·0	1837·73
β	61·8	19·49	1880·00
β	60·1	19·93	1880·07

H 3272 (p. 118).

AC	H	P = 39°.3	D = 24″ est.	1831	7–8	13
	Sm	42·2	25·0	1836·71	7½	13
	β	42·8	33·31	1879·59	8	11·0
	β	42·7	33·65	1879·86	8	10·5

Two small stars nearer than this noted by H., all plain with 6-inch.

γ *Orionis* (p. 121).

Sm	P = 150°.0	D = 178″.0	1838·85
Challis	148·1	180·46	1841·19
β	143·4	178·49	1879·91
β	144·4	177·87	1880·00

In placing this star in Class II., the measure of 1841, quoted by Sm., was overlooked.

109 *P. Orionis.* (p. 124).

H	P = 298°·1	D = 18″ est.	1830
Sm	295·0	20·0	1834·71
β	299·9	25·87	1879·99
β	300·3	26·19	1880·00

β *Leporis* (p. 125).

Sm	P = 67°·5	D = 210″·0	1832·00	II

There are two 10 m. stars in this quadrant as follows:—

AD	β	P = 75°·0	D = 206″·36	1879·88
AE	β	57·6	241·53	1879·88

ε *Orionis* (p. 134).

Sm	P = 67°·9	D = 160″·0	1835·02
β	56·8	180·01	1879·88
β	57·3	179·84	1879·91

124 *Tauri* (p. 135).

BC	Σ	P = 315°·7	D = 5″·97	1830·5
	Sm	315·0	5·0	1835·65
	β	314·1	5·67	1879·85
AB	Sm	240·8	98·0	1835·65
	β	214·8	145·73	1879·85
AD	Sm	170·0	82·0	1835·65
	β	175·7	79·00	1876·85

The close stars BC make Σ 755.

ν *Aurigæ* (p. 139).

H	P = 208°·2	D = 53″·71	1782·68
Sm	201·9	85·0	1833·75
β	207·0	54·97	1877·82
β	206·7	54·32	1879·85

78 *M. Orionis* (p. 138).

Sm	P = 32°·0	D = 45″·0	1836·79
	22·5	50·70	1878·10
β	21·6	51·00	1878·16
β	20·8	50·28	1880·02
β	22·2	50·88	1880·03

B is double (= β 559).

37 *M. Aurigæ* (p. 140).

Sm	P = 357°·0	D = 25″·0	1836·79
β	345·8	17·45	1879·85

A coarse cluster, and many stars in the field. The two measured come nearest to the Cycle description.

β *Aurigæ* (p. 143).

H	P = 35°·8	D = 169″·10	1782·23
Sm	38·2	185·0	1837·70
Ch	38·0	183·45	1841·96
β	39·0	183·72	1879·86
β	38·7	184·03	1880·05

The measure in 1841 by Challis is given in the Cycle. Hence this star properly belongs to Class I.

θ *Aurigæ* (p. 143).

H	P = 286°·0	D = 35″ ±	1782·68
Sm	289·0	30·0	1832·64
OΣ	290·9	43·29	1852·12
Fl	293·3	45·50	1877·79
β	292·7	45·58	1880·05
β	294·4	45·44	1880·14

There is some change from proper motion. The distance of Sm. is much too small.

35 *Camelopardi* (p. 144).

H	P = 13°·8	D = 35″ est.	1830
Sm	14·4	30·0	1833·66
Ma	12·5	39·91	1843·3
De	13·1	39·44	1867·0

5 *Lyncis* (p. 148).

AB	H	P = 139°·5	D = 20″ est.	1830
	Sm	130·0	25·0	1833·77
	β	139·1	30·34	1879·86
AC	S	272·1	95·44	1825·05
	Sm	271·9	96·0	1833·77
	β	272·5	95·93	1879·86

μ Geminorum (p. 148).

Sm	P = 89°·0	D = 80″·0	1831·98
β	76·5	72·94	1880·00
β	76·7	72·38	1880·03

ζ Canis Majoris (p. 149).

Sm	P = 338°·0	D = 167″·0	1833·81
β	337·2	174·15	1879·88

β Canis Majoris (p. 150).

Sm	P = 339°·0	D = 104″·0	1833·76
β	339·0	184·63	1879·88
β	339·1	185·29	1879·91

10 *Monocerotis* (p. 151).

Sm	P = 225°·0	D = 72″·0	1832·99

There are two 10 m. stars in this quadrant. It is impossible to tell which Sm. observed, since his measures fit one about as well as the other.

AB	β	P = 256°·4	D = 76″·99	1879·88
	β	256·8	76·94	1879·91
AC	β	231·0	80·64	1879·88
	β	. . .	81·18	1879·91

γ Geminorum (p. 154).

AB	Sm	P = 335°·0	D = 75″·0	1830·80
	β	335·5	141·74	1880·03
AC	Sm	290·0	110·0	1830·80
	β	294·7	133·04	1880·03

There are several stars in the direction of B, but nothing nearer Smyth's place.

Sirius (p. 158).

Sm	P = 45°·0	D = 150″·0	1835·80

It has been long known that no star was to be found in or near this place. It presents an instance similar to that of Procyon, about which so much has been written, the only difference being that the error in the distance of the star near Procyon is much less than in this case. Mr. George Hunt has made a thorough examination of the vicinity of Sirius, and it is

[illegible] (p. [illegible]).

8m	P = [illegible]	D = [illegible]	[illegible]
β	[illegible]	[illegible]	[illegible] [illegible]

[illegible]

[illegible] (p. [illegible]).

AB	8m	P = [illegible]	ΔR.A. = [illegible]	[illegible]
	β	[illegible]	D = [illegible]	[illegible]
	β	[illegible]	[illegible]	[illegible]
AC	8m	[illegible]	ΔR.A. = [illegible]	[illegible]
	β	[illegible]	—	[illegible]
	β	[illegible]	—	[illegible]

[illegible] (p. 26[illegible]).

8m	P = 94·1	ΔR.A. = 19·[illegible]	1832·[illegible]	[illegible]
β	107·[illegible]	...	1879·32	[illegible]

The difference in R.A. agrees substantially. There is a faint star between in the direction of 94°·7.

8 Ursæ Majoris (p. 262).

8m	P = 127·3	ΔR.A. = 20·4	1832·41
β	124·1	D = 188·59	1879·30

β Corvi (p. 271).

AB	8m	P = 119·5	ΔR.A. = 27·4	1831·34
	β	129·7	...	1879·25
AC	8m	296·5	ΔR.A. = 28·0	1831·34
	β	292·0	...	1879·25

8 Can. Ven. (p. 272).

8m	P = 228·0	D = 297·0	1835·24	10
β	220·5	276·20	1879·26	9

8 Virginis (p. 286).

8m	P = 144·2	ΔR.A. = 5·8	1832·31	10·5
β	142·5	D = 151·71	1879·25	10·5
β	142·2	152·36	1879·34	10·5

ε Virginis (p. 290).

Sm	P = 123°·8	D = 229″·0	1838·42	15
β	120·6	240·49	1879·25	
β	120·3	241·29	1879·34	11·8

53 *Virginis* (p. 292).

H	P = 30° est.	D = 50″ est.		
Sm	35·0	45·0	1833·40	15
β	9·3	70·80	1878·24	12–13
β	10·0	71·03	1879·25	12–13

Sm. called the small star "bluish." The positions of H are estimated from a diagram.

61 *Virginis* (p. 295).

Ḣ	P = 345°· ±	D = 73″·25	1783·0
Sm	340·6	ΔR.A. = 2ˢ·8	1832·31
Kn	22 6	D = 169″·29	1862·29
β	24·8	189·96	1878·35
β	25·8	191·43	1879·25
β	25·8	...	1879·28

There has been considerable change due to the proper motion of the large star, and there is a large error in the repetition of Ḣ's angle by Sm. Kn. has called attention to this, and states that Sm.'s angle should be about 14° or 15°.

α Virginis (p. 296).

Sm	P = 57°·3	ΔR.A. = 19ˢ·4	1833·28	10
β	32·9	D = 144″·82	1879·25	12
β	61·5	...	1879·34	10·5
β	61·9	359·84	1879·35	...

In the first observation a much nearer and fainter star was measured than the Sm. star. Later the proper companion was identified.

μ Hydræ (p. 300).

Sm	P = 80°·9	ΔR.A. = 19ˢ·5	1834·38
β	94·1	D = 138″·68	1879·28

quite sufficient to give his results. The real position of the Smyth 10 m. star is:—

Hunt	P = 47°.5	D = 10″.44 ±	1879.19

The distance, being too great for the screw, was roughly measured by the "comb" of the micrometer.

H 740 (p. 165).

H	P = 13° est.	D = 20″ est.	1825 ±
Sm	10.5	15.0	1835.22
Rad	7.2	20.95	1863.1
β	6.7	20.69	1880.00
β	8.0	20.91	1880.06

There is a very faint companion to B, 12.5 magnitude:—

β	P = 281°.9	D = 8″.67	1880.06

ζ *Geminorum* (p. 169).

AC	Sm	P = 85°.0	D = 65″.0	1831.81
	β	83.4	87.20	1880.00
	β	83.7	87.24	1880.03
AB	Sh	355.5	91.03	1821.23
	Sm	355.0	90.0	1831.81
	De	351.7	93.54	1874.06
	β	351.6	93.67	1880.00

δ *Canis Majoris* (p. 170).

Sm	P = 224°.0	D = 165″.0	1832.90
β	224.0	266.88	1879.98
β	224.1	265.97	1880.0

η *Canis Majoris* (p. 175).

Sm	P = 285°.0	D = 169″.0	1833.82
β	284.9	178.67	1879.98
β	285.0	178.63	1880.03

β *Canis Majoris* (p. 176).

AB	Sm	P = 80°.0	D = 35″.0	1831.84
	β	76.8	123.95	1879.98
	β	76.6	122.35	1880.00
AC	Sm	312.0	105.0	1831.84
	β	309.8	139.09	1879.98
	β	310.1	138.87	1880.00

There are several excessively faint stars on the following side of the primary, two of which were measured, the first being the nearest that could be seen.

P = 82°·3	D = 47″·77	1880·16
22·7	97·02	1879·98

The first is very faint, and will require probably a 12-inch aperture to show it, as it has less than half the light of the companion to Procyon.

H 2435 (p. 195).

H	P = 202°·2	D = 3″ est.	1830
Sm	205·0	4·0	1834·29
β	206·3	6·91	1880·03

P. VIII. 81 (p. 197).

Σ	P = 331°·5	D = 18″·20	1828·71
Sm	325·0	15·0	1838·69
β	331·1	17·95	1880·03

Observed because no other measures have been made since Struve.

θ Cancri (p. 198).

♅	. . .°	D = 44″·87	1783·05
H	P = 61·3	60 est.	1830
Sm	62·4	65·0	1833·31
Kn	59·3	58·41	1862·31
β	59·9	60·70	1879·25
β	59·5	60·72	1880·05

41 *Sextantis* (p. 234).

A and B	H	P = 305°±	D = 20″±		6 ... 17–1
	Sm	310·0	20·0	1838·31	16
	β	303·8	26·95	1878·10	
A and C	Sm	120·4	290·0	1838·31	10
	β	72·5	233·48	1879·33	9–10
	β	72·3	233·42	1879·34	

There is no star in the place given by Sm. of C. That measured by me is undoubtedly the one referred to in the Cycle.

β Ursæ Majoris (p. 237).

Sm	P = 172°·6	D = 75″·0	1831·37	
β	353·9	245·	1879·29	Single distance

There is no star in the place designated by Sm. as belonging to a "pale grey" 11 m. star. The one I have roughly measured is 9–10 m., and seemed most likely to be the one. There is a fainter star a little more than half the distance between the two.

δ Leonis (p. 244).

AB	Sm	P = 50°·0	ΔR.A. = 4ˢ·9	1836·21
	β	44·5	D = 95″·20	1879·25
	β	44·2	95·55	1879·28
AC	Sm	345·9	ΔR.A. = 2ˢ·9	1836·21
	β	345·2	...	1879·25
	β	344·7	...	1879·28

δ Crateris (p. 248).

Sm	P = 94°·1	ΔR.A. = 19ˢ·8	1834·33	11
β	107·8	...	1879·34	10·5

The difference in R.A. agrees substantially. There is a faint star between in the direction of 94°·7.

δ Ursæ Majoris (p. 262).

Sm	P = 127°·3	ΔR.A. = 20ˢ·4	1832·41
β	124·1	D = 188″·59	1879·30

β Corvi (p. 271).

Aβ	Sm	P = 119°·5	ΔR.A. = 27ˢ·4	1831·34
	β	129·7	...	1879·25
AC	Sm	306·5	ΔR.A. = 28·0	1831·34
	β	292·0	...	1879·25

8 *Can. Ven.* (p. 272).

Sm	P = 228°·0	D = 297″·0	1835·24	10
β	220·5	276·20	1879·26	9

δ Virginis (p. 286).

Sm	P = 144°·2	ΔR.A. = 5ˢ·8	1832·31	10·5
β	142·5	D = 151″·71	1879·25	10·5
β	142·2	152·36	1879·34	10·5

ε Virginis (p. 290).

Sm	P = 123°.8	D = 229″.0	1838·42	15
β	120·6	240·49	1879·25	
β	120·3	241·29	1879·34	11·8

53 *Virginis* (p. 292).

H	P = 30° est.	D = 50″ est.		
Sm	35·0	45·0	1833·40	15
β	9·3	70·80	1878·24	12–13
β	10·0	71·03	1879·25	12–13

Sm. called the small star "bluish." The positions of H are estimated from a diagram.

61 *Virginis* (p. 295).

♅	P = 345° ±	D = 73″.25	1783·0
Sm	340·6	ΔR.A. = 2ˢ.8	1832·31
Kn	22·6	D = 169″.29	1862·29
β	24·8	189·96	1878·35
β	25·8	191·43	1879·25
β	25·8	...	1879·28

There has been considerable change due to the proper motion of the large star, and there is a large error in the repetition of ♅'s angle by Sm. Kn. has called attention to this, and states that Sm.'s angle should be about 14° or 15°.

α Virginis (p. 296).

Sm	P = 57°.3	ΔR.A. = 19ˢ.4	1833·28	10
β	32·9	D = 144″.82	1879·25	12
β	61·5	...	1879·34	10·5
β	61·9	359·84	1879·35	...

In the first observation a much nearer and fainter star was measured than the Sm. star. Later the proper companion was identified.

μ Hydræ (p. 300).

Sm	P = 80°.9	ΔR.A. = 19ˢ.5	1834·38
β	94·1	D = 138″.68	1879·28

72 *Virginis* (p. 301).

Σ	P = 16°·1	D = 30″·06	1831·53
Sm	18·5	25·0	1832·26
OΣ	16·8	28·98	1855·22
β	16·7	29·17	1879·25
β	15·6	29·30	1879·34

Sm. does not seem to have been aware that this was one of the Σ stars, and only refers to its being registered by H, but without measures. And yet the small star, which is only 11·5 of Σ's scale, was well enough seen to be described in the Cycle as "violet tint."

75 *Virginis* (p. 303).

H	P = 110°·3	D = 90′ ±	1830	13
Sm	112·0	93·0	1835·32	14
β	109·7	78·53	1879·25	11
β	110·5	78·04	1879·38	11·5

P XIII. 163 (p. 306).

Sm	P = 215°·0	D = 68″·0	1831·26	13
β	228·4	91·61	1879·24	11
β	227·9	91·56	1879·25	...

This faint star is called "ash-coloured" by Sm.

P XIII. 171 (p. 308).

Σ	P = 335°·7	D = 27″·76	1829·35
Sm	336·3	30·0	1830·99
β	335·4	27·93	1878·25
β	336·4	27·68	1878·28
β	334·7	27·51	1879·25

These are all the measures of this pair.

85 *Virginis* (p. 309).

H	P = 317°·8	D = 35″ est.	1830	15
Sm	320·0	30·0	1834·28	16
β	311·9	43·39	1879·28	
β	311·7	43·14	1879·37	11·7

η Bootis (p. 311).

Sh	P = 119°·5	D = 126″·20	1822·66
Sm	117·6	123·7	1832·42
β	110·8	114·75	1879·25
β	111·0	114·50	1879·34

The observations of Sh. were not noticed when this star and the next were entered on the list.

τ Virginis (p. 312).

Sh	P = 290°·0	D = 79″·29	1823·30	9
Sm	291·4	78·6	1831·24	8·5
β	289·6	79·57	1879·28	8·5

α Bootis (p. 315).

Sm	P = 49°·3	ΔR·A· = 15ˢ·1	1835·47	11
β	44·7		1879·25	...
β	44·8		1879·3	10

Very distant; called "lilac" by Sm.

P XIV. 95 (p. 322).

Sm	P = 266°·5	D = 40″·0	1836·49	12
β	268·4	25·54	1879·29	10
β	267·9	24·79	1879·32	10

This is described as "a truly difficult object," and best "seen on averting the eye," but called "bluish."

5 *Ursæ Minoris* (p. 323).

Sm	P = 135°·5	D = 45″·0	1833·61	11
β	129·4	56·45	1879·30	10·5

β Ursæ Minoris (p. 330).

Sm	P = 5°·5	D = 165″·0	1833·64	11
β	342·8	207·82	1879·30	10·5

δ Bootis (p. 338).

Sh	P = 79°·6	D = 105″·33	1822·80
Sm	75·0	110·0	1835·49
Σ	78·9	104·87	1835·60
β	78·7	104·54	1879·25
β	78·7	104·93	1879·34
β	78·4	104·93	1879·47
β	78·6	104·67	1879·48

ι Draconis (p. 344).

Sm	P = 46°·2	D = 117″·0	1833·60	9
β	50·0	254·67	1879·26	8·7

α Serpentis (p. 348).

H	P = 359°·5	D = 60″ est.		14
Lam.	179·7	61·48	1830 ±	
Sm	361·5	50·0	1837·32	15
β	354·5	58·89	1878·35	...
β	353·2	58·68	1879·30	12

The angle from Munich Observations should evidently be increased 180°.

ζ Ursæ Minoris (p. 351).

Sm	P = 210°·5	D = 310″·0	1834·64	11
β	212·9	313·48	1879·30	11·8

δ Ophiuchi (p. 356).

Sm	P = 131°·5	D = 319″·0	1834·45	10
β	127·2	265·70	1879·38	10
β	291·6	64·64	1879·38	12·5

I have measured another star much nearer than that observed by Sm.

ν Coronæ Borealis (p. 358).

AB	Sm	P = 24°·2	D = 86″·9	1831·59
	β	22·3	86·13	1879·29
	β	22·5	86·48	1879·34
AC	Sm	55·2	128·0	1831·59
	β	52·2	123·85	1879·29
	β	52·3	123·37	1879·34
BD	Sm	221·5	10·0	1831·59
	β	222·0	13·05	1879·29
	β	223·4	13·41	1879·34
Aa	β	29·9	56·60	1879·29
	β	29·2	55·37	1879·34

I have measured a faint star, 12 m. nearer the principal star than any before observed.

ν^1 *Coronæ Borealis* (p. 362).

Sm	P = 16°·9	D = 137″·0	1838·57	12
β	15·7	104·68	1879·29	
β	15·5	104·45	1879·30	10

η *Draconis* (p. 365).

AF	Sm	P = 31°·0	D = 190″·0	1833·62
	β	18·4	366·	1879·27
	β	18·8	367·	1879·36

There seems to be no doubt of the identity of the star. My measures are single distances only. Besides the close companion (OΣ 312), there are several faint stars nearer than the one observed by Sm., which are :—

AC	β	P = 83°·5	D = 107″·	1879·27	13
AD	β	158·3	. . .	1879·27	12–13
AE	β	172·1	. . .	1879·27	11

β *Herculis* (p. 366).

Sm	P = 276°·0	D = 278″·0	1835·66	11*m*
β	274·3	256·15	1879·30	9
β	274·3	257·74	1879·34	9

η *Herculis* (p. 372).

Sm	P = 265°·0	D = 141″·0	1835·65
W and S	261·4	113·7	1873·47
W and S	261·6	114·3	1874·55
β	261·6	113·39	1879·27

At one time Σ suspected the large star to be a close pair, but afterwards rejected it as single. Of the correctness of this conclusion there has never been any doubt. Sm. measured the angle and distance in 1842.

η *Ophiuchi* (p. 379).

Sm	P = 260°·5	D = 269″·0	1833·61	13
β	267·4	256·24	1879·34	10·8
β	267·5	256·86	1879·41	10

36 *Ophiuchi* (p. 381).

Sm	P = 289°·9	D = 193″·8	1831·57
Ja	298·3	180·0	1854·07
Flm	306·4	199	. . .
β	305·9	198·03	1879·34

Flammarion gives the annual proper motion 1″·27 in the direction of 202°. The distance found by Jacob appears to be much too small. According to my observation, the distance in 1854 would have been about 192″, and Smyth's position substantially correct. The observation of Jacob is taken from the *Hand Book of Double Stars.*

ε *Ursæ Minoris* (p. 381).

Sm	P = 358°·5	D = 41″·0	1835·55	12
β	6·7	77·08	1879·27	11·5
β	6·4	78·23	1879·36	11·0

The companion called "pale blue."

λ *Herculis* (p. 390).

Sm	P = 295°·0	Δ R.A. = 23ˢ·4	1834·58	10
β	251·8	...	1879·30	10

Nothing in Sm.'s place. The one measured is probably the Cycle star, as it corresponds fairly in magnitude and difference of R.A. There are a good many faint stars nearer.

γ *Draconis* (p. 400).

Sm	P = 123°·5	Δ R.A. = 9ˢ·7	1832·63	12
β	116·3	D = 124″·77	1879·27	10·8

There are several stars nearer than this, measures of which will be found in my first series of observations (*Mem. R. A. S.* xliv.). A later observation of the nearest gives

P = 151°·8	D = 20″·90	1879·27	13

τ *Ophiuchi* (p. 402).

Sm	P = 114°·5	D = 83″·1	1832·55	10
Sm	115·0	82·7	1838·58	...
β	127·0	100·07	1879·34	...
β	126·8	100·53	1879·37	9·5

18 *M. Clyp. Sob.* (p. 415).

Sm	P = 322°·0	D = 35″·0	1835·56	9 .. 11
β	131·7	24·74	1879·37	8·7 .. 10·8
β	134·4	24·89	1879·43	9 .. 11

H 2002 (p. 414).

Sm	P = 250°·0	D = 20″·0	1836·55	8½ .. 10
β	253·6	19·57	1878·42	...
β	252·9	20·09	1879·37	10 .. 10

χ *Draconis* (p. 421).

Sm	P = 54°·3	ΔR.A. = 14ˢ·6	1834·61
β	26·1	D = 144′·22	1879·30
β	26·9	145·00	1879·36

There is a faint companion to the small star, BC, 315° ± ; 8″ ±.

2 *Aquilæ* (p. 426).

Sm	P = 133°·8	D = 55″·0	1831·58
β	130·1	52·26	1879·34
β	130·4	52·52	1879·37

5 *Aquilæ* (p. 427).

Sm	P = 145°·0	D = 30″·0	1838·60
β	145·2	27·25	1879·34

25 *M. Sagittarii* (p. 420).

Sm	P = 149°·5	D = 27″·0	1836·60	8 .. 8

No stars answering this description in the cluster. The star which agrees most nearly in place and magnitude is Lalande 34207. The measures of this and a distant star are :—

β	P = 252°·4	D = 66″·60	1879·34	8 .. 9·3

In the field, *sp* is a pair of 9 m. stars, 175° : 10″ ±.

In the field, *np*, are several 9–10 m. stars, about equal distances from each other. Two of them were measured :—

P = 301·3	D = 30·90	1879·34	9 .. 9·3

Probably the wide stars first observed are the ones mentioned by Sm.

110 *Herculis* (p. 428).

H	P = 108°·9	D = 60″ est.	1830
Sm	110·0	55·0	1831·68
β	92·3	61·16	1879·30
β	91·8	61·07	1879·57

There is a faint star between these, about one third the light of the old companion.

β	P = 95°·5	D = 44″·70	1879·30

P XVIII. 197 (p. 430).

Sm	P = 168°·9	D = 99″·0	1833·66
β	170·4	114·11	1879·34
β	170·6	113·86	1879·37

There is a 13 m. star near A, measured as follows:—

β	P = 356°·8	D = 22″·53	1879·37

ν^1 *Lyræ* (p. 432).

AB	Sm	P = 78°·0	D = 35″·0	1836·81
	β	70·2	36·46	1879·30
	β	70·8	36·01	1879·36
AC	S	124·0	59·84	1825·61
	Sm	124·1	58·6	1836·81
	β	122·1	58·65	1879·30
	β	122·4	58·51	1879·36
CD	Sm	208·0	12·0	1836·81
	β	212·9	18·23	1879·30
	β	212·3	17·52	1879·36

β *Lyræ* (p. 433).

AB	Σ	P = 149°·8	D = 45″·77	1835·2
	Sm	150·1	45·8	1834·73
	Hl	149·2	45·83	1876·7
	β	149·1	45·64	1878·39
AC	Sm	319·5	60·0	1834·73
	β	317·7	66·02	1879·30
	β	317·7	66·49	1879·36
AD	Sm	25·0	71·0	1834·73
	β	19·1	85·95	1879·30
	β	18·6	85·61	1879·36

It did not seem necessary to measure AB again, as the observations are sufficient to show the absence of any change. The measure by Hall is the mean result of 8 nights. This star is inserted here as an illustration of the apparent accuracy of the measures of Sm., when the star had been previously observed. The other companions, C and D, had not been measured at this time by other observers.

2024 *H. Aquilæ* (p. 437).

H	P = 220° est.	D = 10″ est.	1820 +	11 . . 12
Sm	220·0	6·5	1835·57	7½ . . 11
Cin	219·5	11·28	1879·54	8·5 . . 10
β	218·8	11·31	1879·43	9·5 . . 10·8
β	218·6	11·23	1879·54	...

This is H 870, but there is an error of 5^m in the R.A. of H, and it is repeated by Sm. In another observation H gives magnitudes 9 . . 10.

γ *Lyræ* (p. 439).

Sm	P = 23°·5	Δ R.A. = 3ˢ·7	1834·59	11
β	20·9	D = 119″·44	1879·30	10·5

There are several stars nearer than this, besides the OΣ companion.

P XVIII. 299 (p. 440).

H	P = 270° est.	D = 50″ est.	1828	15
Sm	265·0	50·0	1838·55	14
β	275·9	42·30	1879·31	11·5
β	276·6	41·56	1879·35	11·5

There is a distant 9 m. star in 33°·1, and a smaller star, about two-thirds as far, in 15°·2. The principal star is 16 *Lyræ*.

ζ *Aquilæ* (p. 441).

Sm	P = 84°·0	Δ R.A. = 8ˢ·4	1832·47
β	76·9	D = 156″·65	1879·34

δ *Draconis* (p. 445).

Sm	P = 19°·7	Δ R.A. = 2ˢ·7	1834·78
β	27·8	D = 155″·45	1879·35
β	27·7	154·74	1879·66

There is a 12 m. star nearer than this:—

β	P = 354°·3	D = 90″·35	1879·35

δ Aquilæ (p. 446).

Sm	P = 259°·3	D = 96″·5	1833·62
P. Sm	275·6	94·96	1862·74
β	271·7	101·71	1879·34
β	272·0	100·96	1879·37

γ Aquilæ (p. 455).

Sm	P = 255°·0	D = 136″·0	1836·74
β	257·4	132·24	1879·31
β	258·0	132·93	1879·52

56 *Aquilæ* (p. 462).

Sm	P = 72°·5	D = 43″·0	1834·63	[illegible]
β	78·8	46·92	1879·51	11·8

β Aquilæ (p. 464).

Sm	P = 343°·0	D = 175″·0	1834·50	10
β	347·3	151·66	1879·34	10·5
β	347·2	151·77	1879·37	10·0
β	346·9	152·24	1879·43	10·5

α² Capricorni (p. 472).

Da	P = 220°·1	D = 44″·14	1830·60
Sm	221·8	43·0	1838·72
β	221·3	44·52	1879·43
β	221·2	44·09	1879·46
β	220·9	44·35	1879·58

Sm. does not mention the earlier measure.

α² Capricorni (p. 472).

Sm	P = 155°·7	D = 198″·0	1838·72	9
β	156·4	154·39	1879·43	9·5
β	156·1	154·47	1879·36	9·5

M. 29 *Cygni* (p. 478).

Sm	P = 299°·5	D = 55″·0	1835·48
β	295·1	66·27	1879·31

Not easy of identification. These stars are the nearest in position and distance to the observation of Sm., but differ in magnitudes. Sm. called the stars 8 and 11 m., and I have entered those measured by me, 8·5 and 8·8 m. There is a star, 11 m., nearer A than the one given above.

$P = 319°·2 \qquad D = 38''·62 \qquad 1879·31$

1 *Aquarii* (p. 483).

AB	Sm	P = 220°·0	D = 20″·0	1838·69	13
	β	217·3	56·35	1879·43	11·5
	β	217·4	55·39	1879·46	11·5
AC	Sm	45·0	35·0	1838·69	14
	β	38·9	72·97	1879·43	11·3
	β	38·9	72·82	1876·46	11·3

α *Delphini* (p. 483).

Sm	P = 117°·8	D = 105″·0	1835·72	13
β	113·8	80·44	1879·31	10·8
β	113·9	80·90	1879·37	...

Two other stars nearer A than this.

α *Cygni* (p. 485).

Sm	P = 102°·5	D = 108″·5	1837·65
Bond	88·6	95·5	1848·40
β	106·2	75·85	1879·31
β	105·8	75·41	1879·35
β	105·9	75·09	1879·40

There seems to be some error in the single observation by Bond. At my request a search was made for the original record at the Harvard College Observatory, but it could not be found.

η *Cephei* (p. 488).

Sm	P = 29°·0	D = 45″·0	1834·74	13
β	33·5	100·20	1879·35	11·5
β	34·1	100·89	1879·36	11·0

The companion is called "dusky."

χ Delphini (p. 490).

H	P = 22°·0	D = 60″ est.	1828	13
Sm	21·0	55·0	1838·84	14
β	21·5	40·03	1879·50	12
β	22·1	39·91	1879·51	. . .

The small star is said to be "pale lilac."

59 *Cygni* (p. 492).

AC	H	P = 143°·8	D = 20″ est.	1828
	Sm	142·0	30·0	1836·72
	β	140·6	26·73	1879·35

ζ Cygni (p. 497).

Sm	P = 51°·8	D = 105″·0	1835·55
β	67·2	101·39	1879·40
β	67·2	101·11	1879·41

α Cephei (p. 499).

Sm	P = 36°·0	D = 150″·0	1832·74	10
β	23·5	209·48	1879·35	10·8
β	23·4	208·95	1879·47	. . .

The small star is described as "pale sapphire."

β Aquarii (p. 502).

H	P = 322°·8	D = 20″ est.	1828
Sm	320·0	25·0	1833·73
β	320·0	34·26	1877·70
β	318·0	34·60	1879·57

ε Pegasi (p. 507).

AB	H	P = 322°·7	D = 90″·93	1782·89
	Sm	327·0	85·0	1833·67
	β	325·1	81·53	1879·51
	β	325·3	81·18	1879·57
AC	S	323·0	138·51	1825·15
	Sm	324·3	138·1	1833·67
	De	321·7	140·41	1874·8
	β	321·4	140·27	1879·51

The earlier measures are quoted by Sm.

P XXI. 312 (p. 509).

AB	H	P = 93° est.	D = 15″ est.	1825 ±	15
	Sm	95·0	15 0	1838 66	14
	P. Sm	92·5	20·32	1862·74	...
	β	92·7	20·71	1878·39	
	β	93·1	20·06	1879·54	11·0
AC	H	315 est.	20 est.	1825 ±	17
	P. Sm	323·6	21·74	1862·7	...
	β	324·0	24·40	1878·39	...
	β	325·8	24·10	1879·54	11·5

Sm., who could see B only, calls it "blue."

20 *Pegasi* (p. 509).

H	P = 320° est.	D = 40″ est.	1825·8	12
Sm	330·0	35·0	1838·66	14
β	326·1	51·28	1877·78	...
β	324·6	51·09	1879·51	11·5
β	325·9	50·90	1879·57	11·3

The small star was called "blue."

α Aquarii (510).

Sm	P = 44°·0	D = 116″·0	1837·71	13
β	41·5	115·08	1879·50	12·0
β	41·5	114·20	1879·51	11·5

π[1] *Pegasi* (p. 512).

AB	Sm	P = 258°·7	D = 74″·0	1835·56
	β	261·7	72·62	1877·78
	β	261·3	73·00	1879·40
AC	Sm	90·2	185·00	1835·56
	β	89·9	184·74	1879·40

The agreement here is very singular, considering the distance of the components.

2 *Lacertæ* (p. 515).

Sm	P = 6°·9	D = 35″·0	1835·84	13
β	9·5	48·44	1879·46	11·0
β	9·9	47·96	1879·49	10·7

ζ Pegasi (520).

	Sm	P = 15°·5	D = 65″·0	1831·68	13
Aa	β	137·8	64·33	1879·54	11
Ab	β	5·2	177·	1879·51	11

Nothing in the place of Sm. The more distant of the two I have measured is probably the one. Single distance only of that star.

ξ Pegasi (p. 522).

AB	H	P = 122°·8	D = 15″ est.	1828·	18
	Sm	120·9	15·0	1834·79	15
	De	117·7	12·17	1866·8	12·0
	β	112·0	11·99	1878·62	...
	β	111·4	11·87	1878·63	...
	β	113·0	11·99	1879·54	11·8
AC	Sm	32·5	110·0	1834·79	15
	β	21·8	127·03	1879·51	12·5
	β	21·7	127·63	1879·54	12·0

β Pegasi (p. 527).

	H	P = 204°·1	D = 80″ est.	1828·	16–17
	Sm	199·0	75·0	1836·88	15·0
	β	208·0	97·91	1879·49	11·5
	β	208·5	98·86	1879·53	11·0

4 *Cassiopeæ* (p. 534).

AB	Sm	P = 225°·0	D = 97″·5	1834·67
	β	226·2	98·13	1879·47
AC	Sm	260·0	218·0	1834·67
	β	258·3	214·35	1879·47
CD	Sm	256·0	10·0	1834·67
	β	35·3	10·30	1879·47

The comparatively close agreement of the measures, with the exception of the angle of CD, is remarkable, if there are no prior measures. I am not aware of any, but Sm. says :—"When the great Northumberland Equatoreal was mounted at Cambridge, I requested the Rev. James Challis to scrutinize the object, which he kindly did, with similar results to my own."

ι Piscium (p. 536).

Sm	P = 39°·5	D = 199″·0	1835·83
β	33·5	281·	1879·54

Single distance only. There is a very faint star, 13–14 m., much nearer, but too faint to be seen with any moderate aperture, as it would be decidedly fainter than H's 20 m., 288°·2 : 54″.

κ Andromedæ (p. 536).

AB	Sm	P = 186°·0	D = 47″·0	1836·72	14
	β	188·9	46·34	1878·78	11
	β	189·1	47·16	1879·46	11
	β	188·1	46·43	1879·41	11
AC	Sm	295·5	98·5	1836·72	12
	β	294·6	103·17	1879·46	11

The small stars are called "dusky" and "ash-coloured."

P. XXIII. 171 (p. 537).

AB	Σ	P = 103°·8	D = 5″·38	1831·85
	Sm	105·0	5·0	1833·75
	β	98·7	5·58	1879·46
AC	Sm	155·0	175·0	1833·75
	β	129·9	381·	1879·46

Σ did not measure the third star. The difference in accuracy in the measures of Sm. of these two stars is very apparent, and yet, according to his estimate, the distant star is three magnitudes brighter than Σ's companion.

Note.—Since the foregoing observations were forwarded, I have measured two of the few remaining stars of Class II. This substantially concludes the work. The only wide pair not measured is 44 M. *Cancri* (p. 200).

ψ Leonis (p. 220).

Sm	P = 140°·0	D = 260″·0	1834·19
β	139·7	280·38	1880·15
β	139·5	280·66	1880·17
β	139·7	279·73	1880·18

30 *Canis Majoris* (p. 174).

Sm	P = 73°·0	D = 85″·0	1834·83
β	77·9	84·40	1880·19
β	78·0	84·46	1880·21

Although the two near companions of H are easily seen with

6-inch, Sm. does not mention them. I measured them as follows:—

A and B	P = 91°.5	D = 7″.72	1880.19
	90.4	7.97	1880.21
A and C	81.0	14.30	1880.19
	79.6	14.35	1880.21

Notes on a Paper entitled "An Examination of the Double-Star Measures of the Bedford Catalogue," by S. W. Burnham, Esq. By E. B. Knobel, Esq.

The above paper by Mr. Burnham is mainly devoted to an examination of those objects in the Bedford Catalogue designated by the late Admiral Smyth as "stars with comites," of wide double stars, and of stars with faint companions.

Most observers who have bestowed any attention on this class of so-called "double-stars" have noticed discrepancies, in their results, sometimes of considerable amount, compared with the figures given in the "Cycle." My attention was in the first instance drawn to discrepancies in some of the Cycle results, about the year 1867 or 1868, by the Rev. E. Lowe, of Atherstone, who was then engaged in a systematic examination of all the objects in the Bedford Catalogue, with the fine 8½-inch Alvan Clark Refractor, originally belonging to Mr. Dawes, and which now forms the equatorial telescope of the Temple Observatory at Rugby.* Six years ago I measured several of these objects, and finding that my results differed widely from those in the Cycle, I communicated them to a then member of the council of the Royal Astronomical Society, stating that "I could not but think that Smyth's measures were in error." I was strongly counselled to publish these results, but I did not think them sufficiently numerous to be of much value.

Mr. Burnham's micrometrical measurements are very valuable as correcting the, in many cases, erroneous figures given in the Cycle for the position-angle and di[illegible] of these wide objects.

But there are m[illegible]ts connected [illegible]al Smyth's observations of the[illegible]hich have cit[illegible] Mr. Burnham's notice, or w[illegible]ontained in p[illegible]nd MSS. to which he may n[illegible] access.

It is eminently [illegible] in investi[illegible] kind every possible sour[illegible]ation should [illegible] sulted, and that, in [illegible] discrep[illegible]

* If my memory serves [illegible] results [illegible] in the margin of his copy of [illegible] the Rugby Observatory.

endeavour to seek for a clue to them in the author's own writings, or in his known mode and means of observation, rather than in a hypothesis which may not be substantiated by anything therein contained.

The classification by Mr. Burnham of the stars into two groups, of 1. "Double stars accurately measured by Struve, Sir William Herschel, South, and others, before the epoch of Smyth;" and 2. "Double stars and stars with distant companions, which had not been carefully measured by any other observer up to the time of the publication of the Cycle," is more arbitrary than natural, and rather prejudges the whole question, especially as so little evidence is offered in support of the statements with regard to Class I.

The objects in Class II. consist almost entirely of stars with distant or faint companions which were inserted in the Bedford Catalogue for various reasons: either the star had a history, or it had been observed for a special purpose, or it was near something else, or it possessed some peculiarity in name which afforded the author an opportunity of discussing the oriental history of it, or it was a Greenwich star, or a companion was given to the star as a means of future identification, or "the angle and distance of an eligible companion was marked for the purpose of watching the motions of the larger component with an expected heliometer; as this was but a preparation, they are only registered on the lowest weight." *

The objects selected by Mr. Burnham and placed in Class II. when examined with reference to the printed Bedford Catalogue and the MSS. in Admiral Smyth's writing belonging to the Royal Astronomical Society, may be thus summarised:—

They comprise 150 of the celestial objects in the Cycle.

Of these, 79 are titled by Admiral Smyth "stars and comites;" 80 of the objects do not appear to have been micrometrically examined at all, the distance of the components is stated to have been simply estimated, or determined by Δ A.R. (equivalent to an estimation).† In this connection it is most important to take into consideration the weights assigned by Admiral Smyth to the observations discussed by Mr. Burnham. The 150 selected objects comprise 180 determinations of the position and distance of the components. Of these 180 distances, 148 have weight 1 assigned to them, 21 have weight 2, and 11 have higher weights. For the position-angles, 96 have weight 1, 52 have weight 2, and 32 higher weights.

As 82 per cent. of the distances have weight 1 assigned to them, we have first to enquire what value was attached by the

* *Cycle*, vol. i. p. 430.

† "Distance is a mere estimation by Δ A.R. over a small bar in the eyepiece expressly fitted for such cases."—*Cycle*, vol. ii. p. 462.

[illegible] 'The position and distance of the nearest of these was obtained by estima-[illegible] made in a dark field for the angle and the Δ A.R. by a bar for the distance." [illegible] 446.

author to such a weight. The answer is sufficiently clear. In Cycle, vol. i. p. 426, Admiral Smyth states that he has subjoined to the measurements of position and distance "weights of the comparative value of the observations in numbers from one to ten, *the first representing nearly worthlessness*, and the latter perfection." The paragraph already quoted * shows a class of observations to which weight 1 was attached. In vol. i. p. 428 Admiral Smyth writes, "The extremely faint objects were *estimated* either by an annular micrometer or a vertical bar which I had fitted to an excellent eyepiece with a small hole in its centre." Many of these extremely faint objects are included in Mr. Burnham's selections, and they have all weight 1 attached to them. In "Speculum Hartwellianum," p. 229, he says—"In the Cycle there is a slight typographical error in the distance of C—namely, instead of $15''\cdot1$ it should have been $15''\cdot0$, *as the trifling sum of weight* (W. 1) *would indicate*." And at p. 260 he says:—"The distances are *mere guesses to which my lowest weights* (W. 1) *are applied*, but perhaps 0 ought to have been introduced into my scale." Should any further doubt exist as to what value Admiral Smyth attached to an observation with weight 1 appended, it is certainly decided in vol. ii. p. 453, where he says, "And it should be remembered that such mere points of light were scrutinised to prove the power of the instrument for its general work, rather than to establish data for an epoch by estimation only. *Thus the mark* (*w* 1) *serves only to show that the object was identified*." † These words are most clear and explicit, and though it is undoubtedly and unfortunately true that subsequent observers have taken every figure in the Cycle as the result of careful micrometrical measurement, they have done so without considering, and in spite of, the author's distinct statements to the contrary.

It remains to consider how it happens that whereas 82 per cent. of the Distances have W. 1 attached, only 53 per cent. of the position angles have that low weight.

The micrometrical means of Admiral Smyth consisted not only of the Parallel Wire Micrometer described by Mr. Burnham, but also of what are more pertinent to the observations under discussion, of the Annular Micrometer for estimating distances of faint objects,‡ the bar micrometer "expressly fitted" for estimating distances by Δ A.R.§ and the Spherical Crystal Micrometer, which he found "excellent for ascertaining the position of objects too faint for illumination." ‖ The annular micrometer consisted of a steel ring inserted in a disc of glass, the radius of the ring being $472\cdot5$ seconds of arc.¶ The Spherical Crystal Micrometer was a very favourite instrument with Admiral Smyth, and the principles of its construction are described

* *Cycle*, vol. i. p. 430.

† For a further explanation of this value, see notes to η Herculis appended.

‡ *Cycle*, vol. i. p. 428.

§ *Cycle*, vol. ii. pp. 446, 462.

‖ *Cycle*, vol. i. pp. 339, 428.

¶ *Cycle*, vol. i. pp. 339, 385.

in the references above given. His original instrument is in the possession of the Astronomer Royal, who has most kindly lent it to me for the purpose of this enquiry, and to endeavour to throw light upon the following quotations from the Smyth MSS. of the Cycle belonging to the Royal Astronomical Society :—

ζ Ceti. "By spherical crystal. Pos. 139° 24′ from 180° = 40° 26′" (this is a clerical error for 40° 36′).

δ Persei. "By rock crystal. Pos. 224° 40′ − 90° = 315° 0′" (this evidently should be + 90°).

δ Canis Majoris. "By spherical crystal = 314°·0′ − 90° = 224° 0′."

61 Virginis. "Crystal at $\begin{array}{l} 160^\circ\ 35' \\ \underline{180} \\ 340^\circ\ 35' \end{array}$" (180° + 160° 35′).

ι Piscium. "Pos. = 140° 30′ = 39° 30′" (evidently 180° − 140° 30′).

These quotations indicate that the reading had to be added to or subtracted from 90° or 180°, and hence a fruitful source of error.

For the range of this micrometer I find that the spherical crystal revolves through an arc having 45 divisions on each side of zero. The value of one division according to Admiral Smyth (Cycle, vol. i. p. 339) = 0″·27, consequently 12″·15 was the greatest possible distance that could be measured with it on his telescope.

A toothed circle graduated from 0° to 360° is fitted to the micrometer, round which the spherical crystal can be made to revolve. Against the 360° graduation there is a small projecting piece of brass, which evidently served for the purpose of inserting the instrument in the draw tube of the telescope in a definite position with regard to the meridian. It is apparently very doubtful how accurately this circle could be adjusted for measuring position-angles, and it is probable, from the absence of any wire or fiducial line for determining accurately the north and south points of the field, that such adjustment was uncertain.

As this micrometer afforded Admiral Smyth a far more reliable instrument for measuring the position-angles of faint objects than he had for their distances, so we find that the average weight he has given to such position-angles are higher than to distances.

I have investigated in a brief manner, but as carefully as the limited time at my disposal would allow, the employment of the Spherical Crystal Micrometer for determining position-angles. For this purpose I have arranged some artificial wide double stars at definite position-angles, and at distances ranging from 10 to 210 seconds of arc. The position-angles were measured exactly after Admiral Smyth's method, by pointing the double image of A in the direction of B.

As the axis of the spherical crystal can be revolved freely

round the circle in either direction without any stop, the result is that we may obtain *two* readings for every position-angle.

The following table of the micrometer readings corresponding to a series of position-angles may be of interest.

Position-Angles.		Micrometer Readings.	Position-Angles.		Micrometer Readings.
0° or 360°	=	360° or 180°	180°	=	180° or 0°
15	=	345 „ 165	195	=	165 „ 345
30	=	330 „ 150	210	=	150 „ 330
45	=	315 „ 135	225	=	135 „ 315
60	=	300 „ 120	240	=	120 „ 300
75	=	285 „ 105	255	=	105 „ 285
90	=	270 „ 90	270	=	90 „ 270
105	=	255 „ 75	285	=	75 „ 255
120	=	240 „ 60	300	=	60 „ 240
135	=	225 „ 45	315	=	45 „ 225
150	=	210 „ 30	330	=	30 „ 210
165	=	195 „ 15	345	=	15 „ 195

From this we gather the following mode of reducing the readings. Stars in north following and south following quadrants—

Position-angle = 360°− mic. reading,
or 180°− mic. reading.

Stars in north preceding or south preceding quadrants—

Position-angle = 360°− mic. reading,
or 180° + (360°− mic. reading).

The first thing that strikes one is the very great liability to error in observations of positions with this instrument, and knowing how freely it was used by Admiral Smyth, it is enough from distinct instances and inferentially to account for a large number of discrepancies. The quotation already given with regard to ζ Ceti reveals a clerical error in the reduction. With δ Persei the correction to the reading is given as −90° instead of +90°. As this position is *np*, Admiral Smyth's correction of +90° is in error, though taking the reading as 225° the result is the same.

δ Canis Majoris. "By spherical crystal = 314°·0 − 90° = 224°·0."

This is clearly in error. Reading 314° for a star which is *sp.* corresponds to a position of 226°·0 = 180° + (360 − 314°) = 226°.

61 Virginis. The R.A.S. MS. gives "Crystal at 160° 35′
180°
340° 35′"

It will be seen that the only possible position-angles where 180° can be properly *added* to the reading are 270° and 360°. From the table I have given we at once see that a circle reading of 160° 35′ must correspond to position-angles of either 19° 25′ or 199° 25′, and as the companion of 61 Virginis was obviously neither *sp* nor *sf*, the correct position angle of the Cycle is 19° 25′. Under no circumstances could a reading of 160° 35′ place the star in the *np* quadrant. Admiral Smyth has given the sum of 180° and the Micrometer reading, instead of the difference. This affords a satisfactory explanation of the discrepancy Mr. Burnham finds in the measures of this star. Further explanations of probably similar errors are offered in the subjoined notes to the following stars: χ Persei, γ Persei, 41 Sextantis, λ Herculis.

Mr. Burnham remarks upon the Cycle observations of the magnitudes and colours of small stars, which he says "are, to say the least, of very questionable accuracy in many instances." The following evidence with regard to colours is adduced in support of that statement.

52 Piscium. "The companion (14 m) is called 'deep blue.'"

76 M Persei. "The small star is described as 'dusky.'"

55 Andromedæ. "Smyth says the small star 'was only caught by intense attention' *and yet* calls it 'bluish.'"

53 Virginis. "Smyth called the small star (15 m) 'bluish.'"

72 Virginis. "Smyth does not seem to have been aware that this was one of the Struve stars and only refers to its being registered by H I but without measures. *And yet* the small star, which is only 11·5 of Σ's scale, was well enough seen to be described in the *Cycle* as 'violet tint.'"

163 P. XIII. "This faint star is called 'ash coloured' by Smyth."

α Boötis. "Very distant," called "lilac" by Smyth.

95 P. XIV. "This is described as 'a truly difficult object,' and best seen on averting the eye, *but called* 'bluish.'"

ε Ursæ Minoris. "The companion called 'pale blue.'"

η Cephei. "The companion is called 'dusky.'"

χ Delphini. "The small star is said to be 'pale lilac.'"

α Cephei. "The small star is described as 'pale sapphire.'"

312 P. XXI. "Smyth, who could see B only, calls it 'blue.'"

κ Andromedæ. "The small stars are called 'dusky' and 'ash coloured.'"

It is well known that Admiral Smyth devoted a great deal of attention to the colours of double stars; throughout the Bedford Catalogue no star is mentioned without some colour being given to it. In the "Speculum Hartwellianum" a whole chapter is devoted to the subject. This was subsequently reprinted with additions to form Smyth's tract on "Sidereal

Chromatics." In the preface to this tract (p. vi.) Smyth refers to the "Fancy Colours" he had given in the Bedford Catalogue, which he explains. His words are, "I will here cite a sample from my own practice, placing the *inexact epithets* which I have used in Roman print, and what was probably meant is expressed in Italics." Among the "inexact epithets" are found the above terms, "dusky,"* "ash-coloured," "lilac," and "sapphire," all of which are explained.

In his note to the star 55 *Andromedæ*, Mr. Burnham has omitted the important remark of Admiral Smyth with reference to the colour. He says ("Cycle," vol. ii. p. 43), "Is the intense blue which some of these mere points of light present an optical illusion?"

In "Cycle," vol. i. p. 302, and "Sidereal Chromatics," p. 15, Smyth says, "I have been much struck with the beautiful blue tint of several of the smallest stars visible in my telescope. This, however, may be attributed to some optical peculiarity." Again, in "Cycle," vol. ii. p. 290, he says, "B 15, intense blue; this last colour on so small an object is very striking, and an astronomical friend who examined it at my request with powerful means confirms both the tint and its intensity." In vol. ii. p. 115, he says:—"Σ. discovered the delicate companion C which had escaped the gaze of all other observers, and requires the most careful attention even to be perceived by occasional glimpses, but when seen, has a peculiar deep purple tint which strikes singularly on the eye from so excessively minute an object." In "Cycle," vol. ii. p. 522, he remarks that Sir John Herschel observes that a certain minute star "bears illumination well and is *therefore* blue." Mr. Proctor, in his "Orbs Around Us," p. 322, says, "The late Admiral Smyth, who thought he could recognise very decided blue tints among the minuter stars, expressed a doubt whether this might not be due to some idiosyncracy of his eyesight." And in their recently published "Manual of Astronomy," p. 452, Professor Newcomb and Professor Holden say that certain phenomena "are partly due to the physiological fact that *the fainter a star is the more blue it will appear to the eye.*" †

These quotations show that Admiral Smyth was perfectly well aware of the apparent anomaly in recording decided colours to minute points of light; and the important fact, stated by Professor Newcomb and Professor Holden in their "Astronomy," proves that such observations, as have been called in question by Mr. Burnham, were made by an eye very sensitive to colour, and are probably accurate.

In an interesting "Note on a Relation between the Colours and Magnitudes of the Components of Binary Stars" published in the *American Journal of Science*, June 1880, my friend Prof.

* "C is of the nondescript tint called 'dusky.'"—*Spec. Hart.* p. 265.

† "While the last gleamings of refracted light
Died in the fainting violet away."
Thomson, *Poem to the Memory of Sir Isaac Newton.*

Edward S. Holden says :—" We do not find *isolated* stars of decided green, blue, or purple colours. A few such have been recorded, but in most cases erroneously." To which a note is appended :—" For example, Admiral Smyth calls *α Lyræ* a *green* star." Admiral Smyth remarks in "Sidereal Chromatics," p. 17 : —" There is not an instance of a solitary green, purple, blue, or violet star being found." In the Bedford Catalogue, p. 423, and " Sidereal Chromatics," p. 33, the colour of *α Lyræ* is given "pale sapphire ;" and in the preface to " Sidereal Chromatics," p. vii., the word "pale" is stated to mean "deficient in hue," and the word "sapphire" to mean "blue tint." I have entirely failed in finding any reference to *α Lyræ* as a "green" star.

With regard to the stars under discussion a few conclusions may be safely drawn from the internal evidence afforded by Admiral Smyth's writings, independent of the proofs given by more recent micrometrical measurements. Firstly, nearly all these stars, and those with W. 1 attached, were never intended by Admiral Smyth to be considered as accurately micrometrically measured. The difficulties and misapprehensions that have arisen are entirely due to the improper manner in which such observations are printed with a decimal point and a cypher, and with a certain weight appended. It is plain on the face of it that observations of distances of one or two hundred seconds of arc in round numbers, without tenths or hundredths, cannot be the result of accurate micrometrical measurements. Secondly, the instrumental means of Admiral Smyth for these stars were not calculated to yield accurate results. The uncertainty and difficulty in using the spherical crystal micrometer is sufficiently demonstrated in the remarks I have made. The annular micrometer, used for estimating distances, consisted of a ring with a *radius* of 472″·5, and it needs no further remark to show that estimations with such an instrument must be of the vaguest kind. And thirdly, we see by the quotations from the R. A. S. MSS. that Admiral Smyth was careless in his arithmetical reductions, and from the note to *β Aquarii* we perceive that his judgment in estimating distances was unsound.

The following notes on the stars speak for themselves in affording additional information and explanation of the results discussed by Mr. Burnham.

Additional Notes to Stars in Mr. Burnham's Class II.

β Cassiopeæ (p. 2).

Pos. W. 1. Dist. W. 1.

Smyth says :—" A bright star whose acolyte is so small that it is here rather *estimated* than measured." R. A. S. MS. gives distance " Δ A.R. 19ˢ·1 = 201″·0."

147 ℍ III. (p. 3).

Pos. W. 1. Dist. W. 1.

Apart from W. 1 being attached to each element, Smyth particularly mentions that this double star "is here *estimated*." The object is evidently inserted in the "Cycle" because it occupied the spot where Smyth sought for ℍ's faint nebula, and it serves merely to introduce some remarks upon ℍ's idea of nebulous fluid.

γ Pegasi (p. 5).

Pos. W. 1. Dist. W. 1.

The R. A. S. MS. gives the distance as "181″·0 or 182″·0."

ι Ceti (p. 6).

Pos. W. 1. Dist. W. 1.

Smyth expressly states:—"This is an excessively difficult object, being only discernible after long attention, and by occasionally averting the eye to another part of the field of view. The position and distance are therefore only the result of cautious *estimation*."

12 *Ceti* (p. 8).

AB. Pos. W. 1. Dist. W. 1.

AC. Pos. W. 2. Dist. W. 1.

Smyth calls it "a most difficult test object." "B is only discernible by the closest attention under favouring circumstances, though when once caught is tolerably well seen; the detail here given is therefore a *mere estimation*."

52 *Piscium* (p. 10).

Pos. W. 1. Dist. W. 1.

Smyth calls it "a most delicate object whose position and distance are carefully ["cautiously" in R. A. S. MS.] *estimated*."

δ Andromedæ (p. 11).

Pos. W. 2. Dist. W. 2.

α Cassiopeæ (p. 12).

Pos. W. 3. Dist. W. 3.

Smyth quotes ℍ's measure and says:—"Which compared with mine does not show a greater difference of angle than might be expected from the proper motion of an object whose proximity is accidental, but the difference in distance is so remarkable, that it must be imputed to instrumental error." With regard to the

above remark Smyth adds in "Speculum Hartwellianum," p. 217:—"Meaning of course to impugn ℍ's 56″·17 of fifty years before, but it turned out instead to be a gross blunder of my own, only explicable on the supposition of counting a wrong revolution with the comb out of the field with the power used."

P. O. 146 (p. 14).
Pos. W. 3. Dist. W. 1.

H. 1046 (p. 17).
Pos. W. 1. Dist. W. 1.

The "Cycle" says:—"The double star here carefully *estimated.*"

σ^2 *Piscium* (p. 24).
AB. Pos. W. 4. Dist. W. 2.
AC. Pos. W. 3. Dist. W. 1.

Mr. Burnham says:—"Smyth has *the extraordinary conclusion* that 'an inference of binarity is deducible from a comparison of the registered epochs of ℍ and himself,' whereas Smyth commences the sentence by saying '*No safe conclusion* can be deduced from the discrepancies observable in the position of A and B, as the object is most delicate.'"

R. A. S. MS. says:—"I requested Mr. Challis to make a diagram with the large telescope. His diagram confirms mine."

η *Ceti* (p. 27).
Pos. W. 1. Dist. W. 1.

β *Andromedæ* (p. 27).
Pos. W. 1. Dist. W. 1.

The star which Mr. Burnham calls D is clearly Smyth's companion. In the R. A. S. MS. the distance is given "Δ A. R. 8ˢ·8 W. 1, Pos. 299·0 W. 1." The reduction of these elements gives 124″ or 125″ as the distance. The discrepancy is most probably a printer's error of 225 for 125.

H. 634 (p. 31).
Pos. W. 1. Dist. W. 1.

Smyth states that the position and distance are "*mere estimations.*" They differ much from H.

42 ℍ VII. (p. 33).
Pos. W. 1. Dist. W. 1.

The R. A. S. MS. says:—"The position and distance of which are carefully *estimated.*"

40 *Cassiopeæ* (p. 37).

Pos. W. 2. Dist. W. 1.

"A delicate object."

76 *M. Persei* (p. 40).

Pos. W. 1. Dist. W. 1.

R. A. S. MS. says:—"The wide double star *estimated*."

107 *Piscium* (p. 41).

Pos. W. 2. Dist. W. 1.

Smyth says:—"The comes is so minute that light is inadmissible, the position is therefore gained by the spherical crystal micrometer, and the distance carefully *estimated*."

ζ *Ceti* (p. 43).

Pos. W. 3. Dist. W. 2.

R. A. S. MS. states:—"By annular micrometer nearly the same as wire. By spherical crystal Pos. 139° 24′ from 180°. Δ A.R. 7ˢ·8."

At the bottom Pos. is given = 40° 26′—a clerical error for 40° 36′. The distance is clearly computed from Δ A.R.

55 *Andromedæ* (p. 43).

Pos. W. 1. Dist. W. 1.

The small star was Smyth's minimum visibile; "its position and distance are therefore only *estimated*."

α *Trianguli* (p. 44).

Pos. W. 1. Dist. W. 1.

P. I. 222 (p. 47).

AD. Pos. W. 2. Dist. W. 2.

"This group is most difficult to observe, and the results are rather *estimations* than measures."

P. I. 227 (p. 48).

Pos. W. 5. Dist. W. 3.

χ *Persei* (p. 57).

AB. Pos. W. 2. Dist. W. 1.

AC. Pos. W. 5. Dist. W. 3.

Admiral Smyth's position-angle of AC is 43°·2, whereas Mr. Burnham makes it 136°·3 and remarks :—"It is not easy to see how Smyth could make such an error in his own observation, as there is no such mistake found anywhere else in the work." Smyth gives his observation of position W. 5, showing that he considered it had been made with care. A little consideration of the remarks upon the spherical crystal micrometer will show that 43°·2 is the *exact reading* of that instrument corresponding to a position-angle of 136°·8 thus: 180° − 43°·2 = 136°·8. It is most probable here that Smyth has given the reading of the circle without correction.

34 *M. Persei* (p. 66).

Pos. W. 2. Dist. W. 2.

The R. A. S. MS. says, "a double star *about* 15″ apart."

θ Persei (p. 67).

Pos. W. 1. Dist. W. 1.

γ[1] *Fornacis* (p. 72).

Pos. W. 2. Dist. W. 1.

Cycle says :—"As B vanishes under illumination its position is taken by the spherical rock-crystal micrometer, and the distance *estimated*."

γ Persei (p. 75).

Pos. W. 1. Dist. W. 1.

R. A. S. MS. says :—"May be made triple with C, but only measured H's, and that very slightly, for the evening was bad, and the object wide and unequal."

Smyth's position-angle is 226°·8. Mr. Burnham makes it 323°·9. It is probable that Smyth has given the reading of the spherical crystal micrometer without correction. The details of this instrument already given show that 226°·0 corresponds to a position-angle of 314°, which would be fairly satisfactory for a very slight measurement.

β Persei (p. 77).

Pos. W. 2. Dist. W. 1.

R. A. S. MS. has :—"The effulgence of the large star interferes with measures for distance."

52 *Arietis* (p. 77).

AD. Pos. W. 1. Dist. W. 1.

Smyth says :—"The whole [quadruple group] are of most

difficult measurement . : . the details of the latter two (C and D) being of course *mere estimations.*"

τ[4] *Eridani* (p. 79).

Pos. W. 1. Dist. W. 1.

α Persei (p. 80).

Pos. W. 1. Dist. W. 1.

R. A. S. MS. says:—"A is so brilliant as to render measures difficult, they are *mere estimations.*"

δ Persei (p. 82).

Pos. W. 2. Dist. W. 1.

R. A. S. MS. says:—"By rock crystal Pos. 224° 40′ − 90° = 315°·0" (this should be + 90°). "Dist. Δ A.R. 7s·0."

The reduction of the Δ A.R. differs only 3″ from Mr. Burnham's measure; a probable error by Smyth in reducing his observation.

19 *Pleiadum* (p. 84).

Pos. W. 2. Dis. W. 1.

P. IV. 257 (p. 108).

AC. Pos. W. 2. Dist. W. 2.

R. A. S. MS. says:—"Δ A.R. AC = 4s·3."

66 *Eridani* (p. 111).

Pos. W. 1. Dist. W. 1.

R. A. S. MS. has for distance "Δ A.R. = 0s·4."

β Eridani (p. 111).

Pos. W. 2. Dist. W. 1.

R. A. S. MS. has for distance "Δ A.R. = 4s·2."

H. 3269 (p. 112).

Pos. W. 2. Dist. W. 2.

H. 3272 (p. 118).

Pos. W. 1. Dist. W. 1.

γ Orionis (p. 121).

Pos. W. 3. Dist. W. 2.

R. A. S. MS. has for distance "Δ A.R. 3s·6."

109 *P. Orionis* (p. 124).

Pos. W. 1. Dist. W. 1.

β Leporis (p. 125).

Pos. W. 1. Dist. W. 1.

R. A. S. MS. has for distance "Δ A.R. 13^s·1."

Mr. Burnham says:—"There are two 10 mag. stars in this quadrant." The R. A. S. MS. diagram shows both these stars.

ε Orionis (p. 134).

Pos. W. 1. Dist. W. 1.

R. A. S. MS. has for distance "Δ A.R. 9^s·8."

124 *Tauri* (p. 135).

BC.	Pos. W. 7.	Dist. W. 3.
AB.	„ W. 1.	„ W. 1.
AD.	„ W. 1.	„ W. 1.

In the R. A. S. MS. it is evident that BC was measured, but AB and AD were only arrived at by Δ A.R. The MS. gives "Δ A.R. A and B=5^s·5, Δ A.R. A and D=1^s·0."

ν Aurigæ (p. 139).

Pos. W. 1. Dist. W. 1.

R. A. S. MS. says:—"*Mere estimations.*"

78 *M. Orionis* (p. 138).

Pos. W. 2. Dist. W. 1.

37 *M. Aurigae* (p. 140).

Pos. W. 1. Dist. W. 1.

R. A. S. MS. says:—"It is preceded by the double star here measured *or rather estimated.*"

β Aurigae (p. 143).

Pos. W. 2. Dist. W. 2.

With regard to the measure by Challis given by Mr. Burnham, the R. A. S. MS. has:—"I requested Mr. Challis to look to it (print his results)."

θ Aurigae (p. 143).

Pos. W. 2. Dist. W. 1.

35 *Camelopardi* (p. 144).

Pos. W. 2. Dist. W. 1.

5 *Lyncis* (p. 148).

AB. Pos. W. 1. Dist. W. 1.

AC. „ W. 7. „ W. 2.

R. A. S. MS. says:—"A and C are the stars which constitute ℍ VI. B seems not to have been seen till Σ announced it. It is seen readily in the spherical crystal micrometer, illumination being impracticable."

μ Geminorum (p. 148).

Pos. W. 1. Dist. W. 1.

R. A. S. MS. gives distance "Δ A.R. 5ˢ·4."

ζ Canis Majoris (p. 149).

Pos. W. 1. Dist. W. 1.

R. A. S. MS. gives distance "Δ A.R. 6ˢ·7," and says:—"Piazzi's 80 and 81 Hora VI. by whose observations reduced they would be for 1800 Pos. 337°·0. Dist. 178″·0.″ "

It is worthy of notice that though Piazzi's reduced observations agree with Mr. Burnham's results, and were evidently very good, Admiral Smyth does not appear to have made his agree with them.

β Canis Majoris (p. 150).

Pos. W. 1. Dist. W. 1.

R. A. S. MS. gives distance "Δ. A.R. 4ˢ·7."

10 *Monocerotis* (p. 151).

Pos. W. 1. Dist. W. 1.

R. A. S. MS. diagram shows that Mr. Burnham's B is the star measured, as C is also given.

γ Geminorum (p. 154).

AB. Pos. W. 1. Dist. W. 1.

AC. „ W. 1. „ W. 1.

The Cycle gives the distance as 75ˢ·0 and 110ˢ·0, as if seconds of time, and says:—"Followed nearly on the parallel Δ A.R. 40ˢ by a neat 9 mag. star." We may fairly presume that the distances AB and AC were determined by Δ A.R.

α Canis Majoris (p. 158).

Pos. W. 1. Dist. W. 1.

H. 740 (p. 165).

Pos. W. 2. Dist. W. 1.

ζ Geminorum (p. 169).

AB. Pos. W. 4. Dist. W. 2.

AC. „ W. 1. „ W. 1.

R. A. S. MS. says "Distance Δ A.R. A and B=1ˢ·8. Δ A.R. A and C=5ˢ·0."

δ Canis Majoris (p. 170).

Pos. W. 1. Dist. W. 1.

R. A. S. MS. says "Angle by spherical crystal=314°·0−90°. Distance=Δ A.R.=13ˢ·5."

The reading of 314°·0 on the spherical crystal micrometer corresponds to a position of 226°, not 224° as given by Smyth.

30 Canis Majoris (p. 174).

Pos. W. 1. Dist. W. 1.

R. A. S. MS. says:—"Distance = Δ A.R. 5ˢ·6=85″·0. A bright white star in a rich cluster of small ones, and with its comes very splendid among the star dust." The two near companions mentioned by Mr. Burnham are probably included in the "star dust."

η Canis Majoris (p. 175).

Pos. W. 2. Dist. W. 1.

R. A. S. MS. gives distance "Δ A.R.=13ˢ·4."

β Canis Minoris (p. 176).

AB. Pos. W. 2. Dist. W. 1.

AC. „ W. 2. „ W. 1.

R. A. S. MS. says:—"See Baron de Zach in vol. xii."

H. 2435 (p. 195).

Pos. W. 4. Dist. W. 1.

P. VIII. 81 (p. 197).

Pos. W. 2. Dist. W. 1.

It is to be remarked that Smyth does not agree with the earlier measure of Struve.

θ Cancri (p. 198).

Pos. W. 3. Dist. W. 1.

ψ Leonis (p. 220).

Pos. W. 2. Dist. W. 1.

41 *Sextantis* (p. 234).

AB. Pos. W. 1. Dist.

AC. „ W. 2. „ W. 1.

Smyth says:—"The *estimated* angle and distance (AB) are next to *mere guesses*." R. A. S. MS. says:—"Mere estimations." A and C. Pos. 120°·4 W. 2. Mr. Burnham gives it 72°·5. The remarks on the spherical crystal micrometer show that we must be prepared for numerous mistakes in readings and their reduction. If the above position-angle 120°·4 be really the reading of that micrometer it represents a true position-angle of 59°·6, which is close enough for a mere guess.

β *Ursæ Majoris* (p. 237).

Pos. 172·6 W. 2. Dist. 75·0 W. 1.

R. A. S. MS. says the star is *np*, but the position makes it *nf*. The diagram in the MS. shows the companion about 272°, and at a much greater distance than 75″·0. I am, however, inclined to think that the diagrams in the MS. have been made from the recorded observations, and are not eye-sketches.

δ *Leonis* (p. 244).

Pos. W. 1. Dist. Δ A.R. 4s·9 W. 1.

δ *Crateris* (p. 248).

Pos. W. 1. Dist. Δ A.R. 19s·8 W. 1.

δ *Ursæ Majoris* (p. 262).

Pos. W. 2. Dist. Δ A.R. 20s·4 W. 1.

8 *Can. Ven.* (p. 272).

Pos. W. 1. Dist. W. 1.

β *Corvi* (p. 271).

Pos. W. 1. Dist. W. 1.

Smyth says:—"Position and distance here *estimated*."

δ *Virginis* (p. 286).

Pos. W. 2. Dist. Δ A.R. 5s·8 W. 2.

R. A. S. MS. has "Δ A.R. W. 1."

ε *Virginis* (p. 290).

Pos. W. 4. Dist. W. 2.

R. A. S. MS. says:—"A star with a very minute distant comes."

53 *Virginis* (p. 292).

Pos. W. 1. Dist. W. 1.

Smyth says:—"As I could only catch a sight of B by gleams with the equatorial clock driving the telescope, the above results are but *estimations.*"

61 *Virginis* (p. 294).

♅	Pos. 345°	Dist. 73″·25.
Smyth	„ 340·6 W. 2.	Δ A.R.. 2ˢ·8 W. 1.
β	„ 24·8	189″·26.

R. A. S. MS. has in pencil in Smyth's writing:—Chrys. 160° 35′ W. 2;" then below—

$$" = \begin{array}{r} 160^\circ\ 35' \\ 180 \\ \hline 340\ 35" \end{array}$$

and at bottom "=340°·6."

This discrepancy has been explained in remarks on the spherical crystal micrometer. Mr. Burnham, who has perhaps viewed the above figures under the aspect of a preconceived theory, says:—"There is a large error in the *repetition of* ♅'s *angle* by Smyth." The correct reduction here is 180° − 160° 35′ = 19° 25′.

α *Virginis* (p. 296).

Pos. W. 2. Dist. Δ A.R. = 19ˢ·4 W. 2.

μ *Hydræ* (p. 300).

Pos. W. 2. Dist. W. 2.

72 *Virginis* (p. 301).

Pos. W. 3. Dist. W. 1.

R. A. S. MS. says:—"Measures of distance out of the question with my means; it is therefore a *mere estimation.*"

75 *Virginis* (p. 303).

Pos. W. 1. Dist. W. 1.

Cycle says:—"This object was merely looked at from being among ♅'s sweeps, otherwise it is too difficult to measure and *too wide for tolerable estimation.*"

163 *P.* XIII. *Can. Ven.* (p. 306).

Pos. W. 2. Dist. W. 2.

Cycle says:—"This was selected for a trial of the spherical crystal micrometer on the angle of position by getting the double image of A in a line towards B. The distance, being too great for the value of the scale, was obtained by Δ A.R."

R. A. S. MS. says nothing about Δ A.R. but gives for distance "68°·0 W. 2. 70·0 est."

171 *P.* XIII. *Virginis* (p. 308).

Pos. W. 4. Dist. W. 2.

R. A. S. MS. says:—"Though the companion is so minute, it bears the red illumination pretty fairly. Seen well with the rock-crystal prism."

85 *Virginis* (p. 309).

Pos. W. 1. Dist. W. 1.

Cycle says:—"This most difficult object was merely examined from being one of Herschel's sweeps. It is the *minimum visibile* of my telescope, and therefore impossible to measure."

η Bootis (p. 311).

Pos. W. 4. Dist. W. 2.

τ Virginis (p. 312).

Pos. W. 5. Dist. W. 3.

α Bootis (p. 315).

Pos. W. 2. Dist. W. 1.

Cycle gives for distance Δ A.R. 15^s·1. W. 1.
The R. A. S. MS. appends "=298″·0."

P. XIV. 95 (p. 322).

Pos. W. 2. Dist. W. 1.

Smyth says:—"The position was approximated by the spherical rock-crystal micrometer, but the distance being too wide for its range it was *merely estimated*." R. A. S. MS. gives distance "=40″± est."

5 *Ursæ Minoris* (p. 323).

Pos. W. 2. Dist. W. 1.

β Ursæ Minoris (p. 330).

Pos. W. 1. Dist. W. 1.

δ Bootis (p. 338).

Pos. W. 1. Dist. W. 1.

[R.] A. S. MS. gives distance "Δ A.R. 7^s·9."

ι Draconis (p. 344).

Pos. W. 2. Dist. W. 1.

α Serpentis (p. 348).

Pos. W. 1. Dist. W. 1.

R. A. S. MS. has, "excessively difficult; *merely estimated.*"

ζ Ursæ Minoris (p. 351).

Pos. W. 2. Dist. W. 1.

δ Ophiuchi (p. 356).

Pos. W. 1. Dist. W. 1.

ν Coronæ (p. 358).

AB.	Pos. W. 6.	Dist. W. 4.
AC.	„ W. 8.	„ W. 5.
BD.	„ W. 2.	„ W. 1.

R. A. S. MS. says:—"Bears illumination badly."

ν² Coronæ (p. 362).

Pos. W. 1. Dist. W. 1.

η Draconis (p. 365).

Pos. W. 1. Dist. W. 1.

β Herculis (p. 366).

Pos. W. 2. Dist. W. 1.

η Herculis (p. 372).

AC.	Pos. W. 2.	Dist. W. 1.
AB.	„ W. 1.	„ W. 1.

Cycle says:—"Tolerable estimations of an egg-shaped object.'
R. A. S. MS. gives AC distance =" Δ A.R. 9ˢ·5."

Mr. Burnham says:—"At one time Σ suspected the large star to be a close pair, but afterwards rejected it as single. Of the correctness of this conclusion there has never been any doubt. Smyth measured the angle and distance in 1842." Mr. Burnham has ignored the explanation given by Smyth in *Speculum Hartwellianum* on this subject. He says (p. 275), with regard to AB.:—"This first-class specimen of Σ's Vicinissimæ was gazed at in 1842 till an impression arose that it was slightly elongated, but un-notchable, and therefore under half a second in distance, *but the weights assigned show that the idea was vague, and the conclusions worth next to nothing.*"

η Ophiuchi (p. 379).

Pos. W. 1. Dist. W. 1.

36 *Ophiuchi* (p. 381).

AC. Pos. W. 4. Dist. W. 2.

ε *Ursæ Minoris* (p. 381).

Pos. W. 1. Dist. W. 1.

λ *Herculis* (p. 390).

Cycle Pos. = 295°·0 W. 1. Dist. Δ A.R. 23s·4 W. 1.

β „ = 251°·8

Smyth's position makes B *np*, but in R. A. S. MS. it is given *sp*, where Mr. Burnham proves the star to be. We may surmise that this is another case where the reading of the spherical crystal micrometer has been inserted without reduction. 295°·0 would correspond to a position-angle of 245°·0.

γ *Draconis* (p. 400).

Pos. W. 1. Dist. Δ A.R. 9s·7 W. 1.

τ *Ophiuchi* (p. 402).

Pos. AC. { W. 2. W. 3. Dist. { W. 1. W. 2.

Smyth says he made a hopeless scrutiny of AB, and "making nothing of it, I noted the star C *as a future reference*."

18 M. *Clyp. Sobieski* (p. 415).

Pos. W. 1. Dist. W. 1.

Smyth has probably given the position-angle BA instead of AB.

H. 2002 (p. 414).

Pos. W. 1. Dist. W. 1.

χ *Draconis* (p. 421).

Pos. W. 1. Dist. Δ A.R. 14s·6 W. 1.

2 *Aquilæ* (p. 426).

Pos. W. 2. Dist. W. 1.

Smyth says:—"It is a difficult object to manage, from the brightness of A."

5 *Aquilæ* (p. 427).

AC. Pos. W. 1. Dist. W. 1.

R. A. S. MS. has:—"C is very minute, so that its place is only *estimated* for placing it."

25 M. *Sagittarii* (p. 420).

Pos. W. 1. Dist. W. 1.

Mr. Burnham's difficulty in identifying the pair is possibly explained in R. A. S. MS. which says:—"A knot of from 10th to 13th mag. stars are *between* two stars of 8th. This is the position fixed." Note in MS. says:—"See Blue Book, p. 55, for description and position, &c." The diagram in MS. does not accord with the Cycle position-angle.

110 *Herculis* (p. 428).

Pos. W. 1. Dist. W. 1.

Smyth says:—"The acolyte was caught by gleams on steady gazing in a darkened field; it is certainly a *minimum visibile* in my telescope. The details are of course but *mere estimations*."

P. XVIII. 197 (p. 430).

Pos. W. 3. Dist. W. 1.

ν^1 *Lyræ* (p. 432).

AB.	Pos. W. 2.	Dist. W. 1.
AC.	„ W. 8.	„ W. 8.
CD.	„ W. 1.	„ W. 1.

R. A. S. MS. says:—"Δ A.R. AC=$3^s \cdot 2$." This with W. 8 is rather inexplicable.

Cycle gives:—"As B and D are too small to bear any illumination the places above given are mere, though careful, *estimations*."

β *Lyræ* (p. 433).

AB.	Pos. W. 9.	Dist. W. 9.
AC.	„ W. 2.	„ W. 1.
AD.	„ W. 1.	„ W. 1.

Mr. Burnham states that this star is inserted "as an illustration of the *apparent accuracy* of the measures of Smyth when the star had been previously observed. The other companions, C and D, had not been measured at this time by other observers." As a matter of fact, the measures of C and D have W. 1, and were therefore not intended to be considered as accurate measures.

2024 H. *Aquilæ* (p. 437).

Pos. W. 1. Dist. W. 1.

R. A. S. MS. says:—"During the measures a bright Aurora Borealis."

γ *Lyræ* (p. 439).

Pos. W. 1. Dist. Δ A.R. W. 1.

P. XVIII. 299 (p. 440).

Pos. W. 1. Dist. W. 1.

Smyth says:—"The acolyte is of course too minute to admit of illumination, and its place is therefore *merely estimated.*"

ζ *Aquilæ* (p. 441).

Pos. W. 1. Dist. Δ A.R. W. 1.

Smyth says:—"As the comes defies illumination, the position is inferred by pointing the double image of A in the rock-crystal micrometer to the direction of B."

δ *Draconis* (p. 445).

Pos. W. 1. Dist. Δ A.R. W. 1.

δ *Aquilæ* (p. 446).

Pos. W. 1. Dist. W. 1.

Smyth says:—"The position and distance were obtained by *estimations* made in a dark field for the angle, and the Δ A.R. by a bar for the distance."

R. A. S. MS. gives Dist. Δ A.R.$=6^{s}\cdot6=96''\cdot50$

56 *Aquilæ* (p. 462).

Pos. W. 4. Dist. W. 1.

Smyth says:—"The angle of position was readily obtained (W. 4), but the distance is a mere estimation (W. 1) by Δ A.R. over a small bar in the eyepiece expressly fitted for such cases."

β *Aquilæ* (p. 464).

Pos. W. 1. Dist. W. 1.

R. A. S. MS. gives Dist. ="Δ A·R.$=3^{s}\cdot7=175''\cdot0$."

γ *Aquilæ* (p. 455).

Pos. W. 1. Dist. W. 1.

α^1 *Capricorni* (p. 472).

B*b*.	Pos. W. 3.	Dist. W. 1.
A*a*.	,, W. 2.	,, W. 1.

There is a strange and unexplained difference in the accuracy of the distances B*b* and A*a*, both having the same W. 1.

M. 29 *Cygni* (p. 478).

Pos. W. 1. Dist. W. 1.

R. A. S. MS. says:—"Dist. Δ A.R. of A and B about $4^{s}\cdot9$."

ι *Aquarii* (p. 483).

AB. Pos. W. 1. Dist. W. 1 (in R. A. S. MS.).

AC. „ W. 1. „ no weight given.

α *Delphini* (p. 483).

Pos. W. 1. Dist. W. 1,

α *Cygni* (p. 485).

Pos. W. 1. Dist. W. 1.

η *Cephei* (p. 488).

Pos. W. 1. Dist. W. 1.

χ *Delphini* (p. 490).

Pos. W. 1. Dist. W. 1.

R. A. S. MS. has :—"The position and distance of A and B are by comparative *estimations*." Position is given "20°59′ (by crystal)."

59 *Cygni* (p. 492).

Pos. W. 2. Dist. W. 1.

R. A. S. MS. has :—"The colours were proved for me by Mr. Challis."

ζ *Cygni* (p. 497).

Pos. W. 1. Dist. W. 1.

Smyth probably measured distance with micrometer, for R. A. S. MS. gives distance="104″·79=150″·0."

α *Cephei* (p. 499).

Pos. W. 1. Dist. W. 1.

R. A. S. MS. gives distance "Δ A.R.=9s·8 W. 1; 8s·8 W. 2." The small star is called "pale blue," not "pale sapphire."

β *Aquarii* (p. 502).

Pos. W. 2. Dist. W. 1.

Cycle says:—"This is a most difficult object, and one requiring the utmost delicacy of treatment to procure even an estimation of."

R. A. S. MS. says :—"The angle of position by the spherical crystal micrometer, and the distance an *estimation or comparison*," adding "H's is certainly too wide."

We have here an instance to prove that Smyth's judgment was faulty; Mr. Burnham has proved that H's measure was very slightly in error.

ε Pegasi (p. 507).

AB. Pos. W. 1. Dist. W. 1.

AC. „ W. 3. „ W. 3.

P. XXI. 312 (p. 509).

Pos. W. 1. Dist. W. 1.

R. A. S. MS. says :—"Measurement was out of the question, the angle and distance are therefore by careful *estimation*."

20 Pegasi (p. 509).

Pos. W. 1. Dist. W. 1.

R. A. S. MS. says :—"Position and distance are mere *estimations*."

41 Aquarii (p. 510).

Pos. W. 1. Dist. W. 1.

π¹ Pegasi (p. 512).

AB. Pos. W. 3. Dist. W. 2.

AC. „ W. 3. „ W. 1.

8 Lacertæ (p. 515).

Pos. W. 1. Dist. W. 1.

Smyth says :—"The *estimations* were carefully made."

ζ Pegasi (p. 520).

Post. W. 1. Dist. W. 1.

It is not easy to reconcile Smyth's position $15^{\circ}\cdot5$ with Mr. Burnham's $137^{\circ}\cdot8$. $15^{\circ}\cdot5$ in the spherical crystal micrometer corresponds to an angle of $164^{\circ}\cdot5$.

ξ Pegasi (p. 522).

AB. Pos. W. 1. Dist. W. 1.

AC. „ W. 2. „ W. 1.

Cycle says :—"B only to be caught under intense gazing with the most favourable circumstances of atmosphere and instrument. I added C for identification."

β Pegasi (p. 527).

Pos. W. 1. Dist. W. 1.

R. A. S. MS. says :—"B is readily seen at intervals."

4 Cassiopeæ (p. 534).

AB.	Pos. W. 6.	Dist. W. 2.
AC.	„ W. 2.	„ W. 1.
CD.	„ W. 1.	„ W. 1.

R. A. S. MS. gives distance "Δ A.R. A and C=$25^s\cdot3$, A and B=$7^s\cdot4$. The companion of C is very faint at times."

The great discrepancy in the position-angle CD is not explained.

ι Piscium (p. 536).

Pos. W. 1. Dist. W. 1.

R. A. S. MS. gives:—"Position $140^\circ\cdot30=39^\circ\cdot30$; Δ A.R. $10^s=199''$."

This position is $180^\circ-140^\circ\ 30'$.

See remarks on spherical crystal micrometer.

κ Andromedæ (p. 536).

AB.	Pos. W. 1.	Dist. W. 1.
AC.	„ W. 2.	„ W. 1.

Cycle says:—"The *estimations* here given may at best be only ranked as *comparative guesses.*"

R. A. S. MS. says:—"Principally guesses."

P. XXIII. 171 (p. 537).

AB.	Pos. W. 3.	Dist. W. 1.
AC.	„ W. 1.	„ W. 1.

R. A. S. MS. gives "Distance Δ A.R. $8^s\cdot4$ A and C."

Bocking, Braintree,
1880, *June.*

On the Variability of B.A.C. 2472. By W. S. Franks, Esq.

In the May Number of the *Monthly Notices*, Mr. Tebbutt calls attention to the magnitude of this star, which is P vii. 114. On referring to my note-book, I find that it was observed on January 28, 1878, and there estimated as of the seventh magnitude. It is marked eighth magnitude in the large maps of the S.D.U.K.; but it is not contained in the *Uranometria Nova* of Argelander.

Leicester,
1880, *June* 10.

Elements of Comet Schäberle (1880, *b*). By T. H. Safford, Esq.

T. 1880, July, 1·2602 M. T. Washington.

ω	144° 59′·70
☊	282 11·10
i	123 47·3
log *q*	0·25894

They were computed from observations made at

Ann Arbor	April 9
Vienna	April 13
Ann Arbor	April 27
Ann Arbor	May 4

In their computation all the small corrections (aberration, parallax) were neglected: but the ratio of curtate distances at the first and last observations was so varied that the intermediate places were represented as closely as possible, while the extreme places were perfectly represented.

The following are the values of computed — observed, latitudes and longitudes.

	Δλ cos β	Δβ
April 13	+0′·07	−0′·16
27	−0·17	+0·11

The present calculation was peculiar in this, that the two values of log M obtained by Olbers' approximation were naturally quite different, and gave at once limits for the employment of the *regula falsi*. I think it is often well in calculating the orbit of a comet to employ four observations in order to avoid the danger of mistakes.

Observatory, Williams College,
Williamstown, Mass.,
1880, June 3.

Elements and Ephemeris of Schäberle's Comet (1880, *b*). By M. G. Bigourdan.

(*Extracted from the 'Comptes Rendus,' Tome xci., Nos.* 2 *and* 3.)

The following elements are derived from three normal places formed respectively from observations on

(1) April 8 (Paris); 10 (Pola); 11 (Strasburg); 13 (Vienna).
(2) April 26 (Paris); 28 (Rome); 29 and 30 (Paris).
(3) May 14, 16, 17, 18 (Paris).

T	1880, July 1·83846, Paris M.T.	
ϖ	42° 30′ 56″·1	Mean Equinox 1880·0.
☊	257 15 13·3	
i	123 3 36·1	
log *q*	0·258474.	

Ephemeris for Paris Midnight.

1880		R.A. h m s	Dec. ° ′ ″	Log. Δ	Brightness
Aug.	15·5	6 59 59·8	29 1 16N	0·406448	0·75
	17·5	7 0 4·8	28 6 43	0·402808	0·75
	19·5	7 0 5·9	27 51 45	0·398981	0·76
	21·5	7 0 2·9	27 16 20	0·394969	0·77
	23·5	6 59 55·4	26 40 24	0·390774	0·78
	25·5	6 59 43·3	26 3 56	0·386398	0·78
	27·5	6 59 26·4	25 26 52	0·381839	0·79
	29·5	6 59 4·4	24 49 9	0·377100	0·80
	31.5	6 58 37·0	24 10 45	0·372184	0·81
Sept.	2·5	6 58 3·9	23 31 35	0·367093	0·83
	4·5	6 57 24·9	22 51 36	0·361833	0·84
	6·5	6 56 39·5	22 10 44	0·356409	0·85
	8·5	6 55 47·5	21 28 57	0·350825	0·86
	10·5	6 54 48·6	20 46 9	0·345088	0·88
	12·5	6 53 42·3	20 2 16	0·339207	0·89
	14·5	6 52 28·4	19 17 16	0·333192	0·90
	16·5	6 51 6·5	18 31 4	0·327051	0·92
	18·5	6 49 36·3	17 43 34	0·320793	0·94
	20·5	6 47 57·3	16 54 44	0·314434	0·95
	22·5	6 46 9·1	16 4 29	0·307987	0·97
	24·5	6 44 11·3	15 12 45	0·301465	0·99
	26·5	6 42 3·4	14 19 27	0·294889	1·01
	28·5	6 39 45·0	13 24 32	0·288278	1·03
	30·5	6 37 15·7	12 27 57	0·281658	1·04
Oct.	2·5	6 34 35·0	11 29 39	0·275054	1·06
	4·5	6 31 42·4	10 29 35	0·268493	1·08
	6·5	6 28 37·6	9 27 43	0·262007	1·10
	8·5	6 25 20·1	8 24 3	0·255635	1·12
	10·5	6 21 49·6	7 18 36	0·249414	1·14
	12·5	6 18 5·9	6 11 21	0·243386	1·16
	14·5	6 14 8·8	5 2 24	0·237588	1·17

The brightness on April 6·5 (near the time of discovery) is taken as unity.

On the Star No. 894 Greenwich First Seven Year Catalogue. By M. A. Wagner.

(*Communicated by the Astronomer Royal.*)

In his paper on the supposed transneptunian planets Professor Forbes refers to the star No. 894 of the Greenwich Seven Year Catalogue which he thinks might have been one of the hypothetical planets. The star has been observed twice in 1857, April 15 and 16. The results of the individual observations are:—

	App. A.R.			App. N.P.D.			Mean A.R. and N.P.D. 1857·0.	
	h	m	s	°	′	″		
Apr. 15	11	19	6·74	85	4	35·44	4·95	36·16
16	[illegible]	[illegible]	6·99	[illegible]	[illegible]	32·86	5·22	33·64

The difference between the two observations is not greater than is consistent with the note "very faint," added to the second observation. If, indeed, these two observations have belonged to a planet moving in an orbit with a mean distance of 100, and a time of revolution of 1,000 years the Right Ascension of the object ought to have diminished by about $1^s·6$, and the N.P.D. to have increased by a number of seconds. Hence it appears that these observations cannot belong to a planet.

It might be inferred from Professor Forbes' remark that the alleged star is now wanting in the heavens, and, indeed, there is no star to be found at this place in the *Bonner Durchmusterung*. But the *Durchmusterung* is complete only to the ninth magnitude, and a star designated as "very faint" in the Greenwich Transit Circle probably will be of a less magnitude.

Our 6-inch refractor, however, and the heliometer show the star in its proper place. It is perhaps a little less than tenth magnitude. By comparing it with the two stars W. XI. 233 = Schj. 4117 and W. XI. 258 = Schj. 4124, I deduced from observations of May 5 its mean position 1880·0—

$$\alpha = 11^h\ 20^m\ 16^s·2 \qquad \delta = +4°\ 49'\ 58''$$

The Greenwich position from the Seven Year Catalogue reduced to 1880·0, gives—

$$\alpha = 11^h\ 20^m\ 16^s·20 \qquad \delta = +4°\ 50'\ 1''·4$$

In the *Atlas Ecliptique de l'Observatoire de Paris* this star also seems to exist, although a little misplaced. The map for 1852·5 has a star of the eleventh magnitude in $11^h\ 18^m\ 56^s + 5°\ 3'$, which reduced to 1880·0 gives $11^h\ 20^m\ 21^s + 4°\ 54'$, but in this declination there is no star; the general configuration, how-

ever, agrees very well with the heaven. Therefore, if the original entries of the Greenwich Observatory do not state expressly that the star was brighter than tenth magnitude on April 15, 1857, there seems to be no reason for supposing it a variable star.

Pulkowa, 1880, *May* 12.

Coincidence of Sun-spots and Auroræ in Olden Time.

By the Rev. S. J. Johnson.

As with a slowly increasing number of Sun-spots at the present time we may assume the return of fine auroral displays to be drawing near, it may not be devoid of interest to examine what coincidence may be traced between the auroræ seen in this land by our forefathers and the few recorded instances we have of Sun-spots in early times. Meagre though our data may be, it is by no means difficult to trace this coincidence. The ease with which this may be done is more remarkable from the fact that, according to Wolf's tables, there are instances laid down of sixteen years between the assumed maxima of solar spots, and at other times less than half this period.

In a communication to *Nature*, January 13, 1870, an endeavour was made to show that the epochs at which Sun-spots were seen before the invention of the telescope were coincident with the maximum periods of the Sun-spot cycle. Thus it was remarked—

From	321	A.D. to	1860	we have	139	periods of	11·072+	years each
„	321	„	807	„	44	„	11·045	„
„	807·22	„	840·5	„	3	„	11·093	„
„	840·5	„	1096	„	23	„	11·109	„
„	1096	„	1161	„	6	„	10·833	„

But a great number of spots are on record, chiefly in China, as seen with the naked eye, in addition to those mentioned in the said article.

The first instance in our own country occurs less than twenty years after the first recorded eclipse, being the mention by Matthew of Westminster of "fiery lances in the air," in the year 555. The Chinese do not record spots close to this date, but in 577, which would give two periods of eleven years each.

The following instances will be found in the *Chronicon Scotorum* and the *Anglo-Saxon Chronicle*.

A.D. 660. "In the summer, the sky was seen to burn." *Chronicon Scotorum.*

Employing the data above referred to, it will be seen that from 660

R R

to 807 there are thirteen periods of about $11\frac{1}{2}$ years each. The following evidently occurred about the time of the next Sun-spot maximum.

670. "A thin and tremulous cloud in the form of a rainbow appeared at the fourth watch of the night of the fifth day before Easter Sunday (stretching) from east to west in a clear sky. The moon was turned into blood." *Chron. Scot.* The expression "clear sky" may be understood comparatively as it is now generally assumed that the aurora is hardly ever visible in a perfectly clear sky, and often appears mingling with cirrous haze. On one occasion only, I observed an aurora when not a trace of cloud was perceptible. In the instance of A.D. 670, the auroral arch appears to have been very marked, as we witnessed in 1869, 1870, and 1872. The expression concerning the Moon in other parts of the early chronicles, always indicates a total eclipse. On this occasion, however, there was no eclipse, but the Moon must have appeared suffused with the auroral vapour. It is worthy of note that on March 19, 1848, when the Moon shone with such vivid redness during the total phase, though deeply immersed in the shadow of the earth, instead of the ordinary dull copper tint, an aurora was prevailing.

710. "A bright night in autumn," *Chron. Scot.* This somewhat obscure expression seems best explained by an aurora; and if so, would happen four ten-year periods after the luminous arc of 670.

773. "In this year a red cross appeared in the heavens after sunset." *Angl.-Sax. Chron.* It is probable an auroral light is referred to here. The interval, from 773 to 807, when spots were particularly noted, gives three periods of $11\frac{1}{3}$ years each.

793. "In this year, dire forewarnings came over the land of the Northumbrians, and miserably terrified the people; there were excessive whirlwinds and lightnings, and fiery dragons were seen flying in the air." *Sax. Chron.* The "fiery dragons" may give the idea of red auroral streamers. But this would not synchronise with an assumed Sun-spot maximum, and may be best explained by a meteoric display.

890. "The heavens appeared to be on fire at night on the Kalends of January." *Chron. Scot.* There would be five ten-year periods from 840, when Sun-spots were often noticed during the summer months. As the intervals are periods of ten years, not of half or two-thirds of that number, we may regard the coincidence as quite satisfactory, though eleven years is generally assumed as the average Sun-spot interval. Again, conspicuous auroræ are noticed a year or two at least on each side of the maximum time.

944. "Two fiery columns were seen a week before Allhallowtide which illuminated the whole world." *Chron. Scot.* We have here ten average periods of 10·4 years from 840.

979. "A bloody cloud in likeness of fire was seen oftentimes; and that was most apparent at midnight, and was coloured in various rays. Then, when it was about to dawn, it glided away." *Sax. Chron.* We may suppose either twelve average periods of 11·6 years, or thirteen of 10·7 years after 840.

1098. "Before S. Michael's Mass the heaven appeared almost all night as if it were burning." *Sax. Chron.* Two years before this, solar spots were seen with the naked eye. It would, therefore, happen not long after a Sun-spot maximum.

1117. "In the night of the XVIIth of the Kal. of January (Dec. 16th), the heaven was seen very red, as if it were a conflagration." *Sax. Chron.* The interval from 1096 gives two periods of 10·5 years, and in the following year, 1118, the Chinese record Sun-spots.

1122. "There were many shipmen on the sea, and on (fresh) water, who said that they saw in the north-east a great and broad fire near the earth, which at once waxed, in length, up to the sky, and the sky separated into four parts, and fought against it, as if it would quench it; but the fire nevertheless waxed up to the heavens. The fire they saw in the dawn, and it lasted so long till it was light over all. That was on the day the VIIth of the Ides of December (Dec. 7th)." *Sax. Chron.* The account reminds one of Stowe's description of a fine aurora in the time of Queen Elizabeth. It was but a short interval from the last display in 1117, but the Chinese records speak of Sun-spots being seen the next year, in 1123. We must therefore conclude that the Sun's disc was in a very active state about this time, and according to Dr. Wolf there have been intervals of only 7½ years in modern times between the maxima.

1131. "This year after Christmas, on a Monday night, at the first sleep, the heaven was, on the north side, all as though it were a burning fire, so that all who saw it, were so affrighted as they never were before. That was on the IIIrd of the Ides of January (Jan. 11th)." *Sax. Chron.* An interval of nine years from the last. It will be observed that it was witnessed three ten-year periods before the spot of the Spanish Moor Averroës (1161). The Chinese have numerous records of solar spots in this century, the 12th, and also in the 4th. The *Chronicon Scotorum* and *Anglo-Saxon Chronicle* contain no mention of these phænomena.

Abbenhall Rectory, Gloucester,
1880, *June* 9.

On the Refractive and Dispersive Powers of various Samples of Glass. By Mr. J. G. Lohse.

(*Communicated by Lord Lindsay.*)

With a view to obtaining a knowledge of the refractive and dispersive powers of various sorts of glass now in actual use, the prisms have recently been examined at Dun Echt Observatory by Mr. J. G. Lohse. Prisms 1 to 8 were kindly lent by Mr. Hilger with the option of purchasing any or all of them except No. 8, which was a beautiful sample prism of an object-glass in course of construction. Nos. 9 and 10 were supplied by Mr. Grubb as closely resembling the lenses of the Dun Echt 15·06-in. refractor.

The indices of refraction are for vacuum-glass.

The specific gravities are reduced to vacuum and 0° Cent. in terms of water at its greatest density.

No.	Sort of Glass.	Specific Gravity.	B	C	D	E	F	G	H	
1	Light Flint	3·2177	1·57816	1·57955	1·58369	1·58894	1·59355	1·60267	1·61073	not quite so colourless as Nos. 2, 3, and 4.
2		3·1530	1·56416	1·56558	1·56945	1·57449	1·57890	1·58750	1·59502	colourless.
3		3·5165	1·60510	1·60679	1·61147	1·61767	1·62316	1·63399	1·64421	colourless.
4	to	3·6155	1·61470	1·61647	1·62138	1·62787	1·63355	1·64469	1·65491	colourless.
5		4·1408	1·68539	1·68770	1·69388	1·70207	1·71001	1·72501	1·73897	slightly greenish-yellow, closely resembling the colour of chlorine.
6		4·4374	1·70422	1·70664	1·71346	1·72241	1·73068	1·74689	1·76166	decidedly greenish-yellow; chlorine colour.
7	Extra dense Flint	4·9566	1·77362	1·77669	1·78537	1·79697	1·80771	1·82917	1·84913	very strongly greenish-yellow, about 1½ times the intensity of No 6, but nevertheless apparently just as transparent, if not more so, and, considering the much greater dispersive power, a better prism.
8	Crown (Feil)	2·4688	1·50522	1·50621	1·50867	1·51175	1·51451	1·51964	1·52389	colourless.
9	Crown (Chance)	2·4728	1·51151	1·51254	1·51488	1·51804	1·52073	1·52587	1·53023	colourless.
10	Flint (Chance)	3·6619	1·61706	1·61868	1·62365	1·63012	1·63594	1·64775	1·65828	colourless.

Corrections to *Monthly Notices*, present volume, p. 438.

In the column $\Delta\delta$ the minus sign applies to all the quantities.
In the column "Observer" *for* T. G. L. *read* J. G. L.
After the observations *insert*: The observers were J. Gerh. Lohse and Ralph Copeland.

MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. XL. SUPPLEMENTARY NOTICE. No. 9.

The Babylonian Astronomy. By R. H. M. Bosanquet and Prof. A. H. Sayce.

No. 3. *The Venus Tablet.*

The name of the planet *Venus*, in the writings of ancient Babylon is known to us with as great certainty as any word in the inscriptions. In the small list of words preserved by Hesykhius, there is the word δελεφάτ, which he gives as the name of the Babylonian Venus. The word Dilbat occurs in the inscriptions as the name of a planet, and corresponds generally, as to its place in the lists, with *Venus*. The planet Dilbat is further identified with the goddess Istar,* and sometimes also with the goddess Annunit.† Istar in turn, regarded as a star, was called Nin-śi-ànna, "Lady of the defences of heaven." To show clearly our authority for this statement, the following words are written down, exactly representing two corresponding lines of an inscription in two parallel columns.

W. A. I. ii. 59, 20 (2).

Divine Ninśianna.	Divine Istar; a star.

We thus arrive at the name Ninśianna as a name commonly used for Venus.

The tablet which forms the subject of the present communication speaks of the "divine Ninśianna" throughout. The tablet is a good deal broken, but a considerable number of observations survive. The following is the translation of the

* See *Trans. Soc. Bibl. Arch.* iii. 197; also *W. A. I.* (*Western Asiatic Inscriptions*), ii. 49, 11.

† *W. A. I.* ii. 49, 12.

tablet. For the text see *W. A. I.* iii. 63 and *Trans. Soc. Bibl. Arch.* iii. 316.

OBVERSE.

LINE.

1. . . . .

2. . . . . Venus disappeared, and the 2nd day Venus . . . .

3. . . . . for a year are. A destruction takes place.

4. In the month Tammuz, the 25th day, Venus at sunset disappeared.

5. The 7th day* in heaven it is visible; and in the month Ab, the 2nd day, Venus

6. at sunrise is seen. Waters in the land are. Beating of . . . .

7. In the month Adar, the 25th day, Venus at sunrise disappeared.

8. For a year service continues. Gold . . . .

9. In the month (Sivan) the 11th day† Venus at sunset disappeared. The 9th month, the 4th day, in heaven it was visible; and

10. in the month Adar, the 15th day, at sunrise it is seen. King to king peace sends.

11. In the month Marchesvan, the 10th day, Venus at sunrise disappeared. The 2nd month, the 6th day, in heaven it appeared ‡

12. In the month Tebet, the 16th day, at sunset it is seen. The crops of the land are prosperous.

13. In the month Elul, the 26th day, Venus at sunset disappeared. The 11th day, in heaven it appeared.

14. In the second Elul, the 7th day, at sunset it is seen. The heart of the land is good.

15. In the month Nisan, the 9th day, Venus . . . . disappeared. The 5th month, the 16th day, in heaven (it reappeared; and)

16. In the month Elul, the 25th day, at sunset it is seen. The heart of (the land is good).

17. In the month Iyyar, the 5th day, Venus at sunset is seen . . . . in heaven it appeared; and

18. At sunrise it is seen. The crops of the land are good . . . . the 10th day at sunrise . . . .

19. At sunrise it disappeared. The 15th day in heaven . . . . In the month Sebat, the 11th day at sunset . . . .

20. In the month Ab, § the 10th day, Venus at sun . . . . The 1st month,‖ the 16th day . . . .

21. In the month Marchesvan, the 26th day, at sunset it is seen . . . ; Rains in the land are . . . .

* L. 5. The comparison of the interval of 7 days with the preceding and following observations shows that the month was here supposed to be 30 days

† L. 9. It is established, by comparison of passages in which a number of months and days is mentioned, that in all such cases reference is made to the interval between two observations, and not to the year-number of the month. Here the interval permits us to restore the month of the first observation, which is Sivan; it will appear from the subsequent discussion that this date belongs to the appearance at sunset, not to the disappearance.

‡ L. 13-14. Month of 30 days.

§ L. 20-21. Mistake in the month-number of one of the three entries.

‖ Or "after one month and 16 days."

22. In the month . . . . the 20th day, Venus at sunset disappeared. The 2nd month, the 16th day . . . .

23. In the month . . . . the 4th day, at sunset it is seen. Rains in the land are. '

24. In the month . . . . the 6th day, Venus at sunset disappeared The 15th day . . . .

25. In the month . . . . the 20th day, at sunset it is seen. Rains in heaven, floods in the channels are . . . .

26. In the month Adar, the 26th day, Venus at sunrise disappeared. The 3rd month, the 9th day . . . .

27. In the month Sivan, the 20th day, at sunset it is seen. The forces of barbarian soldiery are collected . . . .

28. In the month Adar, the 11th day, Venus at Sun . . . . (disappeared). The 4th day in heaven . . . .

29. The crops of the earth flourish. The heart of the land is good.

30. . . . . wind, rain, snow, ...*

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31. †In the month Nisan, the 2nd day, Venus at sunrise is seen. Deserts in the land are.

32. Up to the 6th day of the month Chisleu at sunrise it is fixed.‡ The 7th day of Chisleu it disappears, and after 3 months in heaven

33. It appears; and the 8th day of the month Adar, Venus at sunset rises and king to king hostility sends.

34. In the month Iyyar, the . . . . day, Venus at sunset is seen Hostile bands in the land are.

35. (Up to the 7th day of) the month Tebet at sunset it is fixed.§ The 8th day of Tebet it disappears; and

36. (The 7th day in heaven) it appears; and the 15th day of Tebet, Venus

37. At sunrise rises; and the crops of the land are flourishing, the heart of the land is good.

38. (In the month Sivan, the 4th day) Venus at sunrise is seen. An inundation in the land.

39. (Up to the 8th day of the month Sebat) at sunrise it is fixed. The 8th day of Sebat it disappears; and

40. After (3) months in heaven appears; and the 9th day of the month Iyyar, Venus

41. At sunset rises; and hostile bands in the country are.

42. In the month Tammuz, the 5th day, Venus at sunset is seen. Hostile bands in the land are. The crops of the land flourish.

43. Up to the 9th day of the month Adar at sunset it is fixed. The 10th day of Adar it disappears; and

44. The 7th day in heaven it appears; and the 7th day of the month Adar, Venus

45. At sunrise rises; and king to king hostility sends.

* L. 30. Here the tablet marks the end of a paragraph.

† L. 31. Change of style and grammatical usage. Probable beginning of the spurious part. See *post.*

‡ L. 32. *I.e.* seen.

§ L. 35. *I.e.* seen.

REVERSE.

1. In the month Ab, the 6th day, Venus at sunrise is seen. Rains in heaven are. A beating takes place.
2. Up to the 10th day of the month Nisan, at sunrise it is fixed. The 11th day of Nisan it disappears, and
3. After 3 months in heaven it is seen; and the 11th day of the month Tammuz, Venus at sunset
4. Rises; and hostile bands in the land are. The crops of the land are prosperous.
5. In the month Elul, the 7th day, Venus at sunset is seen. The crops of the land flourish. The heart of the land is good.
6. Up to the 11th day of the month Iyyar, at sunset it is fixed. The 12th day of Iyyar it disappears; and
7. After 7 days in heaven it reappears; and the 9th day of the month Iyyar, Venus
8. At sunrise rises; and hostile bands in the country are.
9. In the month Tisri, the 8th day, Venus at sunrise is seen. Hostile bands in the land are. The crops of the land flourish.
10. Up to the 12th day of the month Sivan at sunrise it is fixed. The 13th day of Sivan it disappears; and
11. After 3 months in heaven it reappears; and the 13th day of the month Elul, Venus
12. At sunset rises; and the crops of the land are prosperous; the heart of the land is good.
13. In Marchesvan, the 9th day, Venus at sunset is seen. The land a strong woman seizes.
14. Up to the 13th day of the month Ab, at sunrise it is fixed. The 15th day of Ab, it disappears and
15. The 7th day in heaven it reappears; and the 11th day of the month Tammuz, Venus
16. At sunrise rises; and hostilities in the land are. The crops of the land flourish.
17. In the month Chisleu, the 10th day, Venus at sunrise is seen. Want of corn and barley in the land is.
18. Up to the 14th day of the month Ab, at sunrise it is fixed. The 15th day of Ab, it disappears and
19. After 3 months in heaven it reappears; and the 15th day of Marchesvan, Venus
20. At sunset rises; and the crops of the land are prosperous.
21. In the month Tebet, the 11th day, Venus at sunset is seen. The crops of the land flourish.
22. Up to the 15th day of the month Elul, at sunset it is fixed. The 16th day of Elul it disappears; and
23. After 7 days in heaven it reappears; and the 23rd day of the month Elul, Venus
24. At sunset rises, and the crops of the land are prosperous.
25. In the month of Sebat, the 12th day, Venus at sunrise is seen. The crops of the land flourish.

26. Up to the 16th day of the month Tisri, at sunrise it is fixed. The 17th day of Tisri it disappears; and
27. After 3 months in heaven it reappears; and the 17th day of the month Tebet, Venus
28. At sunset rises; and . . . .
29. In the month Adar, the 13th day, Venus at sunset is seen. The king . . . .
30. Up to the 17th day of Marchesvan, at sunset it is fixed. The 18th day of Marchesvan, Venus
31. The 7th day in heaven reappears; and the 25th day of Marchesvan, Venus
32. At sunset rises; and the land a strong woman seizes.
33. Twelve collections of observations of the risings of Venus in parallel columns (Accadian and Assyrian) of Babylon.
34. In the 2nd Elul, the 1st day, Venus at sunset disappears.*
35. The 15th day in heaven it is seen; and in the 2nd Elul, the 17th day, Venus
36. At sunrise is seen. A prodigy in the land is; in the palace . . . .
37. In the month Sivan, the 25th day, Venus at sunrise disappears . . .
38. After 2 months, the 6th day, in heaven it is seen; and in the month of Elul, the 24th day,
39. Venus at sunset is seen. The heart of the country is good.
40. In the month Nisan, the 26th day, Venus at sunset disappears; and
41. After 7 days in heaven it reappears; and the month Iyyar, the 3rd day, Venus
42. At sunrise is seen. Hostile bands in the land are. The crops of the land are prosperous.
43. . . . . . Venus at sunrise disappears, and
44. . . . . . in the month Adar, the 28th day, Venus
45. . . . . king to king an ambassador sends

The two following lines, which complete the tablet, are broken off.

It will be seen that there are no year-dates in this tablet; consequently it might be supposed that no further information is obtainable from it. And, indeed, it has been supposed that the observations were probably isolated, and the whole practically of no interest.

Tested, however, by a simple application of the synodical period, we find that the observations link themselves into a certain number of numerical schemes: so that portions of the tablet in question almost certainly refer to continuous series of phenomena. And the different portions of the tablet present differences in the numerical schemes according to which they are constituted, the discussion of which leads to singular results.

We confine ourselves in the first instance to the numbers of

* L. 34. Here begins a new paragraph.

months concerned. This affords a rough test of the synodic period, without involving hypotheses as to detail.

It will be desirable to recall approximate values of the synodic periods of *Venus* and *Mercury*. For *Venus* the number is about 514^d, say 19·2 mean months, or about 19·7 lunar months; for *Mercury*, less than 4 months. The value in the case of *Mercury* is only needed to assure us that no confusion has taken place with that planet.

We will now form an analysis of the dates of the tablet, mentioning in each case only the characteristic number of the month mentioned. These numbers are:—

1 Nisan.	5 Ab	9 Chisleu.
2 Iyyar.	6 Elul.	10 Tebet
3 Sivan.	7 Tisri.	11 Sebat.
4 Tammuz.	8 Marchesvan.	12 Adar.

Lines of Tablet.	Sunset. Appears.	Sunset. Is seen.	Sunset. Disappears.	Sunrise. Appears.	Sunrise. Is seen.	Sunrise. Disappears.
			Obverse.			
4–7			4	5		12
10–11	3		. . . .	12		8
11–15	10		■	7 (2nd Elul)		1
15–19	6		2	. . . .		. . . .
19			11 (?)	. .	. .	5 (?)
21–22	8					. . . .
24–26		. . . .	. . . .	. . . .		12
27	3		[12]			
31–32				1		9
33–39	12	2	10	10	3	11
40–45	2	4	12	12		
			Reverse.			
1–2					5	1
3–10	4	6	2			3
11–18	6	8	[5]	4		5
19–26	8	10	6	[6]	11	7
27–32	10	12	8	[8]		
33		12 collections of observations, &c.				
34–37			7	7		3
38–43	6		1	2		[ ]
44	12					

(Two lines broken off.)

We have first to consider whether the numbers afford proof or probability of the observations being consecutive. This we answer at once in the affirmative.

The following scheme represents the differences of the month-numbers in the successive rows. This shows all the synodic periods of which both ends are given. It affords a means of appreciating the differences of character in the different parts of the tablet.

Differences of successive Rows.

	Sunset.			Sunrise.		
Lines of Tablet.	Appears.	Is seen.	Disappears.	Appears.	Is seen.	Disappears.
			Obverse.			
4–7						
				19		20
0–11						
	19			19		19
11–15						
	18		20			
15–19						
19						
21–22						
31–32						
				21		26
33–39						
	26	26	26	26	26	26
40–45 and Reverse 1–2						
	26	26	26	26	26	26
3–10						
	26	26	27	26	26	26
11–18						
	26	26	25	26	26	26
19–26						
	26	26	26	26		
27–32						
	12 collections of observations, &c.					
34–37						
				18		19
38–43						
	18					
44						

We see at once that the successions are of such a character as to leave no doubt of the continuity of portions of the scheme.

The first 20 lines contain numbers leading to 6 synodic periods between 18 and 20 months, and one of 17 months. The one of 17 is altered to 19, on noticing that it is preceded by a 2nd Elul, and admitting a second intercalary month. This and the remainder are compatible with the synodic period of *Venus*.

Then follow ten lines too imperfect for analysis.

In l. 31 there is one date which combined with the first corresponding number of the next part gives a period of 21 months; this is within the limits of error.

Then follows a sequence of numbers, commencing in line 32, and continuing uninterruptedly down to line 32 of the reverse.

Corresponding figures in successive lines differ by 2 throughout. Six phases are enumerated in every period, without a single omission; and every month-number, except one, throughout six synodic periods falls in exactly with an obvious numerical scheme. The synodic period thus definitely announced is 26 months. No assumption can get rid of the excess of at least 6 months over the period of *Venus*; and it is clear, not only that this portion of the tablet is a fabrication, but also that it is a fabrication by some person wholly unacquainted with the phenomena. There can be little doubt that the scribe, finding this portion of the ancient Babylonian tablet illegible, fulfilled his duty by reconstructing it out of his inner consciousness, for the benefit of the great astronomical collection of Sargon of Aganè. We may note that the part spoken of as spurious differs in style, grammar, &c., from the rest. But it was picked out by one of us from the numbers only, without any knowledge of these peculiarities.

The next line (33 reverse) is

"Twelve collections of observations of the risings of Venus in parallel columns (Accadian and Assyrian) from Babylon."

It is supposed that this is the colophon of the previous portion of the tablet, which is arranged in twelve well-defined short paragraphs or sentences. The text is not bilingual. We therefore suppose it to be copied or made up from an earlier bilingual text in the library of Babylon; the Accadian original being probably earlier still.

In what is preserved of the remaining lines of the tablet, we have three synodic periods of 18, 19, and 18 months respectively, the last observation of the last pair being incomplete. These are within the limits reconcilable with the true period of *Venus* by means of intercalary months. And we are bound to admit, consequently, that the observations bear *primâ facie* the stamp of genuineness.

Any further examination of the tablet may consequently be confined to the first 21 lines, and to what follows the line "Twelve collections of observations."

We proceed to consider somewhat more in detail the complete observations in the first 20 lines. The following is a scheme of these observations, in which the month-number is replaced by the number of complete months. Thus the 9th day of Nisan is written $0^m\ 9^d$.

	Sunset.		Sunrise.	
Lines.	Appears.	Disappears.	Appears.	Disappears.
	m d	m d	m d	m d
4–7		3 25	4 2	11 25
9–11	2 11	—	11 15	7 10
11–15	9 16	5 26	6 7 (I)	0
15–19	5 25	1 5	—	—

The observations of the next few lines are too imperfect for certain discussion, though they might possibly be reconstructed.

The fact that the months whose length is accidentally given (see notes) are of 30 days, appears to point to a different calendar from that of the lunar months. We have calculated the synodic periods both on the assumption of months of 30 days, and of lunar months; the results are not in either case so completely in accordance as to decide the question. It is not improbable that the months were arbitrarily arranged from time to time, partly according to one rule, partly according to the other.

The observation to which (I) is affixed is in the month 2nd Elul—an intercalary, apparently foreign to the regular calendar, in which the intercalary was at the end of the year, and called *Veadar.**

The synodic period of *Venus* is, according to our calculation, $583^{d}\cdot919864$. For the present purpose it will be sufficient to consider it as 584^{d}.

The synodic periods deduced singly from the differences of the above observations are:—

y m d	y m d	y m d	y m d
		1 7 13	1 7 15
1 7 5	—	1 6 22 (I)	1 4 29
1 8 9	1 7 9		

(1 y. = 12 months simply.)

$$\text{Months of } 29\tfrac{1}{2} \text{ days} \quad \overset{d}{584} = \overset{\text{l.y.}}{1}\ \ \overset{\text{l.m.}}{7}\ \ \overset{d.}{26\tfrac{1}{2}}$$

$$\text{Months of } 30 \text{ days} \quad 584 = \overset{y}{1}\ \ \overset{m}{7}\ \ \overset{d}{14}$$

(1 l y. = 1 lunar year = 354d. 1y. = 360d.)

The first two numbers present coincidences which leave no doubt that during the observations of lines 4–11, the months were counted as of 30 days each.

The subsequent periods present deviations which seem to indicate first that intercalaries were introduced; also either that the text is corrupt (which is very probable) or that lunar months were returned to in the course of the series. The number $1^{y}\ 4^{m}\ 29^{d}$ is preceded by one intercalary month shown in an observation and denoted by (I); there also appears to be another intercalary month before line 12, as is obvious from the number $1^{y}\ 6^{m}\ 22^{d}$; and the two together would raise the $1^{y}\ 4^{m}\ 29^{d}$ to nearly $1^{y}\ 7^{m}$, which is within limits of calendar error. The number $1^{y}\ 8^{m}\ 9^{d}$ is not so easily accounted for. $1^{y}\ 7^{m}\ 9^{d}$ is within the limits of error, if it were not for the intervening intercalary. This makes the last number really $1^{y}\ 8^{m}\ 9^{d}$, some 12^{d} greater than is consistent with lunar months. Further, we

* See Norris's *Assyrian Dictionary*, i. 50.

must notice that appearances of the planet may probably have been delayed by cloudy weather. Therefore it is possible that on this ground certain of the observations may not accurately correspond to the phases in the synodic scheme.

We will now form the differences corresponding to two synodic periods. These are:—

y m d	y m d	y m d	y m d
	3 2 1	3 2 5	3 0 14
3 3 14	—		

	d	d l.y. l.m. d.
Months of	$29\frac{1}{2}$	$2 \times 584 = 3\ \ 3\ \ 23\frac{1}{2}$
	d	y m d
Months of	30	$= 3\ \ 2\ \ 28$

It is clear from the first two numbers that there must, in any case, have been an intercalary month before line 12, as before suggested. This would cause the above numbers to assume the following form:—

y m d	y m d	y m d	y m d
	3 3 1	3 3 5	3 2 14
3 5 14	—		

The first two are within the limits of calendar and other error, especially if we suppose that lunar months were employed during a part of the period. The third is scarcely admissible unless we assume a third intercalary immediately preceding it. In any case, the fourth number is irreconcilable.

One pair of numbers gives us 3 synodic periods—the first and last of the above observations. The resulting number is:—

		y m d
		4 9 10
	d	d l.y. l.m. d
Months of	$29\frac{1}{2}$	$3 \times 584 = 4\ \ 11\ \ 20\frac{1}{2}$
	d	y m d
Months of	30	$= 4\ \ 10\ \ 12$

It must be remembered that one intercalary is actually mentioned in the observations, and there is sufficient evidence of another. Taking these into account we find for the corrected interval given by the two extreme observations:—

y m d
4 11 10

counting the year at 12 months.

This coincides substantially with the number corresponding to lunar months.

We have decided for the present to content ourselves with presenting the above comments on this remarkable series of numbers. They sufficiently indicate the nature of the results

obtainable, without professing to give a final treatment of the subject. We have applied various corrections, and constructed various hypothetical calendars; but the results have not been such as to encourage us to produce them. It is possible that when more is known of the history of this ancient period the difficulties of the text may be more easily dealt with.

We may give one illustration of the application of a correction which seems at first sight feasible and even necessary, and yet has been definitely abandoned, at least for the present.

On the reappearance of *Venus* from the region of the Sun, or on its disappearance into the Sun's rays, it would seem that the vertical distance of *Venus* above the Sun, as the latter dips below the horizon, should determine the critical position when the planet is just seen. And according to the time of year, a definite vertical distance such as 12° corresponds to very different distances along the ecliptic. In order to obtain determinations of the synodic period, it would therefore be necessary to apply corrections which should reduce the dates to accurately corresponding configurations. Thus when the ecliptic is more inclined to the vertical, the appearance is later and the disappearance sooner by a certain amount depending, for a particular phase, on the inclination of the ecliptic. Although we cannot tell precisely when the equinox fell, yet assuming that it was approximately at the beginning of the year, it is easy to calculate rough values of these corrections. We have calculated them for each month, and examined the effect of their application. Their amount is considerable; and they appear to throw the observations into confusion. They spoil the synodic periods that do exist, and do not improve materially those that are faulty.

The explanation of this appears to be, that the visibility of *Venus* depends more on its (crescent) phase than on the vertical distance from the Sun in the horizon. If this be admitted, the distance from the Sun in the ecliptic becomes the only point of importance, and the substantial correctness of synodic periods derived directly from the observations is to some extent explained.

The observations which occupy the last lines of the tablet are as follows:—

	Sunset.		Sunrise.	
Line.	Appears. m d	Disappears. m d	Appears. m d	Disappears. m d
34–37		6 1	6. 15 (I)	2 25
38–43	5 24	0 26	1 3	—
44	11 28 (? Imperfect.)			

Differences 1 y 6 m 25 d and 1 y 6 m 18 d

But on account of the intercalary 2nd Elul the corresponding synodic periods are—

1 y 7 m 25 d and 1 y 7 m 18 d

One of these numbers would decide for the lunar months; the other for the 30 days. Unfortunately, we cannot decide between them.

As to the interval between the disappearance at sunset and the appearance at sunrise.

In two cases this interval is specified to be 7 days (lines 5, 41), but not elsewhere throughout the genuine portion. In line 13 this interval is specified as of 11 days, and in line 35 (rev.) as of 15 days. Throughout the portion which we consider not genuine, this interval is uniformly stated to be 7 days. A slight calculation shows that if we suppose *Venus* to be visible at sunrise or sunset when 12° from the Sun in the ecliptic, this interval should consist of 15 days nearly. It is possible that *Venus* may be visible in tropical regions when nearer to the Sun than this. But in order that the interval may be only 7 days, it is necessary that *Venus* should be visible when only 6° from the Sun in the ecliptic. We beg to call the attention of astronomers in tropical countries to the question at what distances from the Sun *Venus* can be actually seen at sunrise or sunset. But in the mean time we consider the interval of 7 days as highly doubtful, and casting a doubt upon an observation in each case where it occurs. Its uniform occurrence in the spurious part of the tablet is in accordance with this opinion.

As to the interval between the disappearance at sunrise and the appearance at sunset, calculation based on a distance of 12° from the Sun gives 54 days nearly. The numbers from the text are 76 days, 66 days, and 5 months 16 days. The two first numbers accord pretty well with what should happen, the distance from the Sun being taken as greater than 12°, since *Venus* is in the least bright portion of its course. The last number cannot represent an observation.

The number 89 days also occurs in the last lines of the tablet.

The only remaining point worth noticing is that if a tablet such as this were discovered, with first-hand observations, and any king's name, or mark of historical connection which would fix the date within, say, 100 years, it might be possible to fix the date more closely by means of our knowledge of the motions of *Venus*. Nothing of the kind is actually possible with this tablet, as there is no clue whatever to the age of the observations. But, with a view to possible discoveries, it may be well to state shortly the considerations involved.

It is well known that 8 years are nearly equal to 13 revolutions of *Venus*. If this were exactly true, the phases of *Venus* would go through a cycle in 8 years, forming 5 synodic revolutions, and the cycle would repeat itself in successive periods of 8 years. Since, however, the commensurability of mean motions is not exact, the dates of the phases at given points of the cycle do not remain fixed, but shift slowly. If then *Venus* has a definite phase on a given day of the year, the only possible dates are given by the shifting cycle, and it will appear that two such

possible dates for a given day are separated by an interval of 235 years nearly. If, therefore, we can assign the date approximately within, say, half one of these intervals, it is quite possible that we might be able to fix the date more closely by means of the observations of *Venus*.

The numbers are obtained as follows:—

(Herschel's *Outlines of Astronomy*, table at the end.)

Sidereal year	$365^{d}\!\cdot\!256361 = a$
Sidereal rev. of Venus	$224\cdot700787 = b$

in mean solar days. Whence synodic period in days

$$= \frac{ab}{a-b}$$

$$= 583^{d}\!\cdot\!919864$$

8 years (sidereal)	=	$2922^{d}\!\cdot\!05089$
5 synodic periods	=	$2919\cdot59932$
deviation of cycle	=	$2\cdot45157$

To examine the recurrence of the synodic revolutions in the cycle suppose the cycle to be true. Then the synodic period would be in mean months

$$\frac{96^{m}}{5} = 19^{m}\!\cdot\!2$$

and the synodic periods from the beginning of the cycle are—

y	m
0	0·0
1	7·2
3	2·4
4	9·6
6	4·8

after which the cycle begins again.

Now, suppose that at a given epoch *Venus* is in a given phase, and that this takes place at the zero of the cycle. Then (neglecting the eccentricity of the orbits and other corrections) *Venus* can only be in the given phase at the other times of the year shown in the column of months. For instance, this cannot occur between the month 0 and the middle of the month 3. But for ancient and future times the cycle is displaced to the extent above indicated.

The quantity which appears above as 2·4 mean months is, when calculated more accurately, $72^{d}\!\cdot\!070645$. Dividing this by

the deviation in one cycle and multiplying by 8^y, we find for the period of change through one interval of the cycle, $235^y \cdot 182$ (sidereal years).

It would be quite possible in this way to calculate the dates at which the observations of this tablet could have been made; but a conjectural element enters into the reconstruction of the calendar of the observations. And as there is nothing to associate these observations with historical dates, there is no possibility of a real contribution to ancient history in this case.

It is, however, amply proved that these observations do actually refer to *Venus*, except the portion which we have excepted as spurious. It is further shown that the writer of the tablet, which has come down to us from remote antiquity, was himself an ignorant copyist of an earlier tablet or tablets; but of what date are the observations of this earlier tablet or tablets there is no evidence to show, except the antique style, and the fact of their belonging to the collection supposed to have been made by Sargon of Aganè; which tends to refer them to a period older than 1700 years B.C.

The length to which this comment has extended, and other circumstances, compel us to postpone the discussion of the identification of stars, &c.

Addition to a Paper entitled "On the Theoretical Value of the Acceleration of the Moon's Mean Motion in Longitude produced by the Change of Excentricity of the Earth's Orbit." By Sir G. B. Airy, K.C.B., Astronomer Royal.

In a paper lately communicated to the Society, my principal object was to exhibit the power of a new or factorial method of obtaining the disturbance of the Moon's movements depending on the introduction or change of an external force; and I applied it to the disturbance called the "Acceleration of the Moon's Mean Motion," produced by a gradual change in the excentricity of the earth's orbit. In completing the calculation, and estimating for that purpose the external factor of the formula, I limited myself to the imperfect expressions for the change of magnitude of the force, which (historically) had been adopted in the earliest investigations. I now propose to employ the more complete formula; referring, when advantageous, to the preceding paper, and using it to abbreviate this communication where it appears possible to do so.

(19.) The Sun's disturbing action is to be treated here in the same form as in the preceding articles; employing the symbol T for the ecliptic tangential accelerating force, and P for the ecliptic radial force measured from the Earth, and using V and v for the true longitudes of the Sun and Moon, R for the Sun's radius vector, and r for the Moon's radius vector, A for

the semi-axis major of the Sun's apparent orbit, and a for the reciprocal of the non-periodic term in the reciprocal of the Moon's radius vector. The units of measure have been explained in the preceding part of the paper. Then—

$$T \cdot \frac{1}{a} \cdot \frac{r}{a} = -\cdot 0083928^*. \left(\frac{A}{R}\right)^3 \cdot \left(\frac{r}{a}\right)^2 . \sin \overline{|2(v-V)|}$$
$$+ \cdot 0000053 \left(\frac{A}{R}\right)^4 \cdot \left(\frac{r}{a}\right)^3 . \sin \overline{|v-V|};$$

$$P \cdot \frac{1}{a} \cdot \frac{r}{a} = +\cdot 0083928^*. \left(\frac{A}{R}\right)^3 \cdot \left(\frac{r}{a}\right)^2 . \left\{ +\frac{1}{3} + \cos \overline{|2(v-V)|} \right\}$$
$$-\cdot 0000160 . \left(\frac{A}{R}\right)^4 \cdot \left(\frac{r}{a}\right)^3 . \cos \overline{|v-V|};$$

with some very small terms depending on the argument $\overline{|3(v-V)|}$. These latter terms, as will be seen below, can only produce available results by being multiplied by other very small terms of nearly the same form, and the results will be far below our limits of accuracy for other terms. As regards cos $\overline{|v-V|}$ in P (above), its factor is less than $\frac{1}{500}$ of the factor of cos $\overline{|2(v-V)|}$, and it will be multiplied by terms whose coefficients are less than $\frac{1}{10}$ of those of cos $\overline{|2(v-V)|}$, so that its efficient result will be less than $\frac{1}{5000}$ part. And that for T will be less than $\frac{1}{15000}$ part. I therefore reject these terms, and use only the following:

$$T \cdot \frac{1}{a} \cdot \frac{r}{a} = -\cdot 0083928 \times \left(\frac{A}{R}\right)^3 \cdot \left(\frac{r}{a}\right)^2 \times \sin \overline{|2(v-V)|},$$

$$P \cdot \frac{1}{a} \cdot \frac{r}{a} = +\cdot 0083928 \times \left(\frac{A}{R}\right)^3 \cdot \left(\frac{r}{a}\right)^2 \times \left\{ +\frac{1}{3} + \cos \overline{|2(v-V)|} \right\}.$$

(20.) Before entering on the details of treatment of these formulæ, it will be convenient to premise the following remarks.

Our object is, to ascertain the effect produced in our numerical expressions, by a numerical change δE of the numerical quantity E, the excentricity of the solar orbit. It is necessary therefore to know what power of E has been employed in forming our numerical expressions. Now, E is always accompanied, in the first steps of expansion, by an argument containing S. It is true that E^3 or E^5 may have contributed in some instances (not in general) to form a portion of the coefficient of that argument containing S; but, if so, that portion of the coefficient must have received the numerical multiplier E^2 or E^4, either of which is extremely small, and which will have affected

* By a fault of transcription, erroneous numbers are printed in the first investigation, p. 371. The term $-\cdot 0077$ is not wanted here, being included in the further investigations.

a very small part of the coefficient. Thus we may assume, without perceptible error, that the numerical coefficient of

$$\cos\overline{S}|, \quad \text{or} \quad \cos\overline{D-S}|, \ \&c.,$$

contains in itself the factor E; that the coefficient of

$$\cos\overline{2S}|, \quad \text{or} \quad \cos\overline{2D-2S}|, \quad \text{or} \quad \sin\overline{2D-2S}|, \ \&c.,$$

contains in itself the factor E^2, and so on. (This, it will be observed, applies to the developments of R and V, but not always to more complicated functions, in which the trace of S may be destroyed by the union of two arguments; in these, the terms must be multiplied with due attention to their formation.) And, the power of E being thus determined, the variation of the coefficient, depending on δE, will be conveniently expressed thus. Let Q be a numerical coefficient containing the n^{th} power of E, so that $Q = W.E^n$ (W being a numeral, or a trigonometrical function, &c.). Then

$$\delta Q = n.W.E^{n-1}.\delta E = n.W.E^n.\frac{\delta E}{E} = n.Q.\frac{\delta E}{E}.$$

(21.) We shall now proceed to form the numerical value of $\left(\frac{A}{R}\right)^3.\left(\frac{r}{a}\right)^2$, with its variation depending on δE.

First, to form $\left(\frac{A}{R}\right)^3$,

Le Verrier gives for $\frac{R}{A}$,

$$+1\cdot0000000 \overset{E^2}{+\cdot0001406} \overset{E}{-\cdot0167687}.\cos\overline{S}| \overset{E^2}{-\cdot0001406}.\cos\overline{2S}|$$
$$\overset{E^3}{-\cdot0000018}.\cos\overline{3S}|;$$

(in which the symbols E, E^2, E^3, placed above the figures, indicate the power of E from which those numbers have been formed): and from this, by ordinary expansion, we find for $\left(\frac{A}{R}\right)^3$,

$$+\left\{+1.0000000 \overset{E^2}{+\cdot00004215}\right\} + \left\{\overset{E}{+\cdot0503061} \overset{E^3}{+\cdot0000114}\right\}\cos\overline{S}|$$

$$\overset{E^2}{+\cdot0012651}.\cos\overline{2S}| \overset{E^3}{+0000309}.\cos\overline{3S}|;$$

and, forming the variation on the principles just described, and attaching it to the principal terms, we obtain for $\left(\frac{A}{R}\right)^3 + \delta \cdot \left(\frac{A}{R}\right)^3$,

$$+ 1\cdot 0004215 + \cdot 0503275 \cdot \cos \overline{S} + \cdot 0012651 \cdot \cos \overline{2S} + \cdot 0000309 \cdot \cos \overline{3S}$$

$$+ \frac{\delta E}{E} \times \left\{ + 0008430 + \cdot 0503703 \cdot \cos \overline{S} + \cdot 0025302 \cdot \cos \overline{2S} + \cdot 0000927 \cdot \cos \overline{3S} \right\}.$$

Secondly, to form $\left(\frac{r}{a}\right)^2$.

The expression which I have found by expansion from Delaunay's coefficients, including no arguments excepting those which contain D, S, and their combinations, is

$$+ 1\cdot 0046872 + \cdot 0005426 \cdot \cos \overline{D} - \cdot 0148320 \cdot \cos \overline{2D} \overset{E}{+ \cdot 0002763} \cdot \cos \overline{S}$$

$$\overset{E^2}{+ \cdot 0000075} \cdot \cos \overline{2S} \overset{E}{- \cdot 0000880} \cdot \cos \overline{D+S} \overset{E}{+ \cdot 0000021} \cdot \cos \overline{D-S}$$

$$\overset{E}{+ \cdot 0001624} \cdot \cos \overline{2D+S} \overset{E}{- \cdot 0010217} \cdot \cos \overline{2D-S} \overset{E^2}{- \cdot 0000421} \cdot \cos \overline{2D-2S}$$

$$\overset{E}{- \cdot 0000021} \cdot \cos \overline{4D+S} \overset{E}{+ \cdot 0000082} \cdot \cos \overline{4D-S},$$

which gives $\left(\frac{r}{a}\right)^2$, requiring no further change.

Then for $\delta \cdot \left(\frac{r}{a}\right)^2$

we have,

$$+ \frac{\delta E}{E} \times \left\{ + \cdot 0002763 \cdot \cos \overline{S} + \cdot 0000150 \cdot \cos \overline{2S} - \cdot 0000880 \cdot \cos \overline{D+S} \right.$$
$$+ \cdot 0000021 \cdot \cos \overline{D-S} + \cdot 0001624 \cdot \cos \overline{2D+S}$$
$$- \cdot 0010217 \cdot \cos \overline{2D-S} - \cdot 0000842 \cdot \cos \overline{2D-2S}$$
$$\left. - \cdot 0000021 \cdot \cos \overline{4D+S} + \cdot 0000082 \cdot \cos \overline{4D-S} \right\}.$$

Multiplying the expression for

$$\left(\frac{A}{R}\right)^3 + \delta \cdot \left(\frac{A}{R}\right)^3$$

by that for

$$\left(\frac{r}{a}\right)^2 + \delta \cdot \left(\frac{r}{a}\right)^2$$

we obtain the expression for

$$\left(\frac{A}{R}\right)^3.\left(\frac{r}{a}\right)^2 + \delta.\left(\frac{A}{R}\right)^3.\left(\frac{r}{a}\right)^2,$$

as follows:—

Value of $\left(\frac{A}{R}\right)^3.\left(\frac{r}{a}\right)^2$.	Factors of $\frac{\delta E}{E}$ in the Variation of $\left(\frac{A}{R}\right)^3.\left(\frac{r}{a}\right)^2$. (The first line in each bracket gives the factor depending on $\left(\frac{A}{R}\right)^3$; the second gives that depending on $\left(\frac{r}{a}\right)^2$).	
+ 1·0051177	+ ·0008539 + ·0000070	= + ·0008609
+ ·0005407 . cos $\overline{D}$	− ·0000017 − ·0000021	= − ·0000038 . cos $\overline{D}$
− ·0148598 . cos $\overline{2D}$	− ·0000342 − ·0000217	= − ·0000559 . cos $\overline{2D}$
+ ·0000001 . cos $\overline{4D}$	+ ·0000001 + ·0000001	= + ·0000002 . cos $\overline{4D}$
+ ·0508402 . cos $\overline{S}$	+ ·0506071 + ·0002770	= + ·0508841 . cos $\overline{S}$
+ ·0012855 . cos $\overline{2S}$	+ ·0025490 + ·0000220	= + ·0025710 . cos $\overline{2S}$
+ ·0000314 . cos $\overline{3S}$	+ ·0000936 + ·0000006	= + ·0000942 . cos $\overline{3S}$
− ·0000743 . cos $\overline{D+S}$	+ ·0000136 − ·0000880	= − ·0000744 . cos $\overline{D+S}$
+ ·0000158 . cos $\overline{D-S}$	+ ·0000135 + ·0000020	= + ·0000155 . cos $\overline{D-S}$
− ·0000019 . cos $\overline{D+2S}$	− ·0000015 − ·0000022	= − ·0000037 . cos $\overline{D+2S}$
+ ·0000004 . cos $\overline{D-2S}$	+ ·0000008 + ·0000001	= + ·0000009 . cos $\overline{D-2S}$
+ ·0000000 . cos $\overline{D+3S}$	− ·0000001 − ·0000001	= − ·0000002 . cos $\overline{D+3S}$
− ·0002114 . cos $\overline{2D+S}$	− ·0003747 + ·0001619	= − ·0002128 . cos $\overline{2D+S}$
− ·0013963 . cos $\overline{2D-S}$	− ·0003752 − ·0010241	= − ·0013993 . cos $\overline{2D-S}$
− ·0000053 . cos $\overline{2D+2S}$	− ·0000147 + ·0000041	= − ·0000106 . cos $\overline{2D+2S}$

Value of $\left(\frac{A}{R}\right)^3 . \left(\frac{r}{a}\right)^2$.	Factors of $\frac{\delta E}{E}$ in the Variation of $\left(\frac{A}{R}\right)^3 . \left(\frac{r}{a}\right)^2$. (The first line in each bracket gives the factor depending on $\left(\frac{A}{R}\right)^3$; the second gives that depending on $\left(\frac{r}{a}\right)^2$).
$-{\cdot}0000772 . \cos \overline{\lvert 2D-2S \rvert}$	$\left\{\begin{matrix} -{\cdot}0000445 \\ -{\cdot}0001099 \end{matrix}\right\} = -{\cdot}0001544 . \cos \overline{\lvert 2D-2S \rvert}$
$-{\cdot}0000001 . \cos \overline{\lvert 2D+3S \rvert}$	$\left\{\begin{matrix} -{\cdot}0000005 \\ +{\cdot}0000001 \end{matrix}\right\} = -{\cdot}0000004 . \cos \overline{\lvert 2D+3S \rvert}$
$-{\cdot}0000020 . \cos \overline{\lvert 2D-3S \rvert}$	$\left\{\begin{matrix} -{\cdot}0000031 \\ -{\cdot}0000027 \end{matrix}\right\} = -{\cdot}0000058 . \cos \overline{\lvert 2D-3S \rvert}$
${\cdot}0000000 . \cos \overline{\lvert 2D-4S \rvert}$	$\left\{\begin{matrix} -{\cdot}0000001 \\ -{\cdot}0000001 \end{matrix}\right\} = -{\cdot}0000002 . \cos \overline{\lvert 2D-4S \rvert}$
$-{\cdot}0000021 . \cos \overline{\lvert 4D+S \rvert}$	$\left\{\begin{matrix} {\cdot}0000000 \\ -{\cdot}0000021 \end{matrix}\right\} = -{\cdot}0000021 . \cos \overline{\lvert 4D+S \rvert}$
$+{\cdot}0000082 . \cos \overline{\lvert 4D-S \rvert}$	$\left\{\begin{matrix} {\cdot}0000000 \\ +{\cdot}0000082 \end{matrix}\right\} = +{\cdot}0000082 . \cos \overline{\lvert 4D-S \rvert}$
$+{\cdot}0000002 . \cos \overline{\lvert 4D-2S \rvert}$	$\left\{\begin{matrix} +{\cdot}0000002 \\ +{\cdot}0000002 \end{matrix}\right\} = +{\cdot}0000004 . \cos \overline{\lvert 4D-2S \rvert}$
$-{\cdot}0000001 . \cos \overline{\lvert 4D+2S \rvert}$	$\left\{\begin{matrix} -{\cdot}0000001 \\ -{\cdot}0000001 \end{matrix}\right\} = -{\cdot}0000002 . \cos \overline{\lvert 4D+2S \rvert}$

Several of the numbers given above can produce no effect in our ultimate result; but it is less difficult and more satisfactory to include all than to make a selection.

(22.) We now proceed with the formation of the quantities which multiply $\left(\frac{A}{R}\right)^3 . \left(\frac{r}{a}\right)^2$; commencing with the argument $\overline{\lvert 2(v-V) \rvert}$.

Referring again to Delaunay and Leverrier, and putting nt and Nt for the mean longitudes of the Moon and Sun,

$$\begin{aligned} v = nt &- {\cdot}0006169 . \sin \overline{\lvert D \rvert} + {\cdot}0114887 . \sin \overline{\lvert 2D \rvert} \\ &- {\cdot}0032422 . \sin \overline{\lvert S \rvert} - {\cdot}0000363 . \sin \overline{\lvert 2S \rvert} \\ &- {\cdot}0000007 . \sin \overline{\lvert 3S \rvert} - {\cdot}0001192 . \sin \overline{\lvert 2D+S \rvert} \\ &+ {\cdot}0008016 . \sin \overline{\lvert 2D-S \rvert} : \end{aligned}$$

$$V = Nt + 0335409 . \sin \overline{\lvert S \rvert} + {\cdot}0003515 . \sin \overline{\lvert 2S \rvert} + {\cdot}0000050 . \sin \overline{\lvert 3S \rvert} .$$

Therefore,

$$\begin{aligned} v - V = \overline{\lvert nt - Nt \rvert} &- {\cdot}0006169 . \sin \overline{\lvert D \rvert} + {\cdot}0114887 . \sin \overline{\lvert 2D \rvert} \\ &- {\cdot}0367831 . \sin \overline{\lvert S \rvert} - {\cdot}0003878 . \sin \overline{\lvert 2S \rvert} - {\cdot}0000057 . \sin \overline{\lvert 3S \rvert} \\ &- {\cdot}0001192 . \sin \overline{\lvert 2D+S \rvert} + {\cdot}0008016 . \sin \overline{\lvert 2D-S \rvert} . \end{aligned}$$

But $\overline{nt - St} = D$. Making this substitution, and doubling the whole;

$$2\overline{(v-V)} = \left\{\begin{array}{l} \overline{2D} \\ \quad + (- \cdot 0012338 \, . \sin \overline{D} + \cdot 0229774 \, . \sin \overline{2D} \\ \quad - \cdot 0735662 \, \sin \overline{S} - \cdot 0007756 \, . \sin \overline{2S} - \cdot 0000114 \, . \sin \overline{3S} \\ \quad - \cdot 0002384 \, . \sin \overline{2D + S} + \cdot 0016032 \, . \sin \overline{2D - S} \,) \end{array}\right\}$$

We shall call this

$$\overline{2D} + \Delta(2D).$$

Then

$$\sin \overline{2(v-V)} = \sin \overline{2D} \, . \cos \overline{\Delta(2D)} + \cos \overline{2D} \, . \sin \overline{\Delta(2D)} \, ;$$
$$\cos \overline{2(v-V)} = \cos \overline{2D} \, . \cos \overline{\Delta(2D)} - \sin \overline{2D} \, . \sin \overline{\Delta(2D)} \, ,$$

and we must now ascertain the values of cos $\overline{\Delta(2D)}$ and sin $\overline{\Delta(2D)}$.

(23.) The series giving these values are

$$\cos \overline{\Delta(2D)} = 1 - \frac{1}{2}\left\{\Delta(2D)\right\}^2 + \frac{1}{24}\left\{\Delta(2D)\right\}^4 - \&c.$$
$$\sin \overline{\Delta(2D)} = \Delta(2D) - \frac{1}{6}\left\{\Delta(2D)\right\}^3 + \frac{1}{120}\left\{\Delta(2D)\right\}^5 - \&c.$$

Now, it appears on trial of the principal terms, with due regard of the repeated bisections in combinations of arguments, that $\frac{1}{24}\{\Delta(2D)\}^4$ and higher powers are not sensible for the investigation before us. Therefore we have

$$\sin \overline{2(v-V)} = \sin \overline{2D} \, . \left\{1 - \frac{1}{2}\{\Delta(2D)\}^2\right\}$$
$$+ \cos \overline{2D} \, . \left\{\Delta(2D) - \frac{1}{6}\{\Delta(2D)\}^3\right\} ;$$

$$\cos \overline{2(v-V)} = \cos \overline{2D} \, . \left\{1 - \frac{1}{2}\{\Delta(2D)\}^2\right\}$$
$$- \sin \overline{2D} \, . \left\{\Delta(2D) - \frac{1}{6}\{\Delta(2D)\}^3\right\} ;$$

each of which is ultimately to be multiplied by

$$\left(\frac{A}{R}\right)^3 . \left(\frac{r}{a}\right)^2 .$$

(24.) For $\{\Delta(2D)\}^2$, I have thought it desirable to form every product as low as ·0000001 (rejecting a few of the smallest); but for $\{\Delta(2D)\}^3$, I have been satisfied with multiplying the four largest terms of $\{\Delta(2D)\}^2$ by the two largest terms of $\Delta(2D)$. Thus we obtain:—

$\Delta(2D)$	$\{\Delta(2D)\}^2$	$\{(\Delta 2D)\}^3$
	Non-periodic $= (+ \cdot 0029723 \quad + \cdot 0054158 \times \frac{\delta E}{E})$	
$+ \sin \overline{D} \times (- \cdot 0012338 \quad)$	$+ \cos \overline{D} \times (- \cdot 0000284 \quad)$	
$+ \sin \overline{2D} \times (+ \cdot 0229774 \quad)$	$+ \cos \overline{2D} \times (+ \cdot 001349 \quad + \cdot 0002712 \times \frac{\delta E}{E})$	$+ \sin \overline{2D} \times (+ \cdot 0001926 \quad + \cdot 0003716 \times \frac{\delta E}{E})$
	$+ \cos \overline{3D} \times (+ \cdot 0000284$	
	$+ \cos \overline{4D} \times (- \cdot 0002636 \quad + \cdot 0000008 \times \frac{\delta E}{E})$	
$+ \sin \overline{S} \times (- \cdot 0735662 \quad - \cdot 0735662 \times \frac{\delta E}{E})$	$+ \cos \overline{S} \times (+ \cdot 0000883 \quad + \cdot 0002023 \times \frac{\delta E}{E})$	$\times \sin \overline{S} \div (- \cdot 0003753 \quad - \cdot 0009502 + \frac{\delta E}{E})$
$+ \sin \overline{2S} + (- \cdot 0007756 \quad - \cdot 0015512 \times \frac{\delta E}{E})$	$+ \cos \overline{2S} \times (- \cdot 0027056 \quad - \cdot 0054096 \times \frac{\delta E}{E})$	
$+ \sin \overline{3S} \times (- \cdot 0000114 \quad - \cdot 0000342 \times \frac{\delta E}{E})$	$+ \cos \overline{3S} \times (- \cdot 0000570 \quad - \cdot 0001710 \times \frac{\delta E}{E})$	$\times \sin \overline{3S} \times (+ \cdot 0000996 \quad + \cdot 0002988 \times \frac{\delta E}{E})$
	$+ \cos \overline{4S} \times (- \cdot 0000011 \quad - \cdot 0000044 \times \frac{\delta E}{E})$	
	$+ \cos \overline{D+S} \times (- \cdot 0000906 \quad - \cdot 0000906 \times \frac{\delta E}{E})$	

(Continued.)

$\Delta(2D)$	$\{\Delta(2D)\}^2$	$\{\Delta(2D)\}^3$
	$+\cos\overline{D-S}\times(+\cdot0000888\quad+\cdot0000888\times\frac{\delta E}{E})$	
$+\sin\overline{2D+S}\times(-\cdot0002384\quad-\cdot0002384\times\frac{\delta E}{E})$	$+\cos\overline{2D+S}\times(+\cdot0016915\quad+\cdot0016939\times\frac{\delta E}{E})$	
$+\sin\overline{2D-S}\times(+\cdot0016032\quad+\cdot0016032\times\frac{\delta E}{E})$	$+\cos\overline{2D-S}\times(-\cdot0016901\quad-\cdot0016896\times\frac{\delta E}{E})$	
		$+\sin\overline{2D+2S}\times(-\cdot0000933\quad-\cdot0001866\times\frac{\delta E}{E})$
	$+\cos\overline{2D-2S}\times(-\cdot0001358\quad-\cdot0002716\times\frac{\delta E}{E})$	$+\sin\overline{2D-2S}\times(-\cdot0000933\quad-\cdot0001866\times\frac{\delta E}{E})$
	$+\cos\overline{2D-3S}\times(-\cdot0000014\quad-\cdot0000042\times\frac{\delta E}{E})$	
	$+\cos\overline{3D-S}\times(+\cdot0000020\quad+\cdot0000020\times\frac{\delta E}{E})$	
	$+\cos\overline{4D+S}\times(+\cdot0000056\quad+\cdot0000056\times\frac{\delta E}{E})$	$+\sin\overline{4D+S}\times(+\cdot0000195\quad+\cdot0000195\times\frac{\delta E}{E})$
	$+\cos\overline{4D-S}\times(-\cdot0000369\quad-\cdot0000369\times\frac{\delta E}{E})$	$+\sin\overline{4D-S}\times(-\cdot0000195\quad-\cdot0000195\times\frac{\delta E}{E})$
	$+\cos\overline{4D-2S}\times(-\cdot0000013\quad-\cdot0000026\times\frac{\delta E}{E})$	

(25.) From these we form the following expressions for $\cos[\Delta(2D)]$ and $\sin[\Delta(2D)]$, equal respectively to $1-\frac{1}{2}\{\Delta(2D)\}^2$ and $\Delta(2D)-\frac{1}{6}\{\Delta(2D)\}^3$.

$\cos[\Delta(2D)]$			$\sin[\Delta(2D)]$		
Non-periodic	[illegible]	[illegible]			
$+\cos(D)$	[illegible]		$+\sin(D)$	[illegible]	
$+\cos(2D)$	[illegible]	[illegible]	$+\sin(2D)$	[illegible]	[illegible]
$+\cos(3D)$	[illegible]				
$+\cos(4D)$	[illegible]	[illegible]			
$+\cos(S)$	[illegible]	[illegible]	$+\sin(S)$	[illegible]	[illegible]
$+\cos(2S)$	[illegible]	[illegible]	$+\sin(2S)$	[illegible]	[illegible]
$+\cos(3S)$	[illegible]	[illegible]	$+\sin(3S)$	[illegible]	[illegible]
$+\cos(4S)$	[illegible]	[illegible]			

(Continued.)

$\cos\overline{\Delta(2D)}$

$$+\cos\overline{D+S}\times\left(+\cdot0000453+\cdot0000453\times\frac{\delta E}{E}\right)$$

$$+\cos\overline{D-S}\times\left(-\cdot0000444-\cdot0000444\times\frac{\delta E}{E}\right)$$

$$+\cos\overline{2D+S}\times\left(-\cdot0008458-\cdot0008470\times\frac{\delta E}{E}\right)$$

$$+\cos\overline{2D-S}\times\left(+\cdot0008450+\cdot0008448\times\frac{\delta E}{E}\right)$$

$$+\cos\overline{2D-2S}\times\left(+\cdot0000679+\cdot0001358\times\frac{\delta E}{E}\right)$$

$$+\cos\overline{2D-3S}\times\left(+\cdot0000007+\cdot0000021\times\frac{\delta E}{E}\right)$$

$$+\cos\overline{3D-S}\times\left(-\cdot0000010-\cdot0000010\times\frac{\delta E}{E}\right)$$

$$+\cos\overline{4D+S}\times\left(-\cdot0000028-\cdot0000028\times\frac{\delta E}{E}\right)$$

$$+\cos\overline{4D-S}\times\left(+\cdot0000184+\cdot0000184\times\frac{\delta E}{E}\right)$$

$$+\cos\overline{4D-2S}\times\left(+\cdot0000006+\cdot0000013\times\frac{\delta E}{E}\right)$$

$\sin\overline{\Delta(2D)}$

$$+\sin\overline{2D+S}\times\left(-\cdot0002384-\cdot0002384\times\frac{\delta E}{E}\right)$$

$$+\sin\overline{2D-S}\times\left(+\cdot0016032+\cdot0016032\times\frac{\delta E}{E}\right)$$

$$+\sin\overline{2D+2S}\times\left(+\cdot0000156+\cdot0000311\times\frac{\delta E}{E}\right)$$

$$+\sin\overline{2D-2S}\times\left(+\cdot0000156+\cdot0000311\times\frac{\delta E}{E}\right)$$

$$+\sin\overline{4D+S}\times\left(-\cdot0000033-\cdot0000033\times\frac{\delta E}{E}\right)$$

$$+\sin\overline{4D-S}\times\left(+\cdot0000033+\cdot0000033\times\frac{\delta E}{E}\right)$$

(26.) We are now to combine the expressions last found, in order to produce the quantities

$$\mathrm{T} \cdot \frac{1}{a} \cdot \frac{r}{a} \text{ and } \mathrm{P} \cdot \frac{1}{a} \cdot \frac{r}{a},$$

or

$$-\cdot 0083928 \times \left(\frac{\mathrm{A}}{\mathrm{R}}\right)^3 \cdot \left(\frac{r}{a}\right)^2 \times \sin \overline{|2(v-\mathrm{V})|},$$

$$+\cdot 0083928 \times \left(\frac{\mathrm{A}}{\mathrm{R}}\right)^3 \cdot \left(\frac{r}{a}\right)^2 \times \left\{+\frac{1}{3} + \cos \overline{|2(v-\mathrm{V})|}\right\}$$

(of which we are to retain only the terms that multiply $\frac{\delta \mathrm{E}}{\mathrm{E}}$, or, as we may conveniently express it, $\dot{\mathrm{E}}'.t$, where E' is a constant), and to examine the results of substituting the combinations in the Factorial Equations. Our ultimate object is, not to retain every term in the solution of the equations, but only the non-periodic terms (increasing with the time) of the longitude and the radius vector, especially the longitude (which alone will ultimately possess any value for us). And this consideration is to be kept in view in solving the factorial equations. We will now consider separately the two disturbing functions.

(27.) On examining the formation of the selected terms of

$$\left(\frac{\mathrm{A}}{\mathrm{R}}\right)^3 \cdot \left(\frac{r}{a}\right)^2 \times \sin \overline{|2(v-\mathrm{V})|},$$

or

$$\left(\left(\frac{\mathrm{A}}{\mathrm{R}}\right)^3 \cdot \left(\frac{r}{a}\right)^2 \times \cos \overline{|\Delta(2\mathrm{D})|}\right) \times \sin \overline{|2\mathrm{D}|}$$

$$+\left(\left(\frac{\mathrm{A}}{\mathrm{R}}\right)^3 \cdot \left(\frac{r}{a}\right)^2 \times \sin \overline{|\Delta(2\mathrm{D})|}\right) \times \cos \overline{|2\mathrm{D}|},$$

with reference to the expansions of these functions lately exhibited, it appears that the first part is the product of a constant (accompanied with a series of cosines) by a sine, and the second is the product of a series of sines by a cosine. Each of these products will be a series of sines. In order to ascertain whether these will produce any constant term in the ultimate result, it is necessary to refer to the factorial equations. Putting

$$\mathrm{F}t \,.\, \sin \overline{|\mathrm{G}t|}$$

for one of the terms of the expansion of T, considered alone (which is legitimate, because the equations are linear), we have the factorial equations,

$$
Ft\,.\,\sin\overline{Gt} = \left\{
\begin{array}{ll}
+\delta\left(\frac{a}{r}\right) & \times\{\quad \cdot0000 - \cdot0281\,.\,\sin\overline{2D}\,\} \\
+\frac{d}{dt}\left(\delta\frac{a}{r}\right) & \times\{-1\cdot9919 + \cdot0061\,.\,\cos\overline{2D}\,\} \\
+\delta v & \times\{\quad \cdot0000 + \cdot0168\,.\,\cos\overline{2D}\,\} \\
+\frac{d}{dt}(\delta v) & \times\{\quad \cdot0000 + \cdot0268\,.\,\sin\overline{2D}\,\} \\
+\frac{d^2}{dt^2}(\delta v) & \times\{+1\cdot0006 - \cdot0144\,.\,\cos\overline{2D}\,\}
\end{array}
\right\}.
$$

$$
0 = \left\{
\begin{array}{ll}
+\delta\left(\frac{a}{r}\right) & \times\{+2\cdot9964 - \cdot0236\,.\,\cos\overline{2D}\,\} \\
+\frac{d}{dt}\left(\delta\frac{a}{r}\right) & \times\{\quad \cdot0000 - \cdot0528\,.\,\sin\overline{2D}\,\} \\
+\frac{d^2}{dt^2}\left(\delta\frac{a}{r}\right) & \times\{-1\cdot0053 + \cdot0212\,.\,\cos\overline{2D}\,\} \\
+\delta v & \times\{\quad \cdot0000 + \cdot0168\,.\,\sin\overline{2D}\,\} \\
+\frac{d}{dt}(\delta v) & \times\{-1\cdot9888 - \cdot0090\,.\,\cos\overline{2D}\,\}
\end{array}
\right\}.
$$

Using the symbols

$$
\Phi = -\cdot0281\,.\,\delta\left(\frac{a}{r}\right).\sin\overline{2D} + \cdot0061\,.\,\frac{d}{dt}\left(\delta\frac{a}{r}\right).\cos\overline{2D} + \cdot0168\,.\,\delta v\,.\cos\overline{2D}
$$
$$
+\cdot0268\,.\,\frac{d}{dt}(\delta v)\,.\sin\overline{2D} - \cdot0144\,.\,\frac{d^2}{dt^2}(\delta v)\,.\cos\overline{2D};
$$

$$
\Psi = -\cdot0236\,.\,\delta\left(\frac{a}{r}\right).\cos\overline{2D} - \cdot0528\,.\,\frac{d}{dt}\left(\delta\frac{a}{r}\right).\sin\overline{2D}
$$
$$
+\cdot0212\,.\,\frac{d^2}{dt^2}\left(\delta\frac{a}{r}\right).\cos\overline{2D} + \cdot0168\,.\,\delta v\,.\sin\overline{2D} - \cdot0090\,.\,\frac{d}{dt}(\delta v).\cos\overline{2D}
$$

the equations may be conveniently written,

$$
-1\cdot9919\,.\,\frac{d}{dt}\left(\delta\frac{a}{r}\right) + 1\cdot0006\,.\,\frac{d^2}{dt^2}(\delta v) \qquad = Ft\,.\,\sin\overline{Gt} - \Phi;
$$
$$
+2\cdot9964\,.\,\delta\left(\frac{a}{r}\right) - 1\cdot0053\,.\,\frac{d^2}{dt^2}\left(\delta\frac{a}{r}\right) - 1\cdot9888\,.\,\frac{d}{dt}(\delta v) = \qquad 0 \qquad -\Psi
$$

(in which we shall afterwards use, instead of the fractional factors, the integers -2, $+1$, $+3$, -1, -2); and these equations will be most conveniently solved by successive substitution; first, neglecting the small quantities Φ and Ψ; then solving the equations so limited, substituting these limited solutions in Φ and Ψ; and then re-solving; and so on. It is easily found that limited solutions are

$$
\delta\left(\frac{a}{r}\right) = \frac{2G}{G^2-G^4}Ft\,.\cos\overline{Gt}, \qquad \delta v = \frac{3+G^2}{G^2-G^4}Ft\,.\sin\overline{Gt};
$$

and these, on substitution, destroy the term whose factor is Ft, but introduce other terms. Upon making the substitutions

$$\delta\left(\frac{a}{r}\right) = \frac{2G}{G^2-G^4}Ft.\cos\overline{Gt} + c, \qquad \delta v = \frac{3+G^2}{G^2-G^4}Ft.\sin\overline{Gt} + s,$$

we obtain the following equations,

$$-2.\frac{dc}{dt} + \frac{d^2s}{dt^2} = -\frac{2G+2G^3}{G^2-G^4}F.\cos\overline{Gt} - \Phi;$$

$$+3.c - \frac{d^2c}{dt^2} - 2.\frac{ds}{dt} = +\frac{6-2G^2}{G^2-G^4}\ F.\sin\overline{Gt} - \Psi;$$

and, on substituting in Φ and Ψ the first terms of the expressions for $\delta\left(\frac{a}{r}\right)$, δv, and their differentials, we find equations exactly similar to those whose solution is given in Articles 11 and 12. The precise values of the coefficients for $\delta\frac{a}{r}$ and δv are of no importance; at present it is only necessary for us to remark that they are constants for each value of G.

We shall now consider the character of the expressions for $\delta\frac{a}{r}$ and δv: first, for the general value of G; secondly for the the special value $Gt = 2D$.

For the general case, we are to substitute in Φ and Ψ the very approximate values of $\delta\left(\frac{a}{r}\right)$ and δv given by the formulæ

$$\delta\left(\frac{a}{r}\right) = Ct.\cos\overline{Gt}, \qquad \delta v = St.\sin\overline{Gt};$$

and we find

$$\Phi = \begin{cases} +\{+\cdot0281.C \ -\cdot0268.GS\} & \times t.\sin\overline{2D}.\cos\overline{Gt} \\ +\{+\cdot0061.GC-\cdot0144.G^2S-\cdot0168.S\} & \times t.\cos\overline{2D}.\sin\overline{Gt} \\ +\{-\cdot0061.C \ +\cdot0288.GS\} & \times \ \cos\overline{2D}.\cos\overline{Gt} \\ +\{-\cdot0268.S\} & \times \ \sin\overline{2D}.\sin\overline{Gt} \end{cases}$$

$$\Psi = \begin{cases} +\{+\cdot0236.C \ +\cdot0212.G^2C+\cdot0090.GS\} & \times t.\cos\overline{2D}.\cos\overline{Gt} \\ +\{-\cdot0528.GC-\cdot0168.S\} & \times t.\sin\overline{2D}.\sin\overline{Gt} \\ +\{+\cdot0528.C\} & \times \ \sin\overline{2D}.\cos\overline{Gt} \\ +\{+\cdot0424.C \ +\cdot0090.S\} & \times \ \cos\overline{2D}.\sin\overline{Gt} \end{cases}$$

Now, all these terms, as well as the two new terms in the equation lately formed, are periodical; and the form of the equations for c and s is the same as that for $\delta\left(\frac{a}{r}\right)$ and δv; and

therefore, to this second step of approximation as well as in the first, the value of δr contains no non-periodical factor of t. And, as far as can be seen, the same remark applies to succeeding approximations.

For the case when $Gt = 2D$ (which in Article 9 and succeeding Articles is written F) we have only to remark that the equations before us are already solved in Articles 9 and 12. For, as the treatment of A, and of B, and of C, in those equations, is absolutely independent, we have merely to extract (for the effect of the term $-A \cdot bt \cdot \sin \overline{2D}$ in Article 9) the multiples of Ab in Article 12. And, referring to the terms in Article 11 of which y and l are the factors, it will be seen that those terms are periodical.

Therefore the term

$$-\cdot 0083928 \times \left(\frac{A}{R}\right)^2 \cdot \left(\frac{r}{a}\right)^2 \times \sin \overline{2(v-V)}$$

produces no non-periodical term in δr.

(28.) On examining the formation of the terms of

$$\left(\frac{A}{R}\right)^2 \left(\frac{r}{a}\right)^2 \times \cos \overline{2(v-V)},$$

it will be seen that they consist of two parts. One is the series of constants produced by the products of similar cosines or similar sines, to be attached to a term of the same form in

$$+\cdot 0083928 \times \left(\frac{A}{R}\right)^2 \cdot \left(\frac{r}{a}\right)^2 \times \frac{1}{3},$$

of which we shall treat shortly. The other part is a series of products of cosines or sines of different arguments, producing cosines of new arguments.

For the general term $\cos \overline{Gt}$, in the part just mentioned, the same reasoning will apply as that which has been employed for $\sin \overline{Gt}$; the expressions for Φ and Ψ being the same as those which are used in discussing the result of $\sin \overline{Gt}$; and the same conclusion following—namely, that there is no non-periodic term.

In the special case when $Gt = 2D$, the solution has already been effected in Articles 9 and 12, in treating the factor C. And the result is, that it produces no non-periodical term.

Therefore the term

$$+\cdot 0083928 \times \cos \overline{2(v-V)}$$

produces no non-periodic term, except from those non-periodic parts which we shall associate with the non-periodic parts of

$$+\cdot 0083928 \times \left(\frac{A}{R}\right)^2 \cdot \left(\frac{r}{a}\right)^2 \times \frac{1}{3}.$$

(29.) For the non-periodic parts, whatever be their origin, we have again to remark that the solution is given in Articles 9 and 12; where the term $Bb.t$ of Article 9 produces, in Article 12, for $\delta\left(\frac{a}{r}\right)$ the term $-Bbt$, and for δv the term $-Bbt^2$.

(30.) The general result of these investigations is, that we are merely to extract, from the details of the two products

$$+\cdot0083928\times\left(\frac{A}{R}\right)^2.\left(\frac{r}{a}\right)^2\times\frac{1}{3},$$

and

$$+\cdot0083928\times\left(\frac{A}{R}\right)^2.\left(\frac{r}{a}\right)^2\times\cos\overline{2(v-V)},$$

the factors of $\frac{\delta E}{E}$ or $E't$; and to use them in forming the Factorial Equations. To this extraction from details we shall now give our attention.

(31.) Referring to the expansion in Article 22, it will appear that we must consider these three terms:

$$+\frac{1}{3}\left(\frac{A}{R}\right)^2.\left(\frac{r}{a}\right)^2,$$

$$+\left(\left(\frac{A}{R}\right)^2.\left(\frac{r}{a}\right)^2\times\cos\overline{\Delta(2D)}\right)\times\cos\overline{2D},$$

$$-\left(\left(\frac{A}{R}\right)^2.\left(\frac{r}{a}\right)^2\times\sin\overline{\Delta(2D)}\right)\times\sin\overline{2D}.$$

The first part is merely $\frac{1}{3}$ of the development in Article 21, and consists of a constant and a series of cosines.

The second part consists of the product of a constant and a series of cosines by

$$\cos\overline{2D},$$

The third part consists of the product of a series of sines by

$$\sin\overline{2D},$$

Each of these parts will give rise to non-periodic terms.

(32.) For development of the first part, we have merely to take one-third of the non-periodic term in Article 21. Here, and in the development of the following parts, we shall retain only the multiples of $\frac{\delta E}{E}$. Thus we obtain for the first part:

$$+\frac{\delta E}{E}\times+\cdot0002870.$$

For the second and third parts we shall proceed to form the products to which allusion is made in the last sentences of Article 26. The only arguments in

$$\left(\frac{A}{R}\right)^2.\left(\frac{r}{a}\right)^2$$

which can combine with those of

$$\cos \overline{|\Delta(2D)|} \text{ and } \sin \overline{|\Delta(2D)|}$$

to form

$$\cos \overline{|2D|} \text{ or } \sin \overline{|2D|}$$

are the following:

Term of $\left(\frac{A}{R}\right)^3 \cdot \left(\frac{r}{a}\right)^2$.		Term of cos $\overline{\vert\Delta(2D)\vert}$ or sin $\overline{\vert\Delta(2D)\vert}$.
* Non-periodic	will combine with	$\overline{\vert 2D\vert}$
* $\overline{\vert D\vert}$	„	$\overline{\vert D\vert}$
$\overline{\vert D\vert}$	„	$\overline{\vert 3D\vert}$
* $\overline{\vert 2D\vert}$	„	non-periodic
* $\overline{\vert 2D\vert}$	„	$\overline{\vert 4D\vert}$
$\overline{\vert 4D\vert}$	„	$\overline{\vert 2D\vert}$
* $\overline{\vert S\vert}$	„	$\overline{\vert 2D+S\vert}$
* $\overline{\vert S\vert}$	„	$\overline{\vert 2D-S\vert}$
$\overline{\vert 2S\vert}$	„	$\overline{\vert 2D+2S\vert}$
* $\overline{\vert 2S\vert}$	„	$\overline{\vert 2D-2S\vert}$
$\overline{\vert 3S\vert}$	„	$\overline{\vert 2D-3S\vert}$
$\overline{\vert D+S\vert}$	„	$\overline{\vert D-S\vert}$
$\overline{\vert D-S\vert}$	„ . .	$\overline{\vert D+S\vert}$
$\overline{\vert D-S\vert}$	„	$\overline{\vert 3D-S\vert}$
* $\overline{\vert 2D-S\vert}$	„	$\overline{\vert S\vert}$
$\overline{\vert 2D-S\vert}$	„	$\overline{\vert 4D-S\vert}$
* $\overline{\vert 2D-2S\vert}$	„	$\overline{\vert 2S\vert}$
$\overline{\vert 2D-2S\vert}$	„	$\overline{\vert 4D-2S\vert}$

Only those combinations which are distinguished by an asterisk can produce numbers included in the first seven decimals.

(33.) We now exhibit the products. It will be remembered that (except where one factor is non-periodic) each multiplication for cos $\overline{|\Delta(2D)|}$ produces two cosines, of which we shall retain only one, namely, cos $\overline{|2D|}$, and that its coefficient is to be only one-half of the product of the coefficients which form it. In like manner each multiplication for sin $\overline{|\Delta(2D)|}$ produces two sines, of which we retain only sin $\overline{|2D|}$.

Also, that we retain only the factors of $\frac{\delta E}{E}$.

The following are the special multiplications:—

Products giving the factors of $\cos\overline{2D} \cdot \frac{\delta E}{E}$, to be subsequently multiplied by $+\cos\overline{2D}$.

	Term of $\left(\frac{A}{B}\right)^2 \cdot \left(\frac{r}{a}\right)^2$		Term of $\cos\overline{\Delta(2D)}$	Product multiplying $\cos\overline{2D} \cdot \frac{\delta E}{E}$
Non-periodic	$(+1{\cdot}0051177 + {\cdot}0008609 \times \frac{\delta E}{E}) \times$	$+\cos\overline{2D}$	$\times(-{\cdot}0000674 - {\cdot}0001356 \times \frac{\delta E}{E})$	$\left\{\begin{matrix} -{\cdot}0001363 \\ 0000000 \end{matrix}\right\}$
$+\cos\overline{2D}$	$\times(-{\cdot}0148598 - {\cdot}0000559 \times \frac{\delta E}{E}) \times$	Non-periodic	$(+{\cdot}9985138 - {\cdot}0027079 \times \frac{\delta E}{E})$	$\left\{\begin{matrix} +{\cdot}0000402 \\ -{\cdot}0000557 \end{matrix}\right\}$
$+\cos\overline{S}$	$\times(+{\cdot}0508402 + {\cdot}0508841 \times \frac{\delta E}{E}) \times$	$+\cos\overline{2D+S}$	$\times(-{\cdot}0008458 - {\cdot}0008470 \times \frac{\delta E}{E})$	$\left\{\begin{matrix} -{\cdot}0000431 \\ -{\cdot}0000430 \end{matrix}\right\}$
$+\cos\overline{S}$	$\times(+{\cdot}0508402 + {\cdot}0508841 \times \frac{\delta E}{E}) \times$	$+\cos\overline{2D-S}$	$\times(+{\cdot}0008450 + {\cdot}0008448 \times \frac{\delta E}{E})$	$\left\{\begin{matrix} +{\cdot}0000429 \\ +{\cdot}0000430 \end{matrix}\right\}$
$+\cos\overline{2S}$	$\times(+{\cdot}0012855 + {\cdot}0025710 \times \frac{\delta E}{E}) \times$	$+\cos\overline{2D-2S}$	$\times(+{\cdot}0000679 + {\cdot}0001358 \times \frac{\delta E}{E})$	$\left\{\begin{matrix} +{\cdot}0000001 \\ +{\cdot}0000002 \end{matrix}\right\}$
$+\cos\overline{2D-S}$	$\times(-{\cdot}0013963 - {\cdot}0013993 \times \frac{\delta E}{E}) \times$	$+\cos\overline{S}$	$\times(-{\cdot}0000442 - {\cdot}0001012 \times \frac{\delta E}{E})$	$\left\{\begin{matrix} +{\cdot}0000001 \\ +{\cdot}0000001 \end{matrix}\right\}$
$+\cos\overline{2D-2S}$	$\times(-{\cdot}0000772 - {\cdot}0001544 \times \frac{\delta E}{E}) \times$	$+\cos\overline{2S}$	$\times(+{\cdot}0013528 + {\cdot}0027048 \times \frac{\delta E}{E})$	$\left\{\begin{matrix} -{\cdot}0000002 \\ -{\cdot}0000002 \end{matrix}\right\}$

All products, except those for the two first lines, are to be divided by 2.

Products giving the factors of sin $\boxed{2D}\cdot\frac{\delta E}{E}$, to be subsequently multiplied by $-$ sin $\boxed{2D}$.

Term of $\left(\frac{A}{R}\right)^3\cdot\left(\frac{r}{a}\right)^2$			Term of sin $\boxed{\Delta(2D)}$			Product multiplying sin $\boxed{2D}\cdot\frac{\delta E}{E}$
Non-periodic	$(+1{\cdot}0051177$	$+{\cdot}0008609\times\frac{\delta E}{E})\times$	$+\sin\boxed{2D}$	$\times(+{\cdot}0229453$	$-{\cdot}0000619\times\frac{\delta E}{E})$	$\left\{\begin{matrix}-{\cdot}0000622\\+{\cdot}0000197\end{matrix}\right\}$
$+\cos\boxed{S}$	$\times(+\ {\cdot}0508402$	$+{\cdot}0508841\times\frac{\delta E}{E})\times$	$+\sin\boxed{2D+S}$	$\times(-{\cdot}0002384$	$-{\cdot}0002384\times\frac{\delta E}{E})$	$\left\{\begin{matrix}-{\cdot}0000121\\-{\cdot}0000121\end{matrix}\right\}$
$+\cos\boxed{S}$	$\times(+\ {\cdot}0508402$	$+{\cdot}0508841\times\frac{\delta E}{E})\times$	$+\sin\boxed{2D-S}$	$\times(+{\cdot}0016032$	$+{\cdot}0016032\times\frac{\delta E}{E})$	$\left\{\begin{matrix}+{\cdot}0000815\\+{\cdot}0000816\end{matrix}\right\}$
$+\cos\boxed{2D-S}$	$\times(-\ {\cdot}0013963$	$-{\cdot}0013993\times\frac{\delta E}{E})\times$	$+\sin\boxed{S}$	$\times(-{\cdot}0735066$	$-{\cdot}0734078\times\frac{\delta E}{E})$	$\left\{\begin{matrix}+{\cdot}0001025\\+{\cdot}0001029\end{matrix}\right\}$
$+\cos\boxed{2D-2S}$	$\times(-\ {\cdot}0000772$	$-{\cdot}0001544\times\frac{\delta E}{E})\times$	$+\sin\boxed{2S}$	$\times(-{\cdot}0007756$	$-{\cdot}0015512\times\frac{\delta E}{E})$	$\left\{\begin{matrix}+{\cdot}0000001\\+{\cdot}0000001\end{matrix}\right\}$

All products, except that for the first line, are to be divided by 2.

(34.) The non-periodic terms in the three different parts of the effective perturbation-term

$$\text{P, or } +{\cdot}0083928 \times \left(\frac{A}{R}\right)^2 \cdot \left(\frac{r}{a}\right)^2 \times \left\{\frac{1}{3} + \cos \left|\overline{2(v-V)}\right|\right\}$$

may now be collected; and the numerical term to be inserted in the Factorial Equations may be formed.

The first part, or

$$+\frac{1}{3} \times {\cdot}0083928 \times \left(\frac{A}{R}\right)^2 \cdot \left(\frac{r}{a}\right)^2,$$

gives

$$+{\cdot}0083928 \times +{\cdot}0002870 \times \frac{\delta E}{E}.$$

For the second part, the summation of terms in the last table but one, performed in accordance with the rule at the bottom of the table, gives

$$-{\cdot}0001518 \times \cos \left|\overline{2D}\right| \times \frac{\delta E}{E}.$$

This is to be multiplied by

$$+\cos \left|\overline{2D}\right|,$$

which gives for its non-periodic part

$$-{\cdot}0000759 \times \frac{\delta E}{E}.$$

And the second part becomes

$$+{\cdot}0083928 \times -{\cdot}0000759 \times \frac{\delta E}{E}.$$

For the third part, the summation of terms in the last table, performed in like manner, gives

$$+{\cdot}0001298 \times \sin \left|\overline{2D}\right| \times \frac{\delta E}{E}.$$

This is to be multiplied by

$$-\sin \left|\overline{2D}\right|,$$

producing the non-periodic term

$$-{\cdot}0000649 \times \frac{\delta E}{E}.$$

And the third part becomes

$$+\cdot 0083928 \times -\cdot 0000649 \times \frac{\delta E}{E}.$$

The total value of P is thus

$$+\cdot 0083928 \times +\cdot 0001462 \times \frac{\delta E}{E}.$$

Now, by Le Verrier's Elements,

$$\frac{\text{variation of E for one year}}{E} = -\frac{\cdot 000\,000\,4338}{\cdot 016\,769\,27}.$$

And, as one year = 84 units of time (Article 3),

$$\frac{\delta E}{E} = -\frac{1}{84} \times \frac{\cdot 000\,000\,4338}{\cdot 016\,769\,27} \times t.$$

And therefore the total value of P is

$$-\cdot 0083928 \times \cdot 0001462 \times \frac{1}{84} \times \frac{\cdot 000\,000\,4338}{\cdot 016\,769\,27} \times t.$$

For the moment, we shall call this quantity $+Kt$.

(35.) An inspection of the operations in the former part of this paper (Articles 9 to 12) will show that the non-periodical part of the solution of the Factorial Equations is in no way affected by the periodical terms of P and T; and also that the periodical parts there retained in the expressions of the factors are unnecessary. And the use of integral numbers, instead of the fractional numbers in the coefficients entering into those factors, produces only unimportant error in the final result. With these simplifications, our equations become

$$\text{T or} \quad 0 = -2 \,.\, \frac{d}{dt}\left(\delta\frac{a}{r}\right) + \frac{d^2}{dt^2}(\delta v).$$

$$\text{P or} \quad +Kt = +3 \,.\, \delta\left(\frac{a}{r}\right) - \frac{d^2}{dt^2}\left(\delta\frac{a}{r}\right) - 2 \,.\, \frac{d}{dt}(\delta v).$$

It will easily be seen that the assumptions

$$\delta\frac{a}{r} = +Ct, \quad \delta v = +Ht^2,$$

are proper for the solution. Substituting

$$0 = -2C + 2H; \quad Kt = +3 \,.\, Ct - 4 \,.\, Ht;$$

from which

$$H = -K, \quad C = -K;$$

and the value of Ht^2, which is the 'Acceleration' that we are seeking, is $-Kt^2$. To compute the value corresponding to one year, we must make $t = 84$. The value of 'Acceleration' thus found, like all other angular measures here treated, is expressed in terms of radius; to reduce it to seconds, we must divide it by the length of $1''$ in terms of radius $= \cdot 000004848137$. Thus finally we obtain for Acceleration in longitude for one year, expressed in seconds of arc,

$$+\cdot 0083928 \times \cdot 0001462 \times 84 \times \frac{\cdot 0000004338}{\cdot 01676927} \times \frac{1}{\cdot 000004848137};$$

$$= +0''\cdot 00054997:$$

and for Acceleration in longitude for 100 years,

$$+5''\cdot 4997.$$

(36.) The factor $\cos^2 \boxed{\Pi}$, which enters into the original expressions for perturbing forces, has not hitherto been taken into account. Its mean value is $\cdot 99597$; the Acceleration for one year is $+0''\cdot 00054773$; and finally,

The Acceleration in longitude for 100 years $= 5''\cdot 4773$.

I commenced this investigation without intention of mentioning in it any name. But I cannot terminate it without offering to Professor Adams my very hearty congratulations on his success in making a correction so large to a theory so important.

I think that the Lunar Theory is now placed in a difficult position. With the elements formerly received, the ancient eclipses were very well explained. With the modified theory, the agreement cannot be so good, and perhaps is impossible.

I am unwilling to abandon the interpretation of the ancient eclipses, and I think that reconciliation must be sought, in some new secular term, or in some alteration of the mean motions either of the Sun or of the Moon.

I apologise to the Society for two errors in the first part of the paper. Article 16 is totally erroneous, and Article 17 contains a numerical error. Both arose from the same circumstance, the haste in which the paper was closed for the Meeting of the Society.

1880, *July* 19.

On the Method of Least Squares. By J. W. L. Glaisher, M.A., F.R.S.

The present paper contains certain notes on the method of least squares; as they are to some extent connected with my previous paper "On the Solution of the Equations in the Method of Least Squares" (*Monthly Notices*, vol. xxxiv. pp. 311–334), the sections are numbered as if this were a continuation of that paper.

§ 14. The results of the treatment of a system of linear equations by the method of least squares may be stated as follows:—

Let

$$\begin{aligned} a_1x + b_1y + c_1z \dots + f_1t &= n_1, \\ a_2x + b_2y + c_2z \dots + f_2t &= n_2, \\ \dots\dots\dots\dots\dots\dots \\ a_mx + b_my + c_mz \dots + f_mt &= n_m, \end{aligned}$$

be the m equations of condition, $x, y, z, \dots t$ being the μ unknown quantities, and $n_1, n_2, \dots n_m$ the m quantities obtained by observation.* The latter are assumed to be equally good—*i.e.* they are assumed to be such that the mean error of each of them is equal to the mean error ϵ of a standard observation.

The normal equations giving $x_0, y_0, \dots t_0$, the "most probable" values of $x, y, \dots t$ are

$$\begin{aligned} (aa)x_0 + (ab)y_0 + (ac)z_0 \dots + (af)t_0 &= (an), \\ (ba)x_0 + (bb)y_0 + (bc)z_0 \dots + (bf)t_0 &= (bn), \\ \dots\dots\dots\dots\dots\dots \\ (fa)x_0 + (fb)y_0 + (fc)z_0 \dots + (ff)t_0 &= (fn), \end{aligned}$$

where, for example,

$$\begin{aligned} (aa) &\text{ denotes } a_1^2 + a_2^2 \dots + a_m^2, \\ (ab) &\quad ,, \quad a_1b_1 + a_2b_2 \dots + a_mb_m, \end{aligned}$$

so that $(ab)=(ba)$, &c.

* The notation, as in the previous paper, is derived from the case of six unknowns, x, y, z, w, u, t, the coefficients being denoted by a, b, c, d, e, f, with suffixes attached. All the formulæ in the paper, however, have reference to the general case of μ unknowns, $x, y, z, \dots t$, with corresponding coefficients $a, b, c, \dots f$.

The signs of the quantities $n_1, n_2, \dots n_m$ have been changed, so that the equations of condition are written

$$a_1x + b_1y + c_1z \dots + f_1t = n_1, \quad \&c.$$

instead of

$$a_1x + b_1y + c_1z \dots + f_1t + n_1 = 0, \quad \&c.,$$

as it is somewhat more convenient to have the quantities determined by observation on the right hand side of the equations. This change is also made in Oppolzer's *Lehrbuch zur Bahnbestimmung der Kometen und Planeten*, vol. ii. p. 313 (1880). In my previous paper Gauss's and Encke's form (*i.e.* the second of the above forms) was retained.

The values of x_0, y_0, ... are therefore

$$x_0 = \frac{\begin{vmatrix} (an), & (ab), & (ac), & \dots & (af) \\ (bn), & (bb), & (bc), & \dots & (bf) \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ (fn), & (fb), & (fc), & \dots & (ff) \end{vmatrix}}{\begin{vmatrix} (aa), & (ab), & (ac), & \dots & (af) \\ (ba), & (bb), & (bc), & \dots & (bf) \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ (fa), & (fb), & (fc), & \dots & (ff) \end{vmatrix}},$$

$$y_0 = \frac{\begin{vmatrix} (aa), & (an), & (ac), & \dots & (af) \\ (ba), & (bn), & (bc), & \dots & (bf) \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ (fa), & (fn), & (fc), & \dots & (ff) \end{vmatrix}}{\begin{vmatrix} (aa), & (ab), & (ac), & \dots & (af) \\ (ba), & (bb), & (bc), & \dots & (bf) \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ (fa), & (fb), & (fc), & \dots & (ff) \end{vmatrix}},$$

$$z_0 = \&c.$$

The weights p_x, p_y, ... of x_0, y_0, ... are

$$p_x = \frac{\begin{vmatrix} (aa), & (ab), & \dots & (af) \\ (ba), & (bb), & \dots & (bf) \\ \cdot & \cdot & \cdot & \cdot \\ (fa), & (fb), & \dots & (ff) \end{vmatrix}}{\begin{vmatrix} (bb), & (bc), & \dots & (bf) \\ (cb), & (cc), & \dots & (cf) \\ \cdot & \cdot & \cdot & \cdot \\ (fb), & (fc), & \dots & (ff) \end{vmatrix}},$$

$$p_y = \frac{\begin{vmatrix} (aa), & (ab), & \dots & (af) \\ (ba), & (bb), & \dots & (bf) \\ \cdot & \cdot & \cdot & \cdot \\ (fa), & (fb), & \dots & (ff) \end{vmatrix}}{\begin{vmatrix} (aa), & (ac), & \dots & (af) \\ (ca), & (cc), & \dots & (cf) \\ \cdot & \cdot & \cdot & \cdot \\ (fa), & (fc), & \dots & (ff) \end{vmatrix}},$$

$$p_z = \&c.;$$

and the sum of the squares of the residuals, viz. (vv),

$$= \frac{\begin{vmatrix} (aa), & (ab), & \ldots & (af), & (an) \\ (ba), & (bb), & \ldots & (bf), & (bn) \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ (fa), & (fb), & \ldots & (ff), & (fn) \\ (na), & (nb), & \ldots & (nf), & (nn) \end{vmatrix}}{\begin{vmatrix} (aa), & (ab), & \ldots & (af) \\ (ba), & (bb), & \ldots & (bf) \\ \cdot & \cdot & \cdot & \cdot \\ (fa), & (fb), & \ldots & (ff) \end{vmatrix}}.$$

If therefore $\epsilon_x, \epsilon_y, \ldots$ denote the mean errors of $x_0, y_0, \ldots$

$$\epsilon_x^2 = \frac{1}{m-\mu} \frac{\begin{vmatrix} (bb), & (bc), & \ldots & (bf) \\ (cb), & (cc), & \ldots & (cf) \\ \cdot & \cdot & \cdot & \cdot \\ (fb), & (fc), & \ldots & (ff) \end{vmatrix} \begin{vmatrix} (aa), & (ab), & \ldots & (af), & (an) \\ (ba), & (bb), & \ldots & (bf), & (bn) \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ (fa), & (fb), & \ldots & (ff), & (fn) \\ (na), & (nb), & \ldots & (nf), & (nn) \end{vmatrix}}{\begin{vmatrix} (aa), & (ab), & \ldots & (af) \\ (ba), & (bb), & \ldots & (bf) \\ \cdot & \cdot & \cdot & \cdot \\ (fa), & (fb) & \ldots & (ff) \end{vmatrix}^2},$$

$$\epsilon_y^2 = \frac{1}{m-\mu} \frac{\begin{vmatrix} (aa), & (ac), & \ldots & (af) \\ (ca), & (cc), & \ldots & (cf) \\ \cdot & \cdot & \cdot & \cdot \\ (fa), & (fc), & \ldots & (ff) \end{vmatrix} \begin{vmatrix} (aa), & (ab), & \ldots & (af), & (an) \\ (ba), & (bb), & \ldots & (bf), & (bn) \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ (fa), & (fb), & \ldots & (ff), & (fn) \\ (na), & (nb), & \ldots & (nf), & (nn) \end{vmatrix}}{\begin{vmatrix} (aa), & (ab), & \ldots & (af) \\ (ba), & (bb), & \ldots & (bf) \\ \cdot & \cdot & \cdot & \cdot \\ (fa), & (fb), & \ldots & (ff) \end{vmatrix}^2},$$

$\epsilon_z^2 = \text{\&c.};$

where μ is the number of unknowns.

The form of the expressions for $\epsilon_x^2, \epsilon_y^2, \ldots$ is remarkable; the numerators consist of the product of two determinants, of which one contains $\mu - 1$, and the other $\mu + 1$ rows, and the denominator is the square of a determinant of μ rows.

Denoting the last-mentioned determinant, viz.

$$\begin{vmatrix} (aa), & (ab), & \dots & (af) \\ (ba), & (bb), & \dots & (bf) \\ \cdot & \cdot & \cdot & \cdot \\ (fa), & (fb), & \dots & (ff) \end{vmatrix}$$

by ∇, then, in the numerator of the expression for ϵ_x^2, the first determinant is formed from ∇ by omitting the first line and column, and the second determinant is formed from ∇ by bordering it with the elements $(an), (bn), \dots (nn), (nf), \dots (na)$; similarly, the first determinant in the numerator of ϵ_y^2 is derived from ∇ by omitting the second line and second column, and so on. The second determinant and the denominator, ∇^2, are the same in all the expressions.

The foregoing values of x_0, $y_0, \dots$ as determinants and of p_x, $p_y, \dots$ as quotients of determinants may of course be written down at once from the equations which determine them, and no formal proof is needed.* The value of (vv) is taken from § 7 of the previous paper, and a direct proof of this result is given in the next section (§ 15). In § 6 expressions were found for the auxiliaries $(bb.1)$, $(bc.1)$, $(cc.2)$, &c., as quotients of determinants.

§ 15. The method by which it was shown in § 7 that (vv) was equal to

$$\frac{1}{\nabla}\begin{vmatrix} (aa), & (ab), & \dots & (af), & (an) \\ (ba), & (bb), & \dots & (bf), & (bn) \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ (fa), & (fb), & \dots & (ff), & (fn) \\ (na), & (nb), & \dots & (nf), & (nn) \end{vmatrix},$$

depended upon the known equality of (vv) and $(nn.6)$. The expression for (vv) may, however, be very simply established, without assuming this equality, as follows:—

$$\begin{aligned}(vv) &= \Sigma(a_1x_0 + b_1y_0 + c_1z_0 \dots - n_1)^2, \\ &= (aa)x_0^2 + (bb)y_0^2 + \dots + 2(ab)x_0y_0 + 2(ac)x_0z_0 + \dots \\ &\qquad - 2(an)x_0 - 2(bn)y_0 - \dots + (nn), \\ &= x_0\{(aa)x_0 + (ab)y_0 \dots + (af)t_0 - (an)\} \\ &\quad + y_0\{(ba)x_0 + (bb)y_0 \dots + (bf)t_0 - (bn)\} \\ &\quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ &\quad + t_0\{(fa)x_0 + (fb)y_0 \dots + (ff)t_0 - (fn)\} \\ &\quad - (an)x_0 - (bn)y_0 \dots - (fn)t_0 + (nn).\end{aligned}$$

* In a paper entitled "Over het gebruik van determinanten bij de methode der kleinste kwadraten," printed in the *Nieuw Archief voor Wiskunde*, Deel I. pp. 179–188 (1875), Mr. Van Geer gives the determinant values of x_0, $y_0, \dots$ and of their weights. The determinants which form the denominators of the latter are, through some inadvertence, erroneous, although the equations from which they are derived are correctly stated.

In virtue of the normal equations which x_0, y_0, . . . satisfy, each line in this expression vanishes except the last, and therefore

$$(vv) = -(an)x_0 - (bn)y_0 \dots - (fn)t_0 + (nn),$$

$$= -\frac{(an)}{\nabla}\begin{vmatrix} (an), & (ab), & \dots & (af) \\ (bn), & (bb), & \dots & (bf) \\ \cdot & \cdot & \cdot & \cdot \\ (fn), & (fb), & \dots & (ff) \end{vmatrix} - \frac{(bn)}{\nabla}\begin{vmatrix} (aa), & (an), & \dots & (af) \\ (ba), & (bn), & \dots & (bf) \\ \cdot & \cdot & \cdot & \cdot \\ (fa), & (fn), & \dots & (ff) \end{vmatrix} - \dots + (nn),$$

$$= (-)^{\mu}\frac{1}{\nabla}\begin{vmatrix} (an), & (bn), & \dots & (fn), & (nn) \\ (aa), & (ab), & \dots & (af), & (an) \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ (ba), & (bb), & \dots & (bf), & (bn) \\ (fa), & (fb), & \dots & (ff), & (fn) \end{vmatrix},$$

$$= \frac{1}{\nabla}\begin{vmatrix} (aa), & (ab), & \dots & (af), & (an) \\ (ba), & (bb), & \dots & (bf), & (bn) \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ (fa), & (fb), & \dots & (ff), & (fn) \\ (na), & (nb), & \dots & (nf), & (nn) \end{vmatrix}.$$

It will be noticed that the analysis employed in this section merely amounts to the use of the following almost obvious theorem :—taking three letters only for simplicity, if x, y, z be given by the equations

$$\alpha_1 x + \beta_1 y + \gamma_1 z = k_1,$$
$$\alpha_2 x + \beta_2 y + \gamma_2 z = k_2,$$
$$\alpha_3 x + \beta_3 y + \gamma_3 z = k_3,$$

then

$$px + qy + rz - k = \frac{\begin{vmatrix} p, & q, & r, & k \\ \alpha_1, & \beta_1, & \gamma_1, & k_1 \\ \alpha_2, & \beta_2, & \gamma_2, & k_2 \\ \alpha_3, & \beta_3, & \gamma_3, & k_3 \end{vmatrix}}{\begin{vmatrix} \alpha_1, & \beta_1, & \gamma_1 \\ \alpha_2, & \beta_2, & \gamma_2 \\ \alpha_3, & \beta_3, & \gamma_3 \end{vmatrix}}.$$

If the signs of n_1, n_2, . . . n_m be changed, the signs of the expressions x_0, y_0, . . . are changed, but the values of (vv), p_x, p_y, . . . ϵ_x^2, ϵ_y^2, . . . remain unaltered.

§16. *Theorem*: The determinant ∇, viz.

$$\begin{vmatrix} (aa), & (ab), & \dots & (af) \\ (ba), & (bb), & \dots & (bf) \\ \cdot & \cdot & \cdot & \cdot \\ (fa), & (fb), & \dots & (ff) \end{vmatrix} \quad \dots\dots (1)$$

is equal to the sum of the squares of p determinants, p denoting the number of combinations of m things μ together.

To prove this, write at length the determinant ∇,

$$\begin{vmatrix} a_1^2 + a_2^2 \dots + a_m^2, & b_1a_1 \dots + b_ma_m, & \dots, & f_1a_1 \dots + f_ma_m \\ a_1b_1 + a_2b_2 \dots + a_mb_m, & b_1^2 \dots + b_m^2, & \dots, & f_1b_1 \dots + f_mb_m \\ \cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot & & & \\ a_1f_1 + a_2f_2 \dots + a_mf_m, & b_1f_1 \dots + b_mf_m, & \dots, & f_1^2 \dots + f_m^2 \end{vmatrix}$$

and from the $[m]^\mu$ determinants of which this is the sum select that which is formed from the first terms of the constituents of the first column, the second terms of the second column, . . . and the μ-th terms of the last column: this determinant

$$= a_1b_2 \dots f_\mu \begin{vmatrix} a_1, & a_2, & \dots & a_\mu \\ b_1, & b_2, & \dots & b_\mu \\ \cdot & \cdot & \cdot & \cdot \\ f_1, & f_2, & \dots & f_\mu \end{vmatrix}.$$

Similarly the determinant formed from the second terms of the first column, the first terms of the second column, and the same constituents as before in the case of the other columns, is

$$a_2b_1c_3 \dots f_\mu \begin{vmatrix} a_2, & a_1, & a_3, & \dots & a_\mu \\ b_2, & b_1, & b_3, & \dots & b_\mu \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ f_2, & f_1, & f_3, & \dots & f_\mu \end{vmatrix},$$

which is

$$= -a_2b_1c_3 \dots f_\mu \begin{vmatrix} a_1, & a_2, & \dots & a_\mu \\ b_1, & b_2, & \dots & b_\mu \\ \cdot & \cdot & \cdot & \cdot \\ f_1, & f_2, & \dots & f_\mu \end{vmatrix},$$

and in this way it is easily seen that the coefficient of the determinant

$$\begin{vmatrix} a_1, & a_2, & \dots & a_\mu \\ b_1, & b_2, & \dots & b_\mu \\ \cdot & \cdot & \cdot & \cdot \\ f_1, & f_2, & \dots & f_\mu \end{vmatrix} \quad \dots\dots (2)$$

is $\Sigma \pm a_1 b_2 c_3 \ldots f_\mu$, the summation referring to all permutations of the suffixes 1, 2, . . . μ: but this quantity is itself equal to the determinant (2), so that the resulting expression is equal to the square of (2), and, interchanging the lines and columns in the determinants of the form (2), we have the result

$$\begin{vmatrix} (aa), & (ab), & \ldots & (af) \\ (ba), & (bb), & \ldots & (bf) \\ \cdot & \cdot & \cdot & \cdot \\ (fa), & (fb), & \ldots & (ff) \end{vmatrix} = \Sigma \begin{vmatrix} a_1, & b_1, & \ldots & f_1 \\ a_2, & b_2, & \ldots & f_2 \\ \cdot & \cdot & \cdot & \cdot \\ a_\mu, & b_\mu, & \ldots & f_\mu \end{vmatrix}^2 \quad \ldots \quad (3)$$

where the summation symbol refers to the suffixes which in the different determinants are the $[m]^\mu \div [m]^\mu$ sets of μ numbers that can be formed from the m numbers 1, 2, . . m.

Using a recognised notation, the determinant (2) may be conveniently written

$$(a_1 b_2 \ldots f_\mu),$$

and the theorem is that the determinant (1) is equal to $\Sigma\,(a_1 b_2 \ldots f_\mu)^2$.

§17. It can be easily seen, in the same manner, that the determinant

$$\begin{vmatrix} (an), & (ab), & (ac), & \ldots & (af) \\ (bn), & (bb), & (bc), & \ldots & (bf) \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ (fn), & (fb), & (fc), & \ldots & (ff) \end{vmatrix} \quad \ldots \quad (4)$$

is equal to

$$\Sigma \begin{vmatrix} a_1, & b_1, & \ldots & f_1 \\ a_2, & b_2, & \ldots & f_2 \\ \cdot & \cdot & \cdot & \cdot \\ a_\mu, & b_\mu, & \ldots & f_\mu \end{vmatrix} \begin{vmatrix} n_1, & b_1, & \ldots & f_1 \\ n_2, & b_2, & \ldots & f_2 \\ \cdot & \cdot & \cdot & \cdot \\ n_\mu, & b_\mu, & \ldots & f_\mu \end{vmatrix}$$

or, as this expression may be written,

$$\Sigma(a_1 b_2 \ldots f_\mu)(n_1 b_2 \ldots f_\mu).$$

the Σ having the same meaning as at the end of the last section, and the number of terms—*i.e.* of products of pairs of determinants—being as before $= [m]^\mu \div [\mu]^\mu$.

§18. The values of x_0, y_0, . . . are therefore

$$x_0 = \frac{\Sigma(a_1 b_2 \ldots f_\mu)(n_1 b_2 \ldots f_\mu)}{\Sigma(a_1 b_2 \ldots f_\mu)^2},$$

$$y_0 = \frac{\Sigma(a_1 b_2 \ldots f_\mu)(a_1 n_2 \ldots f_\mu)}{\Sigma(a_1 b_2 \ldots f_\mu)^2},$$

$$z_0 = \&c.;$$

and it thus appears that the values of $x, y, \ldots$ given by the method of least squares are in fact those obtained by combining linearly in the manner indicated the values found by solving each set of μ equations which can be formed from the m given equations. Expressing this more in detail, we have the following rule:—

From the m equations of condition we can form p sets of μ equations, p denoting for brevity $[m]^{\mu} \div [\mu]^{\mu}$. Solve each of these sets of equations by the determinant method, and let the resulting values be

$$\begin{array}{ll} \text{for } x, & \dfrac{\alpha_1}{\lambda_1}, \dfrac{\alpha_2}{\lambda_2}, \ldots \dfrac{\alpha_p}{\lambda_p}, \\[2ex] \text{for } y, & \dfrac{\beta_1}{\lambda_1}, \dfrac{\beta_2}{\lambda_2}, \ldots \dfrac{\beta_p}{\lambda_p}, \\[2ex] \text{\&c.,} & \end{array}$$

where $\alpha_1, \ldots \beta_1, \ldots \lambda_1, \ldots$ are the actual determinants which occur in the solutions (*i.e.* so that any factors common to both numerator and denominator in any of the fractions are not to be thrown out); then

$$x_0 = \frac{\lambda_1\alpha_1 + \lambda_2\alpha_2 \ldots + \lambda_p\alpha_p}{\lambda_1^2 + \lambda_2^2 \ldots + \lambda_p^2},$$

$$y_0 = \frac{\lambda_1\beta_1 + \lambda_2\beta_2 \ldots + \lambda_p\beta_p}{\lambda_1^2 + \lambda_2^2 \ldots + \lambda_p^2},$$

$$z_0 = \text{\&c.}$$

The rule may also be stated in a slightly different manner thus:— solve each set of equations and let the system of values be

$$\begin{array}{ll} \text{for } x, & A_1, A_2, \ldots A_p, \\ \text{for } y, & B_1, B_2, \ldots B_p, \\ \text{\&c.;} & \end{array}$$

then

$$x_0 = \frac{\lambda_1^2 A_1 + \lambda_2^2 A_2 \ldots + \lambda_p^2 A_p}{\lambda_1^2 + \lambda_2^2 \ldots + \lambda_p^2},$$

$$y_0 = \frac{\lambda_1^2 B_1 + \lambda_2^2 B_2 \ldots + \lambda_p^2 B_p}{\lambda_1^2 + \lambda_2^2 \ldots + \lambda_p^2},$$

$$z_0 = \text{\&c.},$$

where $\lambda_1, \lambda_2, \ldots \lambda_p$ are as before: that is, they are the determinants whose constituents are the coefficients which occur on the left-hand side of the different sets of equations.

§19. To illustrate the formulæ, take the simple case of $m=4, \mu=3$.

By §16,

$$\begin{vmatrix} (aa), & (ab), & (ac) \\ (ba), & (bb), & (bc) \\ (ca), & (cb), & (cc) \end{vmatrix} = (a_1b_2c_3)^2 + (a_1b_2c_4)^2 + (a_1b_3c_4)^2 + (a_2b_3c_4)^2,$$

by §17,

$$\begin{vmatrix} (an), & (ab), & (ac) \\ (bn), & (bb), & (bc) \\ (cn), & (cb), & (cc) \end{vmatrix} = \begin{aligned} &(a_1b_2c_3)(n_1b_2c_3) + (a_1b_2c_4)(n_1b_2c_4) \\ &+ (a_1b_3c_4)(n_1b_3c_4) + (a_2b_3c_4)(n_2b_3c_4), \end{aligned}$$

and by §18,

$$x_0 = \frac{(a_1b_2c_3)(n_1b_2c_3) + (a_1b_2c_4)(n_1b_2c_4) + (a_1b_3c_4)(n_1b_3c_4) + (a_2b_3c_4)(n_2b_3c_4)}{(a_1b_2c_3)^2 \quad + \quad (a_1b_2c_4)^2 \quad + \quad (a_1b_3c_4)^2 \quad + \quad (a_2b_3c_4)^2},$$

$$y_0 = \frac{(a_1b_2c_3)(a_1n_2c_3) + (a_1b_2c_4)(a_1n_2c_4) + (a_1b_3c_4)(a_1n_3c_4) + (a_2b_3c_4)(a_2n_3c_4)}{(a_1b_2c_3)^2 \quad + \quad (a_1b_2c_4)^2 \quad + \quad (a_1b_3c_4)^2 \quad + \quad (a_2b_3c_4)^2},$$

$$z_0 = \frac{(a_1b_2c_3)(a_1b_2n_3) + (a_1b_2c_4)(a_1b_2n_4) + (a_1b_3c_4)(a_1b_3n_4) + (a_2b_3c_4)(a_2b_3n_4)}{(a_1b_2c_3)^2 \quad + \quad (a_1b_2c_4)^2 \quad + \quad (a_1b_3c_4)^2 \quad + \quad (a_2b_3c_4)^2}.$$

§20. As a numerical example of the process, I now consider the system of four equations which Gauss himself employed to illustrate the method of least squares, and which has generally been adopted as the standard example by writers on the subject. These equations are

$$\left.\begin{aligned} x - y + 2z &= 3 \quad &\text{(i)} \\ 3x + 2y - 5z &= 5 \quad &\text{(ii)} \\ 4x + y + 4z &= 21 \quad &\text{(iii)} \\ -x + 3y + 3z &= 14 \quad &\text{(iv)} \end{aligned}\right\};$$

and the normal equations derived from them are

$$\left.\begin{aligned} 27x_0 + 6y_0 \qquad &= 88 \\ 6x_0 + 15y_0 + z_0 &= 70 \\ y_0 + 54z_0 &= 107 \end{aligned}\right\},$$

which give

$$x_0 = \frac{49154}{19899}, \quad y_0 = \frac{2617}{737}, \quad z_0 = \frac{12707}{6633}.$$

Taking the first three equations (i), (ii), (iii), the values of x, y, z, given by them are

$$x = \frac{P_1}{P}, \quad y = \frac{P_2}{P}, \quad z = \frac{P_3}{P},$$

where

$$P = \begin{vmatrix} 1, & -1, & 2 \\ 3, & 2, & -5 \\ 4, & 1, & 4 \end{vmatrix} = 35,$$

$$P_1 = \begin{vmatrix} 3, & -1, & 2 \\ 5, & 2, & -5 \\ 21, & 1, & 4 \end{vmatrix} = 90,$$

$$P_2 = \begin{vmatrix} 1, & 3, & 2 \\ 3, & 5, & -5 \\ 4, & 21, & 4 \end{vmatrix} = 115,$$

$$P_3 = \begin{vmatrix} 1, & -1, & 3 \\ 3, & 2, & 5 \\ 4, & 1, & 21 \end{vmatrix} = 65.$$

Similarly solving equations (i), (ii), (iv),

$$x = \frac{Q_1}{Q}, \quad y = \frac{Q_2}{Q}, \quad z = \frac{Q_3}{Q},$$

where the values of the determinants are

$$Q = \begin{vmatrix} 1, & -1, & 2 \\ 3, & 2, & -5 \\ -1, & 3, & 3 \end{vmatrix} = 47,$$

$$Q_1 = 122, \quad Q_2 = 167, \quad Q_3 = 93.$$

Solving the systems (i), (iii), (iv), and (ii), (iii), (iv), and denoting the values of x, y, z, by

$$\frac{R_1}{R}, \quad \frac{R_2}{R}, \quad \frac{R_3}{R}$$

and

$$\frac{S_1}{S}, \quad \frac{S_2}{S}, \quad \frac{S_3}{S},$$

the values of the determinants R, R_1, . . . S, S_1, . . . are found to be

$$R = 33, \quad R_1 = 78, \quad R_2 = 113, \quad R_3 = 67$$
$$S = -124, \quad S_1 = -304, \quad S_2 = -444, \quad S_3 = -236.$$

The unreduced values obtained from the four sets of equations are thus

$$\text{for } x, \quad \frac{90}{35}, \quad \frac{122}{47}, \quad \frac{78}{33}, \quad \frac{304}{124},$$

$$\text{for } y, \quad \frac{115}{35}, \quad \frac{167}{47}, \quad \frac{113}{33}, \quad \frac{444}{124},$$

$$\text{for } z, \quad \frac{65}{35}, \quad \frac{93}{47}, \quad \frac{67}{33}, \quad \frac{236}{124},$$

and therefore

$$x_0 = \frac{90 \times 35 + 122 \times 47 + 78 \times 33 + 304 \times 124}{(35)^2 + (47)^2 + (33)^2 + (124)^2},$$

$$y_0 = \frac{115 \times 35 + 167 \times 47 + 113 \times 33 + 444 \times 124}{(35)^2 + (47)^2 + (33)^2 + (124)^2},$$

$$z_0 = \frac{65 \times 35 + 93 \times 47 + 67 \times 33 + 236 \times 124}{(35)^2 + (47)^2 + (33)^2 + (124)^2},$$

giving

$$x_0 = \frac{49154}{19899}, \quad y_0 = \frac{70659}{19899}, \quad z_0 = \frac{38121}{19899},$$

which agree with the values given by the normal equations.

In general, if the values obtained by solving the equations be given in any form such as, *e.g.*, vulgar fractions in their lowest terms, so that the values of x are

$$\frac{18}{7}, \quad \frac{122}{47}, \quad \frac{26}{11}, \quad \frac{76}{31},$$

then these quantities are to have the respective weights

$$P^2, \quad Q^2, \quad R^2, \quad S^2,$$

viz.

$$x_0 = \frac{\frac{18}{7} \times (35)^2 + \frac{122}{47} \times (47)^2 + \frac{26}{11} \times (33)^2 + \frac{76}{31} \times (124)^2}{(35)^2 + (47)^2 + (33)^2 + (124)^2},$$

and the weights are the same in the case of the corresponding values of y and z.

§21. In the absence of any method such as that of least squares, if we had to determine the best values of $x, y, \ldots$ from a system of m linear equations, it would be natural to first solve every set of μ equations which could be formed from the m equations, and to compare the different values of $x, y, \ldots$ thus obtained. The question to be decided would be how to combine the different values, and perhaps the method which would first suggest itself would be to take the arithmetic mean of the values found for x as the adopted value of x, and similarly in the case of

$y, z, \ldots$; it is clear, however, that this mode of treatment would not be satisfactory, as certain of the sets of equations would be better suited for the accurate determination of $x, y, \ldots$ than others.

It appears from § 18 and 22 that in proceeding according to the method of least squares we assign to each system of values of $x, y, \ldots$ a weight proportional to the square of the determinant whose constituents are the coefficients in the set of μ equations from which the values are derived.

It is evident that this determinant affords a good measure of the precision with which $x, y, \ldots$ are determined by the set of equations, representing as it does the common denominator in the values of these quantities. In the method of least squares the square of the determinant is taken as the weight, and in consequence the sign of the determinant is immaterial, and the common denominator of $x_0, y_0, \ldots$, being a sum of squares, is always positive.

§ 22. The determinant which forms the numerator of (vv), viz.

$$\begin{vmatrix} (aa), & (ab), & \ldots & (af), & (an) \\ (ba), & (bb), & \ldots & (bf), & (bn) \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ (fa), & (fb), & \ldots & (ff), & (fn) \\ (na), & (nb), & \ldots & (nf), & (nn) \end{vmatrix} (\mu + 1 \text{ rows}) \quad . \quad . \quad . \quad . \quad (5)$$

is of the same form as ∇ and only differs from it by including the letters n: the determinant (5) is therefore

$$= \Sigma \begin{vmatrix} a_1, & b_1, & \ldots f_1, & n_1, \\ a_2, & b_2, & \ldots f_2, & n_2, \\ \cdot & \cdot & \cdot & \cdot \\ a_\mu, & b_\mu, & \ldots f_\mu, & n_\mu \\ a_{\mu+1}, & b_{\mu+1}, & \ldots f_{\mu+1}, & n_{\mu+1} \end{vmatrix}^2 \quad . \quad . \quad . \quad . \quad . \quad (6)$$

i.e. it is equal to the sum of the squares of all the determinants whose constituents are the coefficients and right-hand members in the different sets of $(\mu+1)$ equations which can be formed from the m equations; the number of such determinants is $[m]^{\mu+1} \div [\mu+1]^{\mu+1}$. It thus appears that (vv) cannot vanish unless every one of these determinants is equal to zero—*i.e.* unless the m equations are all consistent with one another, and equivalent to only μ independent equations. If $(vv)=0$, the mean (or probable) errors of $x_0, y_0, \ldots$ also vanish; and this is as it should be, for it is clear that the mean (or probable) errors of $x_0, y_0, \ldots$ can only be zero when the m equations determine $x, y, \ldots$ uniquely—*i.e.* are equivalent to only μ independent equations.

§ 23. If the number of equations exceeds the number of un-

knowns by unity—*i.e.* if $m=\mu+1$—then the expression (6) consists of only a single determinant, so that (vv) is a complete square, and therefore $\sqrt{(vv)}$ is a linear function of $n_1, n_2, \ldots n_{\mu+1}$, its value being

$$\frac{1}{\nabla}\begin{vmatrix} a_1, & b_1, & \ldots f_1, & n_1 \\ a_2, & b_2, & \ldots f_2, & n_2 \\ \cdot & \cdot & \cdot & \cdot \\ a_\mu, & b_\mu, & \ldots f_\mu, & n_\mu \\ a_{\mu+1}, & b_{\mu+1}, & \ldots f_{\mu+1}, & n_{\mu+1} \end{vmatrix}$$

If in this determinant capital letters be used to denote the minors of the corresponding italic letters, then

$$\sqrt{(vv)} = \frac{n_1N_1 + n_2N_2 \ldots + n_{\mu+1}N_{\mu+1}}{\sqrt{(N_1^2+N_2^2 \ldots N^2_{\mu+1})}},$$

$$x_0 = -\frac{A_1N_1 + A_2N_2 \ldots + A_{\mu+1}N_{\mu+1}}{N_1^2+N_2^2 \ldots + N^2_{\mu+1}},$$

$$y_0 = -\frac{B_1N_1 + B_2N_2 \ldots + B_{\mu+1}N_{\mu+1}}{N_1^2+N_2^2 \ldots + N^2_{\mu+1}},$$

$$z_0 = \&c.$$

In the case of the foregoing example

$$\sqrt{(vv)} = \frac{1}{\sqrt{\nabla}}\begin{vmatrix} 1, & -1, & 2, & 3 \\ 3, & 2, & -5, & 5 \\ 4, & 1, & 4, & 21 \\ -1, & 3, & 3, & 14 \end{vmatrix}$$

$$= \frac{14P-21Q+5R-3S}{\sqrt{(P^2+Q^2+R^2+S^2)}}$$

$$= \frac{14\times35-21\times47+5\times33+3\times124}{\sqrt{\{(35)^2+(47)^2+(33)^2+(124)^2\}}}$$

$$= \frac{40}{\sqrt{(19899)}}.$$

§ 24. The determinants which form the denominators of $p_{,}, p_{,,}, \ldots$ are also sums of squares of determinants; for example, the denominator of $p_{,}$, viz:

$$\begin{vmatrix} (bb), & (bc), & \ldots & (bf) \\ (cb), & (cc), & \ldots & (cf) \\ \cdot & \cdot & \cdot & \cdot \\ (fb), & (fc), & \ldots & (ff) \end{vmatrix} \quad (\mu - 1 \text{ rows}) \quad \ldots \ldots \quad (7)$$

$$= \Sigma \begin{vmatrix} b_1, & c_1, & \ldots f_1 \\ b_2, & c_2, & \ldots f_2 \\ \cdot & \cdot & \cdot \\ b_{\mu-1}, & c_{\mu-1}, & \ldots f_{\mu-1} \end{vmatrix}^2 \quad \ldots \ldots \quad (8)$$

the number of determinants being $[m]^{\mu-1} \div [\mu - 1]^{\mu-1}$.

Thus, in the example, the denominator of $p_{.c}$

$$= \begin{vmatrix} -1, & 2 \\ 2, & -5 \end{vmatrix}^2 + \begin{vmatrix} -1, & 2 \\ 1, & 4 \end{vmatrix}^2 + \begin{vmatrix} -1, & 2 \\ 3, & 3 \end{vmatrix}^2$$

$$+ \begin{vmatrix} 2, & -5 \\ 1, & 4 \end{vmatrix}^2 + \begin{vmatrix} 2, & -5 \\ 3, & 3 \end{vmatrix}^2 + \begin{vmatrix} 1, & 4 \\ 3, & 3 \end{vmatrix}^2$$

$$= 1 + 36 + 81 + 169 + 441 + 81$$

$$= 809,$$

and therefore

$$p_x = \frac{19899}{809}.$$

It may be observed that if the determinant which forms the denominator in any of the expressions for the weights is equal to zero, so also is the numerator, for if each of the determinants in (8) vanishes, then each of the determinants which form the right-hand side of (3) vanishes also.

§ 25. Since all the determinants involved in the statement of results in § 14 are equal to sums of squares, it follows that they can never be negative. In § 5 it was shown that all the products $(aa)(bb.1)$, $(aa)(bb.1)(cc.2)$, &c., are of the form (1), and it is therefore evident that the auxiliaries $(bb.1)$, $(cc.2)$, &c., cannot be negative. A shorter proof of this fact was, however, given in § 7.

§ 26. The case, $\mu = 2$, of the theorem in § 16 is a very well known result; but the only place I know of in which the general theorem is ennunciated occurs in an investigation by Professor C. Niven of the vibrations of a dynamical system where the particles are subject to small frictional forces, printed in the "Cambridge Senate-House Problems and Riders for 1878," pp. 188–191. The proof given in § 16 is the same as Professor Niven's.

Professor Cayley, to whom I communicated the results of §§ 16–18 some time ago was, I found, already acquainted with them; but it seems possible that they may not have been published before, as it would not be easy to express them without the aid of determinants, and the only paper I have met with in which an explicit use of determinants is made in connection with the solution of equations in the method of least squares is that of Mr. Van Geer referred to in the note in § 14.

On the Possible Performance of an Object-Glass for Star-Gazing. By Edward Sang, Esq.

It having been proposed to compute the curvatures for an object-glass, with a view to obtaining the least possible aberration in the image of a star, the preliminary question arose as to how the computations should be conducted.

In the preparation of formulæ for the amount of spherical aberration, the sines and cosines of arcs are represented by two or three terms of the series which truly express them, and therefore such formulæ are only approximative, and the results obtained by them are to be regarded as guides to more accurate determinations. Our ultimate resort is to trace strictly the course of each pencil of light. When the thicknesses are taken into account, the application of the formulæ becomes as laborious as the direct trigonometrical calculation itself; wherefore it was determined to follow the trigonometrical method throughout.

The computation thus takes the form of a series of trials applicable only to the particular case in hand, and we have so to arrange these trials as to make them exhaustive, and so also as to throw light on analogous cases.

The proposition as it occurs in practice is this:—"Given two discs of glass, to construct of them an object-glass which shall give the best possible result." In the present instance that result is to be the formation of the image of an exceedingly minute luminous object; the correlative matters of the flatness of the field of view and of the performance towards the edge of that field not being taken into account. Now, the refractions by the two kinds of glass are data in the problem and fix the amount of the secondary chromatic aberration; wherefore our enquiry must be mainly directed to that part of the total error, which depends on the sphericity of the surfaces—that being the only matter under the control of the constructor.

The case actually proposed was to make an aplanatic combination from two discs, one of *hard-crown*, the other of *dense flint* glass, having an aperture of 7·5 inches, with a thickness in the rough of ·75; the indices of refraction being given in Chance's list as under, and the focal distance to be about 100 inches.

Glass (Chance's).	Density.	C	D	F	G
Hard Crown	2·485	1·5146	1·5172	1·5232	1·5280
Soft Crown	2·55	1·5119	1·5146	1·5210	1·5263
Light Flint	3·21	1·5700	1·5740	1·5839	1·5922
Dense Flint	3·66	1·6175	1·6224	1·6348	1·6453
Extra Dense	3·85	1·6450	1·6504	1·6643	1·6761

The investigation was divided into two parts: one in which the extreme bands C and G alone were considered; the other, in which all the four were taken into account.

The first step was to compute, as for an absolutely thin lens, the curvatures needed to bring together the images for the exreme bands C and G, the result being:—

$$\frac{1}{r_1} - \frac{1}{r_2} = -{\cdot}0460\ 9225,$$

$$\frac{1}{r_3} - \frac{1}{r_4} = +{\cdot}0222\ 1714\ ;$$

so that the thickness of the crown-glass lens may conveniently be fixed at ·70, that of the flint-glass one at ·54, measured on the axis of the system.

These equations show no connection between the second and third curvatures; the internal surfaces may be made coincident, or the glasses may have a lens of air between them, which may be either convex or concave; hence our trials must be arranged in sets, according to the arbitrarily assumed character of this intermediate lens.

Having established the relation between the second and first curvatures, indicated by the equation

$$\frac{1}{r_2} = \frac{1}{r_1} + {\cdot}0460\ 9225,$$

and that between the third and second curvatures by

$$\frac{1}{r_3} = \frac{1}{r_2} + \text{arbitrary constant},$$

the fourth curvature has to be computed so as to make the foci for the bands C and G coincident for those pencils of light which pass close to the axis; this computation is so simple as to need no remark. The influence of the thicknesses of the lenses is seen in the resulting curvature and position of the focus. In this way the assumption of the first curvature determines the whole arrangement in any one of the sets of trials.

It remains for us to compute the aberrations for those pencils of light which pass at the margin of the lens.

This computation involves only the very simplest cases of the resolution of triangles, and would need no special notice but for the circumstance that, in consequence of the angles being small,

the interpolation of the arc from its logarithmic sine, or contrariwise, is laborious. With Taylor's table to each second of arc, the labour is very great. In the reprint of Taylor's given in Shortrede's Tables, I caused to be inserted tables of proportional parts, but even these fail to give us help, because for our purpose second and even third differences are needed. To meet the requirements of the case, I changed entirely the plan of operation, and, measuring the arcs by the same unit as their sines, constructed a canon of logarithmic sines for each ten-thousandth part of the radius up to ·3000. In this way the interpolations were rendered incomparably more easy, and the accuracy was correspondingly augmented so that the results have all the precision that is attainable by the use of seven-place tables.

It was natural to begin our trials with the second and third surfaces coincident. Assuming then $r_2 = r_3$ and taking for $\frac{1}{r_1}$ values ranging from $+ \cdot 005$ to $- \cdot 045$, the distances of the foci from the fourth surface were computed for the achromatised central pencil, and for the marginal pencils corresponding to the bands C and G. The results are shown in the accompanying figure 1, enlarged to make the parts visible.

The numbers on the horizontal scale show the focal distances in inches (enlarged two times); those on the vertical scale, the curvatures, $\frac{1}{r_1}$, of the first surface, + indicating concavity toward the object, − indicating convexity.

The vertical line at 100 shows the position of the achromatised focus for an absolutely thin lens; the strong curved line with its convexity to the left shows the focus for the central achromatised ray of the actual lens, and the two lines convex to the right indicate the foci for the marginal pencils of the bands C and G. These are well apart for the case

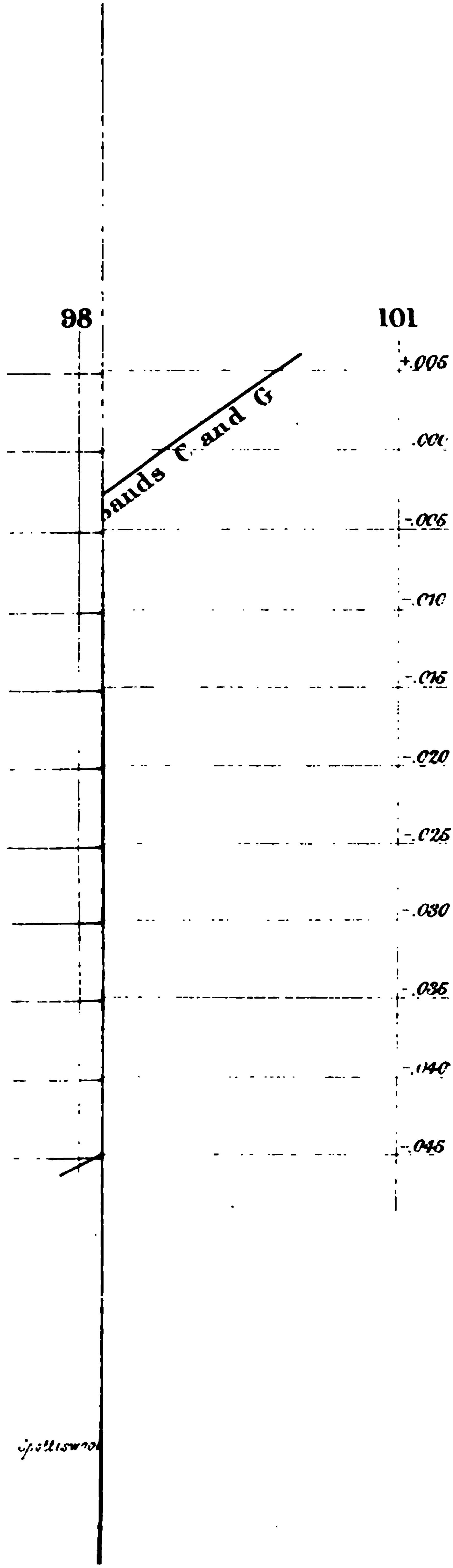
$$\frac{1}{r_1} = + \cdot 005;$$

they become closer and cross each other about

$$\frac{1}{r_1} = - \cdot 040,$$

but their separation is too small to be shown even on the enlarged scale. The horizontal distances between one of these curves and the curve for the central pencil give the respective amounts of longitudinal aberration, and the points of crossing show the conditions for aplanatism for the particular band. Thus the spherical aberration of the band G is *nil* when

$$\frac{1}{r_1} = - \cdot 0065,$$


98
101
+.005
.000
-.005
-.010
-.015
-.020
-.025
-.030
-.035
-.040
-.045
ands C and G

98

99

that for band C when

$$\frac{1}{r_1} = -\cdot 0078.$$

The marginal line for C again crosses at $-\cdot 0230$, that for G at $-\cdot 0238$: so that our problem has two approximate solutions, neither of them satisfactory; the separation of the foci for the marginal pencils C and G being in the latter solution $\cdot 0296$ or about $\frac{1}{34}$th of an inch.

Having failed to obtain complete aplanatism with the interior surfaces coincident, we proceed to try the effect of a separation. Assuming as the basis of our second set of trials

$$\frac{1}{r_2} = \frac{1}{r_2} + \cdot 001,$$

and making the lenses touch at the edge, we get a separation of $\cdot 007$ at the centre, and allow for this in our calculations; it is not necessary to carry these beyond the limits

$$\frac{1}{r_1} = -\cdot 020 \quad \text{and} \quad \frac{1}{r_1} = -\cdot 030.$$

The results are shown in the lower part of the second figure. Here the line for the central achromatised pencil is slightly, while the lines for the marginal pencils are considerably removed from the lens, so that if the curves were continued, the crossings would be much more apart than in the first figure. The separation, also, of the foci for the marginal pencils C and G is somewhat greater than before, and it is needless to proceed further in this direction.

On reversing the mode of variation by supposing

$$\frac{1}{r_3} = \frac{1}{r_2} - \cdot 001,$$

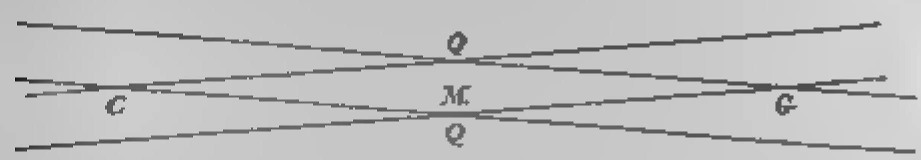
the lenses are made to touch at the centre, and their edges would need to be kept apart by means of a thin ring. The results are shown in the upper part of the second figure and are contained within the limits

$$\frac{1}{r_1} = -\cdot 010 \quad \text{and} \quad \frac{1}{r_1} = -\cdot 020.$$

The central line is here nearer to the lens; the marginal lines, however, much more so, so much that they do not cross the central line at all, and thus, under this supposition, we cannot correct the spherical aberration. For the purpose of comparing

these results the second figure is made in the same form as the first one. The contrast shows us that for these two kinds of glass the best result is to be got (best in so far as the bands C and G are concerned) by making the curvature of the first surface about $-\cdot 0238$, with the second and third surfaces coincident; in which case the fourth surface is sensibly flat.

FIG. 3.



If the curvatures be so adjusted as that the focus for the central achromatised pencil be midway between the foci for the marginal pencils C and G, as shown in figure 3, and if from C and G we draw lines to the edge of the lens, which lines have, in this case, an inclination of $\cdot 0375$ to the axis, the least section of the compound cones thus formed will be on the focal plane passing through M. Now, the distance C G is $\cdot 0296$; wherefore the radius M Q of this least section is

$$\cdot 0148 \times \cdot 0375 = \cdot 000555 :$$

that is, its diameter will be about the 900th part of an inch, in our case. This subtends, at the distance of 100 inches, an angle of $2''\cdot 29$ of the ancient division of the quadrant.

If, instead of making the central foci C and G to agree, we arrange the curvatures so as to place G before C to the extent $\cdot 0148$, the central foci for C and G will agree, alternately, with the marginal foci for G and C, and the diameter of the above false disc will be halved. Thus we see that, when the aperture of the lens is $\cdot 075$ of its focal length, the best performance attainable with these two kinds of glass and for the bands C and G, gives a false disc $1''\cdot 145$ in diameter.

Hitherto we have considered only the extreme bands C and G; the bringing of these together does not necessarily produce achromatism, nor, indeed, can we expect that a better result might not have been reached by so treating some other pair. In combining more than two bands we cannot bring all the foci into one, and the best effect is to be got by making the sum of the squares of their mutual distances as small as possible. Also, if we desire to take into consideration the breadths and intensities of the parts of the spectrum represented by the various bands, we must attach weight-coefficients to them, so that if C represent the focus, and c the intensity of the band C, and similarly of the others, the sum

$$cd\,.\,CD^2 + cf\,.\,CF^2 + cg\,.\,CG^2 + df\,.\,DF^2 + dg\,.\,DG^2 + fg\ \ FG^2$$

is to be made a minimum.

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There is no difficulty in making this calculation for the central pencil. For our present purpose it may suffice to regard the four bands as of equal intensity; and thus we may substitute for the condition $CG^2=0$ this improved one

$$CD^2+CF^2+CG^2+DF^2+DG^2+FG^2 = \text{minimum}.$$

Proceeding exactly as before, we obtain for the conditions of optimism in a thin lens, the equations

$$\frac{1}{r_1}-\frac{1}{r_2} = -{\cdot}0459\ 2642,$$

$$\frac{1}{r_2}-\frac{1}{r_4} = +{\cdot}0220\ 8918,$$

not differing much from those formerly used.

Using the first of these equations with the same thicknesses, ·70 and ·54, as before; making the interior surfaces coincident, and computing the fourth surface so as to produce the optimum effect for the central pencils; and thereafter calculating the positions of the foci for the marginal pencils, we obtain the results exhibited in figure 4. Here the longitudinal scale has been magnified eight times: that is, each inch in the drawing represents one-eighth part of an inch in focal length. Of the eight lines shown in the figure, the four marked with capital letters indicate the foci for the central pencils of the several bands; those marked in small letters serve for the marginal pencils.

A glance at this diagram shows that the best effect is to be got with a front curvature between −·02315, where the line *gg* intersects CC, and −·02330 where *ff* cuts FF. The interval between the foci for the marginal pencils of the bands F and G is then ·1259, so that the diameter of the false disc is ·004721 which subtends at the lens an angle of 9″·8—say ten seconds.

We might now proceed to make a difference between the curvatures of the second and third surfaces, but seeing that the chromatic aberration much exceeds the error from sphericity, and seeing how closely the present resembles the former case, we can hardly expect any improvement, and thus, for these two kinds of glass, we may hold that the diameter of the false image of a minute point cannot be made less than ten seconds.

Although these enquiries have had special reference to one particular combination of glasses, they lead to some general conclusions. The residual error is seen to arise from two sources: the first and the most important being in the dispersive powers, the second in the sphericity of the surfaces of the lenses; which second is only less important than the other because it is, to a certain degree, under our control.

It is, then, of the utmost importance to select from among the different kinds of glass that are available that pair which gives the greatest concentration of the foci; and our endeavours towards the improvement of the refracting telescope should be mainly directed to the dispersive powers of the glasses. If, in our first example, the indices for the bands D and F had been such as to bring all the four foci for the central pencil together, the false disc would have only been 1″·14, or about one-eighth part of what, in the actual state of matters, is possible.

The importance of a proper choice among the kinds of glass may be seen by comparing the results from different combinations of the crown and flint glasses given in Chance's list. The following table shows, for an achromatic thin lens of 100 inches in focal distance, the mean longitudinal aberration from the mean focus, for the six combinations.

Mean Longitudinal Dispersion from Mean Focus.

	Hard Crown.	Soft Crown.
Light Flint	·05782	·03296
Dense Flint	·05712	·03868
Extra Dense	·05828	·04204

In the general problem "to obtain the best result from two given discs of glass," the focal length has to be considered. On computing, in several cases, the spherical aberration for pencils intermediate between the centre and the margin of the lens, these were found—as, indeed, was to have been expected—to be almost exactly proportional to the square of the distance of the incident ray from the centre. If, then, keeping the aperture unchanged, we augment the focal distance in any ratio, we reduce the longitudinal aberration in the same ratio; with twice the focal length the longitudinal spherical aberration is halved; but the inclination to the axis is also halved, wherefore the lateral aberration is reduced four times; consequently, since the distance is doubled, the angle subtended by the false disc is reduced to one-eighth part: that is, in the triplicate ratio of the focal length. So much for the spherical aberration.

On augmenting the focal length, the longitudinal chromatic aberration is augmented in the same ratio; wherefore the lateral aberration is unchanged, and the angle subtended by it is reduced in the ratio of the focal length; hence in this respect also it is advantageous to have a long telescope.

But every bit of glass is liable to internal irregularities, which the manufacturer takes many precautions to remove or to palliate. A ray of light in proceeding through the lens suffers deflection at each change of consistency, and the total effect is an indistinctness or blur in the image. This blur preserves the same angular extent whatever may be the focal length.

Hence the total imperfection of the image consists of three parts: one, depending on the quality of the glass, constant; a second, proportional inversely to the focal length and arising from the distribution of the spectra; and a third, the spherical aberration, inversely proportional to the cube of the length.

If we make the length so great that the second and third imperfections are less than the constant one, we have gained all that is possible; thus we see that the exertions of the glass-maker to produce metal of uniform composition enable us, when we are in search of the best possible result, not to shorten but to lengthen the tube.

The investigation into the action of a telescope does not stop at the focus of the principal lens. Properly speaking, no image is formed there; the rays of light go onwards through the eye-piece, and it is only on the retina that there is an image. The action of the eye-lenses is quite as important as that of the object-glass, and it may be possible, by a proper arrangement of the curvatures, to remedy the unavoidable defects already noticed; the eye-piece should be suited specifically to the objective, and the whole instrument has to be treated as an integral system. The second part of the general subject may be treated in a future essay.

Before venturing critically to interpret the appearance of any object as seen through a refractor, the observer would need to know the character of the imperfections of the image. The false disc may be bright in the middle, fading to become imperceptible at the edge, or it may exhibit a coloured fringe; and in all cases the imperfection causes an overlapping of the images of two near objects, giving rise now to the seeming projection of a star upon the moon's disc, then to the phenomenon of Bailey's beads.

The light of an exceedingly faint object diffused over an appreciable disc by one lens may be so faint as to be unperceived, while by another lens, more happily arranged, the brightness may be enough to make the object visible.

An exact knowledge of the indices of the two glasses, and of their curvatures, would enable us to estimate the amount and character of the diffusion; hence the importance of having, along with all lenses of any magnitude, a prism of glass from the same melting, or, better, one cut from a corner which had been left projecting on the rough disc.

6 *Molendo Terrace, Edinburgh,*
1880, *June* 28.

Note on the Nebula near Merope. By Prof. W. Tempel.

(*Extract from a letter to the Foreign Secretary, communicated by Lord Lindsay.*)

The English scientific papers, as also the *Monthly Notices*, have recently directed some attention to the nebula in the neighbourhood of *Merope*, and it is but natural that I should be much interested in anything written or observed in reference to this nebula, which I was the first to discover, on October 19, 1859, when in Venice (*Astr. Nachr.*, No. 1290).

This nebula has already, so to say, a history; for by some its existence has been thoroughly recognised, whilst by others my assertion of it has been contradicted (*Astr. Nachr.*, No. 1393, p. 13).

Amongst the former are to be found various illustrious astronomers, such as Schmidt, Winnecke, Anwers, Schönfeld, &c., who, with instruments of comparatively small size, have attested to the existence of the nebula to the south of *Merope*, and described its form exactly as I saw it from the beginning, and have always continued to see it.

Prof. Schönfeld says in his catalogue, Part II., p. 80: "Anwers 18 = G. Cat. 768. Tempel's nebula near *Merope* very distinct, and immediately conspicuous, even without more accurate indications of its position. Anwers' notices (74) as to its extension and form very correct."

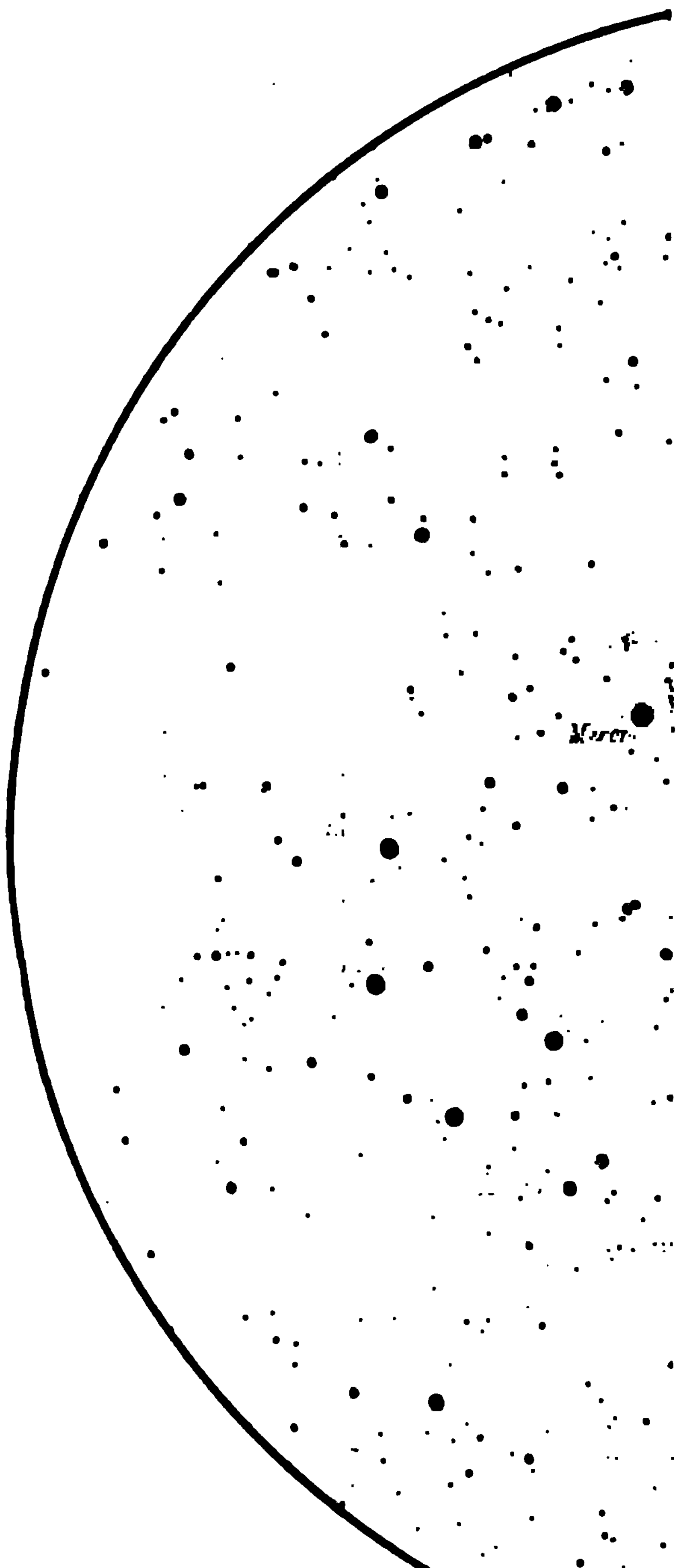
On the other side were ranged astronomers not less distinguished, like D'Arrest, Padre A. Secchi, and the observers at Parsonstown, who failed to see the nebula with their great telescope, and consequently doubted its existence.

But all ambiguity has been since cleared up, for on fitting the large telescopes with eye-pieces of a low magnifying power the nebula becomes distinctly visible, and is shown by them with an image equal in clearness to that given by the smaller instruments.

This fact is no more than is logically to be expected: that what was to be seen with a small telescope should be at least equally visible with a large one. Indeed, any discrepancy ought rather to be the other way, since a small instrument cannot be expected to show all the detail revealed by one of larger size.

It is now ascertained beyond question that the nebula exists. It has been observed with the great Washington Refractor (*Inst. of the Observatory*, p. 45), and again with the Reflector of Lord Rosse (*Observatory*, vol. i., p. 370); and anyone publishing statements as to its non-existence merely uses vain words, and proclaims himself wanting in knowledge of the history of nebulæ and of the management of telescopes.

The description written by Goldschmidt in the year 1864 seems to show that a certain amount of nebulosity surrounds the entire group of the *Pleiades*; but this is an optical illusion, and



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in another letter I shall, with your permission, proceed to prove this by the description of a false image in the telescope, a curious phenomenon hitherto unexplained.

My present sketch is, in its main features, the same as that published at Milan (*Pubb. dell' Oss. R. di Brera*, No. 5), only that all the minute stars and the exact form of the nebula have been added here with my *Amici No. I.*, with a magnifying power of 113.

To the right near *Merope* the nebula is so sharply defined that its curvature is clearly traceable, whilst its remaining outline is pale and indistinct. I could only succeed in distinguishing two nuclei, or nodules, and they but little more luminous than the rest.

A glance at the sketch is sufficient to show that in various parts many of the stars are omitted, which, for want of time and owing to pressure of other work, I have as yet been unable to put in. These may amount to some hundreds; indeed, even in the portions which seem thick with stars, many remain to be added, because the scale of the drawing is too small.

Being unable to foresee when it will be in my power to complete a more accurate work, I send this sketch, which is sufficiently so to compare with the drawings of others.

The circle shows the diameter of the field of view of my Steinheil, with an eye-piece of 24.

Comparing my drawing with that of Mr. Maxwell Hall, the two will be found to agree perfectly. That of Mr. Common, on the other hand, has evidently been executed with a telescope of insufficient power to show the *Merope* nebula. This will be obvious to anyone who examines the portions where Mr. Common has drawn three nebulous masses, where minute stars are shown in my sketch; invisible in his telescope as separate points of light, they must have appeared as nebulosity.

My sketch shows such a number of double stars that I have not hitherto been able to find them all in the catalogues, and it is worthy of note that the magnifying power is lower than that ordinarily used in the observation of double stars, showing that *Amici No. I.* gives images so clearly defined as to separate closely-united stars, even when they were not specially looked for, or previously known by me to be thus divisible.

Royal Observatory in Arcetri,
Florence, 1880, *May* 22.

Observations of Comet I. 1880, *made at the Royal Observatory, Cape of Good Hope.* By David Gill, Her Majesty's Astronomer at the Cape.

The following are the results of observations of Comet I. 1880, made here during the period of the visibility of the Comet. It is a matter of much regret that observations of the nucleus

could not be obtained nearer to perihelion. The view of the Comet was cut off to the south-west by Table Mountain, and the drawings of the tail, made February 2 to February 9, were made at Sea Point, situated to the west of Table Mountain.*

From the amount of haze near the horizon and the small optical power of the portable instrument employed, the nucleus could not be observed at Sea Point, and it was not until February 9, that anything like a nucleus could be made out at the Royal Observatory, and then only by glimpses through cloud.

From February 10 to February 15 (both inclusive) observations were obtained with the $8\frac{1}{2}$ foot Equatoreal (6·9 inches' aperture), but so faint and ill-defined was the object, and so unsatisfactory our instrumental means, that, notwithstanding the greatest care, I do not think the places can be relied upon nearer than within 10″ of arc. On February 10 the parallel-wire micrometer was employed, but afterwards it became necessary to use the ring micrometer.

The observations of the tail of the Comet were made by myself. The observations of the nucleus were made by myself and Mr. Finlay, first assistant—about an equal number of observations being made on each evening by each observer.

The places of the stars of comparison, *a*, *b*, *c*, *d*, *e*, *f*, were determined by early morning meridian observations in the end of June and beginning of July of the present year. It was impossible, from daylight, to obtain meridian observations of these stars sooner. The mean place of the comparison star, $7\frac{1}{2}$ mag., employed by Mr. Ellery on February 14, is also given, derived from meridian observations made in the end of June of the present year.

The corrections of this star to apparent place for February 14 are

$$\text{To R.A.} \quad +0^{s}.52$$

$$\text{N.P.D.} \quad +3''.92$$

Monday, Feb. 2. A small portion only of the tail was seen projecting over south shoulder of Table Mountain. The drawing has no pretension to accuracy.

Tuesday, Feb. 3. This drawing was made from Mr. Henry Solomon's Garden, at Green Point. The night was cloudy, and very hazy, so that the star *β Pis. Aust.* could not be seen, but only guessed at. The only really satisfactory part of the drawing is the position of the tail, as defined by the stars *θ* and *l Gruis*. This is accurately shown for 8.30. At a little past 9 o'clock the star *θ Gruis* was fairly immersed in the tail, and the star *l* was near its southern side.

* See Plate in the April No. facing p. 300.—ED.

Wednesday, Feb. 4. There is a little doubt about the precise position of the tail relative to β *Pis. Aust.* The star was nearly lost in haze near the horizon, but the position of the tail relative to ϕ *Gruis*, θ *Phœnicis*, τ *Phœn.*, is extremely exact; and relative to η, ξ, and ζ *Phœnicis* is fairly exact—as it there became very faint, but could still be traced beyond α *Hydri* without a definite outline. Epoch. $8^h\ 33^m$ Cape M. T.

Thursday, Feb. 5. Owing to haze near the horizon there is a slight uncertainty about the position of the tail relative to γ *Pis. Aust.*, though not much. The position of the tail relative to ϵ *Phœn.* is very exact. A line joining ϵ *Phœn.* and μ *Phœn.* is exactly bisected by the northern edge of the tail, whilst ρ *Phœn.* just marked the limit of the southern border of the tail. After this the tail could be traced, as shown, to α *Reticuli*, but the border could not be laid down with great precision. Epoch $8^h\ 40^m$ Cape M. T.

Friday, Feb. 6. The position of the tail very precisely shown relative to δ *Pis. Aust.*, and a small star between δ and Fomalhaut, but owing to haze the position relative to ϵ *Pis. Aust.* could not be very precisely shown. The position of tail is shown with great accuracy relative to β *Sculp.*, κ and μ *Phœn.* The north border of the tail was well marked by β and δ *Phœn.*, which were only just involved in it. The southern border of the tail seemed to pass half-way between q^1 and q^2, and ϕ *Erid.* and η *Horlog.* were just involved in it. The tail could be traced as shown, without any well-defined boundary passing undoubtedly to N. of α *Retic.* and within α *Doradus* beyond which it was just visible nearly up to *Canopus.* The greater part of the drawing was made at $8^h\ 30^m$, the faint part near *Canopus*, at 9^h.

Saturday, Feb. 7. It is obvious from our present knowledge of the path of the nucleus, that I have been deceived on this date, by the haze and the faintness of the nucleus, into supposing that, as no condensation or nucleus, could be made out, the tail must extend and could be traced as far as ϵ *Pis. Australis.*

Feb. 8 and 9. The drawings were made by glimpses through cloud, and are both unreliable. That of Feb. 9 is undoubtedly wrong—it would appear that θ has been mistaken for μ Sculptoris.

Observations of Comet I. 1880, at the Royal Observatory, Cape of Good Hope. (Observers, Gill and Finlay.)

	Cape Mean Time.	Right Ascension.	Parallax Factor.	No. of Obsns.	N.P.D.	Parallax Factor.	No. of Obsns.	Comparison Star.
d	h m s	h m s			° ′ ″			
Feb. 10	8 50 2	0 3 58·59	+0·0665	3				a
	8 52 49	„	...		123 43 15·53	+0·4760	2	a
11	8 33 4	0 20 22·16	+0·0661	4	„	...		b
	8 45 42	0 20 31·53	+0·0664	4	123 31 30·78	+0·4434	4	Lacaille 94
12	8 42 18	0 36 28·06	+0·0658	5	123 11 33·93	+0·4145	4	c
	8 42 18	0 36 28·29	+0·0658	5	123 11 23·44	+0·4145	5	d
13	8 30 57	0 51 39·32	+0·0644	7	122 44 20·93	+0·3888	5	Lacaille 290
14	8 23 5	1 6 7·89	+0·0627	1	122 11 17·23	+0·3435	1	σ Sculptoris
	8 42 51	1 6 19·25	+0·0641	3	122 10 52·86	+0·3806	3	e
	8 42 51	1 6 19·79	+0·0641	3	122 10 57·19	+0·3806	3	f
15	8 24 31	1 19 54·92	+0·0616	10	121 32 52·17	+0·3340	10	Lacaille 384

The observations are not corrected for Parallax, Mean R.A. and N.P.D. of the stars of comparison.

Star.	R.A. 1880·0	N.P.D. 1880·0	No. of Obsns.	Whence derived.
	h m s	° ′ ″		
a (8 mag.)	0 2 52·80	123 42 17·01	2	Cape Obs. 1880
b (8 mag.)	0 17 34·16	123 31 59·26	3	„ „
Lacaille 94	0 21 58·49	123 40 11·20		Cape 1878, & Melb.
c (8 mag.)	0 34 32·91	123 13 7·58	2	Cape Obs. 1880
d (8 mag.)	0 41 50·65	123 6 48·87	2	„ „
Lacaille 290	0 57 41·58	122 43 25·35		„ 1878
σ Sculptoris	0 56 42·44	122 11 53·71		Cape 1878, & Melb.
e (9 mag.)	1 7 49·07	122 12 37·30	2	Cape Obs. 1880
f (8·7 mag.)	1 11 10·04	122 2 32·98	2	„ „
Lacaille 384	1 17 55·88	121 34 17·09		„ 1878

Mean R.A. and N.P.D. of ✱ observed with Comet at Melbourne on Feb. 14. (7·8 mag.) and $1^h\ 0^m\ 20^s·02$. $\Delta = 122°\ 29'\ 46''·79$.

On a Method of determining the Pressure on the Solar Surface. By Prof. E. Wiedemann.

In a paper published in *Wied. Ann.* v., p. 503 (1878), I have shown how experiments on interference bands produced by two rays of light of large difference in phase may give an approximate measure of the mean free path of a molecule in a gas, or rather of the time elapsing between two encounters. I should like to draw the attention of astronomers to the fact that we may in this way obtain information as to the pressure in a luminous gas, whether in our laboratory on the solar surface or in the tail of a comet; for the time elapsing between two encounters of a molecule is almost independent of the temperature, and chiefly depends on the pressure.

In order to make the determination, we must decompose the light sent out by the gaseous body—say a protuberance—by means of the spectroscope, and separate a ray of light, which must be as homogeneous as possible. If we produce Newton's rings between two adjustable pieces of glass and count the number of interference bands which are visible we shall obtain the required information. In the above-mentioned paper I have discussed the number of interference bands seen by J. J. Müller in a sodium flame, and shown that the result agrees well with the

length of the mean free path as calculated according to the kinetic theory of gases. Various methods of producing the interference bands besides the one mentioned may of course be employed.

Note on a Disappearance of Jupiter's Satellites in 1611.
By the Rev. S. J. Johnson.

After calling attention to the fact that on the morning of October 15, 1883, there will be another instance of Jupiter appearing without any satellites exterior to his disc, though for a much shorter period than on the last occasion, August 21, 1867, Mr. Johnson proceeds—'A like instance seems to have occurred to Galileo, unless it arose from want of optical power. On March 15, 1611, after giving the configurations of the satellites at 1h. 30m., he proceeds as follows—"Vix conspici poterant planetæ occidentales. Horâ tandem 3 nullus apparebat, sed omnes, ob maximam vicinitatem cum Jove, latitabant. Postea, ad horam usque 7, multoties Jovem intuitus sum, nullusque planetarum apparuit; ulterius eum jam ad orizontem tendentem non observavi."' (G. Galilei, *In Jovis satellites lucubrationes.*)

Abbenhall Rectory,
Mitcheldean, Gloucester, Mar. 3.

Admiral Sir E. Ommanney, writing from Yarmouth, Isle of Wight, August 16, last, records that the luminous bodies called *Perseids* were observed in that locality about the time predicted, August 10, 12, and 13. They were most numerous, August 10, between the hours of 9 P.M. and 11 P.M.; the radiant point was scarcely definable. In general the trajectory of the luminous bodies was short, their visibility only for a brief moment, and the track very attenuated; but a few were large and vivid, one meteor was especially conspicuous which made a traverse from the N.E. to the tail of the Great Bear, expanding into a globular form previous to disappearance. Several meteors were seen at intervals on the night of the 11th, and on the 13th.

Errata.

Page 497, eleventh line from the commencement, *for* 'Double stars and stars with distinct companions' *read* 'Double stars and stars with distant companions."

Page 514, in the measure of Smyth's companion to Sirius, *for* 10″·44 *read* 1044″.

Page 563, tenth line from bottom, last word, *for* 'the' *read* 'ten.'

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LIST OF PRESENTS

RECEIVED DURING THE SESSION OF 1879–80,

AND OF

BOOKS PURCHASED WITH THE TURNOR FUND

DURING THE SAME PERIOD,

FORMING

APPENDIX XXXI.

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